

UCLA

UCLA Electronic Theses and Dissertations

Title

Narrow Transitions in Yb+ for Quantum Information and Precision Measurement

Permalink

<https://escholarship.org/uc/item/4624m6p3>

ISBN

9798265484680

Author

McMillin, Patrick Joseph

Publication Date

2025-12-11

Peer reviewed|Thesis/dissertation

UNIVERSITY OF CALIFORNIA

Los Angeles

Narrow Transitions in Yb^+ for
Quantum Information and Precision Measurement

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Physics

by

Patrick J. McMillin

2025

© Copyright by
Patrick J. McMillin
2025

ABSTRACT OF THE DISSERTATION

Narrow Transitions in Yb^+ for
Quantum Information and Precision Measurement

by

Patrick J. McMillin

Doctor of Philosophy in Physics

University of California, Los Angeles, 2025

Professor Wesley C. Campbell, Chair

The metastable (“*m*-type”) qubit defined on zero-field hyperfine clock states in the long-lived $^2\text{F}_{7/2}^o$ manifold in Yb^+ gives a promising pathway to low crosstalk, high fidelity multiqubit operations using the same ion species via the recently proposed “*omg* blueprint” [ACC21] for atomic quantum processing. The *m*-type qubit has been shown to have excellent initialization and readout fidelity when paired with heralded state preparation, but coupling these states to motion is not straightforward using conventional techniques. The structure of further excited Yb^+ contains many odd-parity metastable states that are accessible directly from the $^2\text{F}_{7/2}^o$ manifold, furnishing new optical-frequency (“*o*-type”) qubit capabilities. We report isotope shifts and lifetimes for three near-infrared and visible *E2* transitions from the $^2\text{F}_{7/2}^o$ manifold to higher metastable states and measurements of their hyperfine structure. These narrow transitions can form the basis for engineering a state-dependent force for the *m*-type qubit. The addition of these previously unobserved transitions to Yb^+ at accessible wavelengths may assist in the search for isotope shift non-linearities.

The dissertation of Patrick J. McMillin is approved.

Eric R. Hudson

Paul Hamilton

Justin R. Caram

Wesley C. Campbell, Committee Chair

University of California, Los Angeles

2025

To Sydney

TABLE OF CONTENTS

1	Introduction	1
2	Ion Traps	3
2.1	Basic Principle of Operation	3
2.2	The Equations of Motion	4
2.3	Oblate Ring Trap	5
2.4	New Ion Trap	7
2.4.1	Trap Electrode Polishing	8
2.4.2	Vacuum Chamber and Construction	8
2.4.3	Bakeout	10
2.4.4	Helical Resonator	11
2.5	Electronics for the Trap	12
2.6	Excess Micromotion Compensation	13
3	Atoms and Light	20
3.1	Hyperfine Structure	20
3.1.1	Hamiltonian from the Vector Approach	21
3.1.2	Hamiltonian from the Tensor Approach	23
3.1.3	Quick Note on Clock States	24
3.2	Zeeman Structure (Weak Field)	25
3.3	Electric Dipole Transitions	26
3.3.1	Doppler Cooling	28

3.4	Electric Quadrupole Transitions	29
3.5	Magnetic Dipole Transitions	33
3.6	Magnetic Dipole Decays	34
4	Ytterbium Ions and Our Experiment	36
4.1	Photoionization of Neutral Yb	36
4.2	Structure of Yb^+	38
4.2.1	J_1K and J_1J_2 Coupled States in Yb^+	40
4.3	Hyperfine Structure in $^{171}\text{Yb}^+$	41
4.4	Cooling Yb^+	41
4.4.1	$^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$	43
4.5	State Preparation and Measurement of $^{171}\text{Yb}^+$	44
4.5.1	State Preparation	45
4.5.2	Single Qubit Rotations	46
4.5.3	Measurement	49
4.6	Lasers and Light Collection	52
4.6.1	Quad Cavity	52
4.6.2	369nm laser	53
4.6.3	411nm laser	54
4.6.4	935nm and 976nm lasers	54
4.6.5	760nm lasers	55
4.6.6	399nm laser	55
4.6.7	Ti:Sapph laser	55
4.6.8	1108nm laser	56

