

Single-Qubit Cheat Sheet

Qubit States

$$|0\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |1\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{Any pure state can be written as } |\psi\rangle = \cos(\frac{\theta}{2})|0\rangle + e^{i\phi}\sin(\frac{\theta}{2})|1\rangle$$

Unitary Single-qubit Operators

$$\begin{aligned} Z &= |0\rangle\langle 0| - |1\rangle\langle 1| &\doteq \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ X &= |1\rangle\langle 0| + |0\rangle\langle 1| &= \sigma_- + \sigma_+ \doteq \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ Y &= i|1\rangle\langle 0| - i|0\rangle\langle 1| &= i\sigma_- - i\sigma_+ \doteq \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ H &= \frac{1}{\sqrt{2}}(|0\rangle\langle 0| + |1\rangle\langle 0| + |0\rangle\langle 1| - |1\rangle\langle 1|) &= \frac{Z+X}{\sqrt{2}} = e^{i\pi/2} R_X(\pi) R_Y(\pi/2) \doteq \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \\ S &= |0\rangle\langle 0| + i|1\rangle\langle 1| &= e^{i\pi/4} R_Z(\pi/2) \doteq \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \\ T &= |0\rangle\langle 0| + \sqrt{i}|1\rangle\langle 1| &= e^{i\pi/8} R_Z(\pi/4) \doteq \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{i} \end{pmatrix} \\ \sigma_\phi &= e^{i\phi}|1\rangle\langle 0| + e^{-i\phi}|0\rangle\langle 1| &= e^{i\phi}\sigma_- + e^{-i\phi}\sigma_+ \doteq \begin{pmatrix} 0 & e^{-i\phi} \\ e^{i\phi} & 0 \end{pmatrix} \\ \boldsymbol{\sigma} &= X \hat{\mathbf{e}}_X + Y \hat{\mathbf{e}}_Y + Z \hat{\mathbf{e}}_Z \\ R_{\hat{\mathbf{n}}}(\theta) &= e^{-i\frac{\theta}{2}\hat{\mathbf{n}} \cdot \boldsymbol{\sigma}} &= \cos\left(\frac{\theta}{2}\right) \mathbb{1} - i \sin\left(\frac{\theta}{2}\right) \hat{\mathbf{n}} \cdot \boldsymbol{\sigma} \end{aligned}$$

Non-unitary Single-qubit Operators

$$\begin{aligned} \sigma_+ &= |0\rangle\langle 1| \doteq \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\ \sigma_- &= |1\rangle\langle 0| \doteq \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \end{aligned}$$

Commutation Algebra

$$\begin{aligned} XY &= iZ & YX &= -iZ & [X, Y] &= 2iZ \\ YZ &= iX & ZY &= -iX & [Y, Z] &= 2iX \\ ZX &= iY & XZ &= -iY & [Z, X] &= 2iY \end{aligned}$$