Physics 17 Class Notes
The Quantum Alligator Detector

WCC

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Setup

The following is a re-packaging of the Elitzur-Vaidman bomb tester [1], a wonderful example of what is sometimes known as *interaction-free measurement*.

We will consider a scenario in which we are tasked with finding an alligator in our dark laboratory without disturbing it by shining light on it[1]. The tool we have devised for this is an ideal single-photon light source and an ideal Michelson interferometer as shown in Fig. 1.

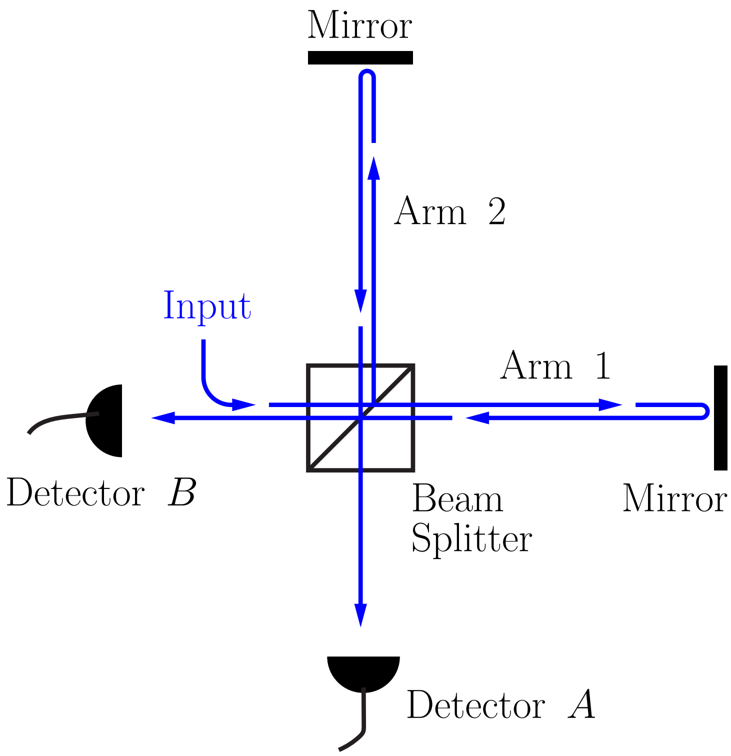

Figure 1: Michelson interferometer for quantum alligator detection. The beamsplitter is perfectly balanced.

[1]To make things more colorful, one can imagine that the alligator has a terrible hangover after a long night out and she just wants to sit in the perfect darkness and recover. Should she be disturbed by having any clumsy physicists shine light on her, she will immediately go on a vicious rampage and eat everyone in the lab.

We have carefully arranged the relative path lengths of our interferometer such that the paths that lead to detector A interfere perfectly destructively, while those that lead to detector B interfere perfectly constructively. Since we will be sending single photons in and since interferometers of this type exhibit only single-particle interference, we can state with certainty the following:

1. Any photon sent into the empty interfetometer will make detector B click with 100% certainty.
2. Detector A will *never* click as long as the interferometer is empty.

Here, by *empty* I mean only that there is no alligator in the interferometer.

Our description of how the empty interferometer works is as follows. When the photon first encounters the beamsplitter, its wavefunction is split into two parts, each going into one, unique arm of the interferometer. After reflecting from the two mirrors, these two parts of the wavefunction then encounter the beamsplitter once more, interfering with one another in the two output ports such that the interference in the port headed toward detector A is perfectly destructive while that in the port headed toward detector B is perfectly constructive.

Next, we consider walking around the darkened lab with our device and sending in single photons while monitoring the two detectors. As per statement 2 above, detector A cannot click unless something happens, whereas a click on detector B is just what we expect if nothing has disturbed our apparatus.

Now imagine that the alligator has wandered into arm 2 of the interferometer, completely blocking the mirror. We can now consider the three possible outcomes from sending a single photon into the interferometer with the alligator inside:

(i) The photon hits the alligator, in which case neither detector clicks, alerting us to the presence of the alligator as we are being eaten. Alas!

(ii) The photon makes detector B click, in which case we likely don't suspect anything is amiss and we keep testing.

(iii) The photon makes detector A click. Something has changed the way the interferometer behaves, but whatever it was certainly didn't absorb our photon (after all, one of our photon detectors *clicked*). We slowly tiptoe out of the lab, unconsumed, leaving the alligator in peace.

We can work out the probabilities for the three outcomes as follows.

When the photon first encounters the beamsplitter, its wavefunction is split into two parts, each going into one, unique arm of the interferometer. However, unlike the empty interferometer case, the alligator is now there to detect which arm of the interferometer the photon is in (whether she knows it or not). Since the beamsplitter was perfectly balanced, it is 50/50 probability which arm of the interferometer this detection reveals as containing the photon. As such, we conclude that there is a 50% chance that the alligator finds a photon in arm 2 of the interferometer. This is the probability of outcome (i),

$$\mathcal{P}_{\mathrm{i}} = 0.5. \tag{1}$$

There is also, however, a 50% chance that the alligator's measurement came up null – she concludes that there are no photons hitting her[2]. In this case, even if she has no idea that she is in an interferometer or that we just sent in a photon, we are aware of this information, and we now *know for certain* that the photon is *not* in arm 2, and it must therefore have gone into arm 1. *Note that this is different from saying it went into both – the measurement has changed the wavefunction, even though the measurement result was null.*

If we consider this case in which the photon definitely went into arm 1 only, the photon acts as if arm 2 never existed and we just have a single photon that bounces off a mirror and then encounters a 50/50 beamsplitter. Note that there is no wavefunction reconverging on the beamsplitter from arm 2 anymore – that port of the beamplittler is empty and there is no interference. From here, there is a 50% probability that the photon in arm 1 is transmitted by the beamsplitter on its second encounter with it and it then makes

[2] quite to her satisfaction

detector B click. So we have 50% probability that the alligator's measurement gave a null result, multiplied by a subsequent 50% probability that the photon in this case was transmitted, as opposed to reflected, by the beamsplitter, which is outcome (ii). We therefore have

$$\mathcal{P}_{\text{ii}} = 0.5 \times 0.5 = 0.25. \tag{2}$$

Last, by symmetry with outcome (ii), we see that the remaining 25% of the time, the photon goes into arm 1 *and* is reflected when it encounters the beamsplitter the second time, causing a click in detector A:

$$\mathcal{P}_{\text{iii}} = 0.25. \tag{3}$$

This third outcome (iii) is the most interesting of the three. In this case, we have detected the presence of the alligator, but the alligator has never seen a photon from our test, and has no empirical evidence that a photon was sent into the interferometer at all.

References

[1] Avshalom C. Elitzur and Lev Vaidman. Quantum mechanical interaction-free measurements. *Foundations of Physics*, 23(7):987, 1993.