4.6.9	Imaging Systems	56
4.6.10	Wavemeter Calibration	57
4.7	Experimental Control	58
4.7.1	Our ARTIQ Crate	58
4.8	Microwave System	59
5	Spectroscopy of $^{173}\text{Yb}^+$	61
5.1	Trapping and Cooling $^{173}\text{Yb}^+$	61
5.1.1	Initial Scheme: Sidebands Only	62
5.1.2	Current Scheme: Carrier and Sideband	63
5.2	Spectroscopy Results	64
5.2.1	$^2\text{D}_{3/2} \rightarrow ^3[3/2]_{1/2}^o$ Spectroscopy	64
5.2.2	$^2\text{S}_{1/2} \rightarrow ^2\text{D}_{5/2}$ Spectroscopy	66
5.2.3	$^2\text{F}_{7/2}^o \rightarrow ^1[3/2]_{3/2}^o$ Spectroscopy	66
5.3	Future Spectroscopy	68
5.3.1	More Transitions Out of the $^2\text{F}_{7/2}^o$	68
5.3.2	Refining the $^2\text{D}_{5/2}$ Measurements	69
5.3.3	Toward Microwave Spectroscopy in the $^2\text{F}_{7/2}^o$	69
5.4	Towards SPAM in $^{173}\text{Yb}^+$	70
5.4.1	Microwave Assisted Optical Pumping	70
5.4.2	Narrowband Optical Pumping	72
5.4.3	Readout of the Ground Qubit in $^{173}\text{Yb}^+$	72
6	High Fidelity State Preparation and Measurement	74

6.1	Electron Shelving	74
6.1.1	Shelving in $^{171}\text{Yb}^+$	74
6.1.2	Shelving Scheme	76
6.2	Shelving Duration and Infidelity	79
6.3	Measurement of M1 Decay	81
6.3.1	861nm Repump Scheme	82
6.4	SPAM Experiment	85
6.4.1	SPAM Sequence	85
6.5	Potential Improvements	91
7	The Metastable Qubit in Yb^+	92
7.1	<i>omg</i> Blueprint for Yb^+	92
7.2	Heralded State Preparation and Single Qubit Rotations	93
7.3	SPAM in the <i>m</i> -type Qubit	96
7.3.1	<i>m</i> -type Qubit State Detection Limits	98
7.4	<i>m</i> -type Laser Driven Gate Possibilities	100
7.4.1	Coupling to Even-Parity States	100
7.4.2	Coupling to Odd-Parity States	101
8	Narrow Transitions from $^2\text{F}_{7/2}^o$	102
8.1	Selection of States	102
8.2	Magnetic Dipole Decay Pathways	104
8.2.1	Finding the Reduced Matrix Element	104
8.3	Initial Detection of Transitions	107

8.3.1	Aligning the Beam	107
8.3.2	Search Space	108
8.3.3	Method of Search	109
8.4	Frequency Determinations	110
8.4.1	860nm Transition	110
8.5	King Plots	111
8.5.1	Isotope Shift and Linearity	111
8.5.2	Nonlinearity of King Plots in Yb	112
8.6	Hyperfine Splittings in $^{171}\text{Yb}^+$	113
8.7	Transition Moments	116
8.7.1	Atomic Scale Units	116
8.7.2	Measurement of the Transfer Rate	118
8.7.3	Estimating Rabi Frequencies	121
	References	122

LIST OF FIGURES

2.1	Motion of an ion in 1D. The long timescale motion is due to the ion’s “secular” motion, whereas the fast, small motion is the “micromotion”. This plot is made with $q = 0.1$, $w = 2\pi \times 1\text{MHz}$, and $\Omega = 2\pi \times 41\text{MHz}$	6
2.2	Camera image of the oblate ring trap. The trap has a total of 8 pie-slice shaped DC compensation electrodes (4 above, and 4 below the trap), and RF is delivered to the central ring. Tapered trenches were used to couple light to the trap, and imaging was done from above and below.	7
2.3	(a) A tilted side-on view of the ion trap. The spacing between the needles is 1.5mm. (b) Axial view of the trap. The rod width is 480um, the spacing between adjacent rods is 510um, and the distance from a rod to the center of the trap is 460um.	9
2.4	Camera image of the 4 rod trap in the vacuum chamber. All Tungsten rods are connected with T1 connectors. There are a total of 4 in-vacuum antennae. Two of the antennae are straight, un-terminated wires which run along the trap axis below the trap cage. The other two are circular, one about 1” in diameter, and the other is about 3” in diameter. On the left side of the image, the oven can be seen. The oven is electrically isolated form the chamber by including ceramic washers and screws between the oven holder and the groove-grabber. We placed a stainless steel mesh on both of the recessed viewports in an attempt to reduce the effects of charging from the windows.	15
2.5	Pinout diagram of the 15-pin DSUB connector for our ion trap. Each pin is labeled on the diagram. The pins within a trapezoid are “through” connections, meaning they’re both connected to the same element in the chamber (oven, DC electrode, antenna).	16

2.6	End-on view of the trap from the RF feedthrough side. The DC rods and needle electrodes are labeled with their corresponding pins on the DSUB connector from figure 2.5. The needle on the other side of the trap is not shown, however it is connected to pin 9. The RF 1 (2) rod is connected to the right (left) pin on the RF feedthrough.	16
2.7	Assembled vacuum chamber before baking. The octagon can be seen on the right. There are 6 2.75" windows on the chamber, and 2 custom-made recessed viewports on the top and bottom of the octagon. There is a 2-pin feedthrough opposite from the pumping port. Our TiSub pump is situated in the large vacuum nipple, out of line-of-sight from any other vacuum component. In the same 5-way cross as the TiSub, we have our ion gauge and our 15-pin electrical feedthrough. On the other 5-way cross, the ion pump, NEG, and angle valve are connected. The last port is blanked.	17
2.8	Camera image of the inside of the RF resonator. The two large helical "pick-up" coils are seen held in place by 3D printed coil holders. The ends of the coils are connected to SMA connectors which are connected to bias tees for applying small offset voltages.	18
2.9	Plot of the measured radial secular frequency as a function of applied trap power. Power values are calculated by assuming a flat gain from the amplifier of +40dBm. Secular frequencies are measured by applying "tickle" scans to the ion via weak signals on the trap electrodes.	19
3.1	Plot of relative transition strengths for $ \Delta m_J = 0, 1$, and 2 electric quadrupole transitions as a function of angle γ (the angle between the polarization vector $\hat{e}$ and the magnetic field $\vec{B}$), and ϕ (the angle between the magnetic field $\vec{B}$ and the laser's $\vec{k}$ vector).	33

4.1	Our 1 + 1 REMPI scheme for Yb. The first step is exciting neutral Yb atoms with 399nm light on the strong $^1S_0 \rightarrow ^1P_1^o$ transition. Then light at 369nm is used to promote the excited electron to the continuum, leaving behind a Yb ion in the $^2S_{1/2}$ state.	37
4.2	Energy level diagram of relevant states in Yb^+ . Laser driven transitions are shown as solid lines between states, and select E1 decays are shown as dashed lines. . .	40
4.3	(a) Energy level diagram for Doppler cooling $^{171}\text{Yb}^+$ References for the hyperfine splittings may be found in table 4.4. (b) Energy level diagram for Doppler cooling $^{173}\text{Yb}^+$	43
4.4	(a) Diagram for optical pumping of the ground state hyperfine qubit in $^{171}\text{Yb}^+$. References for the hyperfine splittings may be found in table 4.4. Light is applied resonant to the $^2S_{1/2}, F = 1$ manifold to the $^2P_{1/2}^o, F = 1$ manifold transition in order to “pump” population to the qubit $ 0\rangle$ state. The presence of the $^2D_{3/2}$ state and its repump scheme are omitted for simplicity. (b) Example optical pumping rate plot. Population in the $ 1\rangle$ qubit state is measured after a variable time of optical pumping light. The time constant τ is extracted from a simple $e^{-t/\tau}$ fit model.	46
4.5	(a) Example of Rabi flopping on the ground state hyperfine qubit prior to optimization of experimental parameters. (b) Full range microwave linescan of the $ 0\rangle$ qubit state to all states in the $^2S_{1/2}, F = 1$ manifold. From this scan, we can see the separation between the Zeeman sensitive transitions and the “clock” transition is about 7MHz. In practice, to better determine the qubit transition we perform narrower scans about the qubit transition.	47

4.6	(a) Example Ramsey experiment. The fringe traces out an oscillation at the frequency of the detuning used in the $\pi/2$ pulses. (b) Simplified pulse sequence for Ramsey experiment in our lab. A frequency-detuned $\pi/2$ pulse is followed by a variable wait time, and an identical $\pi/2$ pulse is applied after the wait time. .	48
4.7	(a) Readout diagram for measuring the qubit state in $^{171}\text{Yb}^+$. Photons are scattered by the ion if the qubit is in state $ 1\rangle$, whereas no photons will be scattered if the qubit was prepared in state $ 0\rangle$. The presence of the $^2\text{D}_{3/2}$ state and its repump scheme are omitted for simplicity. (b) Histogram showing the number of photons collected in each experiment after preparing both the qubit $ 0\rangle$ state (red), and $ 1\rangle$ state (blue). The fidelity is presented with a 1σ Wilson confidence interval, discussed in section 4.5.3.1	50
4.8	Layout of the quad cavity mirrors as viewed from the side <i>opposite</i> of the electronics (TEC, thermistors, piezo, and ion pump cable) output.	53
4.9	Layout of our ARTIQ crate. We opted to include 3 DDS cards for driving AOMs, a microwave generator for our EOM tones, a TTL card for sending/receiving signals, and a DAC card for application of DC voltages.	59
5.1	Diagram of applied laser tones for our initial trapping of $^{173}\text{Yb}^+$. We apply symmetric sidebands at $1/2$ the hyperfine splitting of the ground state with the laser detuned symmetrically between the hyperfine levels in the ground state, addressing the $F = 3$ manifold in the $^2\text{P}_{1/2}^o$ state. A broad 935nm laser tone is applied to address the majority of the hyperfine manifolds in the $^2\text{D}_{3/2}$ state. . .	63

5.2	Linescans of the 935nm transitions in $^{173}\text{Yb}^+$. Each scan was fit to a separate Lorentzian lineshape function. Each scan is labeled as $F \rightarrow \mathcal{F}$, where F represents the hyperfine manifold in the $^2\text{D}_{3/2}$ state, and $\mathcal{F}$ represents the hyperfine manifold of the excited $^3[3/2]_{1/2}^o$ state. The 935nm laser had its frequency shifted upward by a 320MHz AOM for all of these measurements, but all measurements were taken with an uncalibrated wavemeter.	65
5.3	Results from quantum jump spectroscopy of the $^2\text{S}_{1/2} \rightarrow ^2\text{D}_{5/2}$ transition in $^{173}\text{Yb}^+$. Theoretical predictions are given by the solid, shorter lines, while the measured frequency interval is given by the lighter bars spanning the x-axis. The x-axis represents the number of m_F sublevels in the corresponding hyperfine manifold.	67
5.4	Diagram of microwave assisted optical pumping in $^{173}\text{Yb}^+$. We apply light resonant with the $^2\text{S}_{1/2}, F = 3 \rightarrow ^2\text{P}_{1/2}^o, F = 3$ transition, pumping population into the $^2\text{S}_{1/2}, F = 2$ manifold. Then microwave pulses clear out all $m_F \neq 0$ sublevels, and the process is repeated.	71
5.5	Diagram of narrow-band optical pumping in $^{173}\text{Yb}^+$. We apply light resonant with the $^2\text{S}_{1/2}, F = 3 \rightarrow ^2\text{P}_{1/2}^o, F = 3$ transition, pumping population into the $^2\text{S}_{1/2}, F = 2$ manifold. Then 411nm pulses clear out all $m_F \neq 0$ sublevels to the $^2\text{D}_{5/2}, F = 2$ manifold, to be repumped into the $^1[3/2]_{3/2}^o, F = 1$ manifold, which decays back to the $^2\text{S}_{1/2}, F = 2$ manifold, for the process to be repeated.	73

6.1	Diagram of the shelving scheme used for ground state hyperfine qubit SPAM. Thick purple lines represent the 411nm transitions used, which are degenerate due to the near-identical Lande-g factors of the $^2S_{1/2}, F = 1$ and $^2D_{5/2}, F = 3$ manifolds. Light purple lines show the other allowed (off-resonant) $ \Delta m_F = 2$ transitions from the ground state. Dashed black lines indicate decays from the upper states to the $^2F_{7/2}^o$ state. Zeeman structure of the $^2F_{7/2}^o$ state has been omitted for simplicity, along with the $^2D_{3/2}$ state and M1 decay pathway.	77
6.2	Population remaining in the $^2S_{1/2}$ manifold in $^{174}\text{Yb}^+$ as a function of 411nm illumination time. The data are fit to a simple exponential decay function, yielding a time constant $\tau = \frac{2}{\Gamma_D \mathcal{B}_{DF}} \approx 17\text{ms}$, where Γ_D is inverse of the lifetime of the $^2D_{5/2}$ state, and $\mathcal{B}_{DF}$ is the branching ratio from the $^2D_{5/2}$ to the $^2F_{7/2}^o$ state.	78
6.3	Population returning to the $^2S_{1/2}$ manifold from the $^2F_{7/2}^o$ state in $^{174}\text{Yb}^+$ as a function of 760nm illumination time. Our rate back to the ground state can be improved with more 760nm laser intensity and employing to a better laser lock scheme.	79
6.4	Full diagram of the shelving and deshelling scheme used for ground state hyperfine qubit SPAM. Hyperfine splittings are omitted but can be seen in table 4.4. Effects on the branching ratios due to hyperfine selection rules are also omitted.	80
6.5	Plot of measured SPAM infidelity as a function of shelving time. Solid lines represent the theoretical rate models. The blue line includes the M1 decay from the $^2D_{5/2}$ state to the $^2D_{3/2}$ state during shelving, where the orange line ignores this effect. Red datapoints are taken without correcting the M1 error, while the purple datapoint is taken when correcting for the M1 decay with an additional repump laser at 861nm (discussed in section 6.3).	82

6.6	Full repump scheme to correct for M1 decays during shelving. This scheme includes 3 repump tones. The first addresses the $^2D_{3/2}, F = 2 \rightarrow ^1[3/2]_{3/2}^o, F = 2$ transition at 861nm. The second addresses the $^2D_{3/2}, F = 1 \rightarrow ^3[3/2]_{1/2}^o, F = 0$ transition at 935nm. And finally we apply 976nm light to address the $^2D_{5/2}, F = 2 \rightarrow ^1[3/2]_{3/2}^o, F = 2$ transition.	84
6.7	Results of randomized benchmarking of single qubit gates on the hyperfine qubit in $^{171}\text{Yb}^+$. The achieved SPAM error is $\epsilon_{SPAM} = 9.8(6) \times 10^{-3}$, and the achieved error per single qubit gate is $7.4(1.0) \times 10^{-5}$. Red hexagonal datapoints indicate the average result per sequence length, where the black/gray points indicate the results of each Pauli and Clifford randomization of that sequence length.	88
6.8	Histogram of counts recorded for the $^{171}\text{Yb}^+$ hyperfine qubit prepared in $ 0\rangle$ (blue) and $ 1\rangle$ (red) when using the shelving detection scheme. The ultimate infidelity achieved using this technique was $2.1_{-0.4}^{+0.5} \times 10^{-4}$	89
7.1	Diagram for $F = 3$ manifold preparation in the $^2F_{7/2}^o$ state in our experiment. We use the $\Delta m_F = \pm 1$ 411nm transitions from the ground state qubit $ 0\rangle$ state to optimize population in the $^2F_{7/2}^o, F = 3, m_F = 0$ state. Optical pumping using 369nm light is also used to bring any population pumped into the $^2S_{1/2}, F = 1$ manifold back into the $ 0\rangle$ state quickly.	95
7.2	Microwave Rabi flopping of the m -type qubit in $^{171}\text{Yb}^+$. Blue datapoints are results from un-Heralded state preparation, while red datapoints are taken with Heralded state preparation. We can see that the contrast of the un-Heralded process is limited by the initial population in the $^2F_{7/2}^o, F = 3, m_F = 0$ state.	97
7.3	Results of the m -type SPAM experiment when Heralded state preparation of both the $ 1\rangle_m$ and $ 0\rangle_m$ states is performed. The infidelity achieved is $1.5_{-0.8}^{+2.4} \times 10^{-4}$. A total of 6,868 experiments were performed: 3,392 $ 1\rangle_m$ experiments, and 3,456 $ 0\rangle_m$ experiments	98

8.1		109
8.2	Number of suspected 860nm transitions in $^{174}\text{Yb}^+$ which pass our thresholding of requisite dark time form a calibration dataset as a function of laser frequency. Blue bands correspond to theoretically estimated $\Delta m_J = 1$ transitions, and yellow bands correspond to $\Delta m_J = 2$ transitions. Position of the bands are determined relative to the center of the observed spectrum, and calculated based off of the magnetic field in the trap.	112
8.3	King plot of measured transitions. Plotted is the spectroscopy laser frequency vs the 411nm transition frequency for $^{176}\text{Yb}^+$ (purple), $^{174}\text{Yb}^+$ (green), $^{172}\text{Yb}^+$ (blue), $^{170}\text{Yb}^+$ (red).	114
8.4	Simple diagram of the hyperfine spectroscopy scheme for the 860nm transition. The centroid of the transition is shown as ν_c , and the transition from the lower ($F = 3$) manifold in the $^2\text{F}_{7/2}^o$ state to one of the excited manifolds ($F = f$) in the $^3[7/2]_{9/2}^o$ state.	115
8.5	Transfer rate experiment for the 860nm transition in $^{174}\text{Yb}^+$. Population in the $^2\text{F}_{7/2}^o$ state is measured after a variable length spectroscopy pulse. Data are fit to a 2- and 3-level rate equation model. The 3-level model accounts for the M1 lifetime of the state (calculated to be 4s), whereas the 2-level model ignores this decay. The results of both models are consistent for this data. The presented errorbar is the error in the fit.	120

LIST OF TABLES

4.1	Mole Fractions and nuclear spin of each stable isotope of Yb [LBB03].	38
4.2	Summary of relevant information (energy levels, total radiative lifetimes, and the decay pathways with branching ratios) for the commonly used states in our work. Only E1 decays are considered for decay pathways. Values with an asterisk (*) refer to calculated values from the DREAM database [QP20]. Energy levels are from measurements made in [Meg67].	39
4.3	Coupling orders and term symbols for J_1K and J_1J_2 coupling schemes.	39
4.4	Hyperfine A coefficients for relevant states in $^{171}\text{Yb}^+$	42
8.1	Table showing the states considered for narrow transitions from the $^2\text{F}_{7/2}^o$ state in Yb^+ . The table presents the electron configuration, term symbol, total angular momentum quantum number J , energy level, and wavelength of the transition from the $^2\text{F}_{7/2}^o$ state based on values from the NIST ASD [KYR24].	103
8.2	Table showing the calculated M1 lifetimes of the considered excited states for narrow transitions from the $^2\text{F}_{7/2}^o$ state in Yb^+ . The table presents the term symbol, energy level, wavelength of the transition from the $^2\text{F}_{7/2}^o$ state based on values from the NIST ASD [KYR24].	108
8.3	Absolute frequency measurements of the newly observed transitions.	113
8.4	Measured hyperfine A coefficients of the excited states in $^{171}\text{Yb}^+$	116

ACKNOWLEDGMENTS

Firstly, thank you very much to my advisor, Wes. He always encourages us to try new things and won't shy away from strange ideas and new challenges. He's been an excellent advisor throughout my time at UCLA, from help with physics to supporting us through "interesting times" from early 2020 onward. I've learned to always take a lesson away from anything I try, no matter the outcome, and to not let the setbacks get me down.

I couldn't have asked for a better group of people to call labmates over the years, starting with Tony Ransford, Conrad Roman, and Thomas Dellaert. I learned so much from Tony and Conrad in the early (and later) years of my time at UCLA, and they were always up for answering any questions, no matter how simple or complicated. Thomas was always there to help keep me moving along and not getting caught up on the little details, and was always willing to take walks to help me get a different perspective. Hassan Farhat and Will Liu have been excellent labmates and amazing friends since they joined the lab, constantly looking for new challenges and willing to get to work on any problem. The new Yb^+ experiment is going to be well taken care of in their hands, hopefully I didn't leave *too* many cryptic code blocks for them to discover down in the basement. Thank you to Drew Parks, who is heading up the A-level Yb^+ experiment, for your great questions and your helpful occasional reminders that mountain biking is terrifying.

The postdocs who have joined the Campbell group in the past few years, Andrei Tretiakov and Patrick Mueller, have been incredible resources of information and motivation. They're great sources for new ideas and stories.

A big thank you is also owed to all of the UCLA AMO groups. Thanks to Justin Christensen who taught me so much about ion traps during group lunches, and Matt Boguslawski who was always willing to talk about optics, lasers, and life. Thank you Sam Vizvary, Zach Wall, Michael Bareian, Clayton Ho, Grant Mitts, and Josh Rabinowitz for letting me borrow random optics and chatting about why the ions weren't behaving. It was always incredibly

helpful to have conversations with Ricky Elwell, he always knows how to balance laughter and learning. Thank you to Changling Zhao, Randy Putnam, Sami Khamis, and Yongjia He for always brightening the days even when we're all stuck underground.

A huge thank you to my whole family for supporting me through graduate school. Thanks to my dad, mom, sister, and uncle for listening to me ramble about work on the phone for so many years, and always being curious about what I'm up to. I'm endlessly thankful to my family in Pomona: Kristy, Mark, Cory, and Zack for always being there to help me in whatever way, and welcoming me into their family.

Without a doubt I couldn't have even started this journey, let alone finished it, without the love and support of my fiance Sydney. I'm so lucky to have had her by my side throughout my time at UCLA, and far before that too. Since high school she's put up with my silly jokes and bad planning, and I can't be more thankful to have her there to keep me sane. I'm so excited to continue to grow alongside her, and get started on the next part of our lives together.

VITA

- 2016–2018 Luchko group at California State University, Northridge. Simulations of small nitroxide probes in supercooled water and simulations of host-guest complexes using molecular dynamics.
- 2017 ATLAS Project at CERN, Switzerland. Analysis of QuickStatus actions on ATLAS Pixel read-out drivers
- 2018 B.S. (Honors in Physics), California State University, Northridge.
- 2018–2020 Teaching Assistant at the University of California Los Angeles for undergraduate Mechanics, Electromagnetism, and Quantum Optics classes and laboratories
- 2020 M.S. (Physics), University of California Los Angeles.

PUBLICATIONS

D. Xiao, J. Li, W.C. Campbell, T. Dellaert, P. McMillin, A. Ransford, C. Roman, and A. Derevianko, “Hyperfine structure of $^{173}\text{Yb}^+$: Toward resolving the ^{173}Yb nuclear octupole moment puzzle”, *Physical Review A* 102, 1 (2020).

A. Ransford, C. Roman, T. Dellaert, P. McMillin, and W.C. Campbell, “Weak dissipation for high-fidelity qubit-state preparation and measurement”, *Physical Review A* 104, 3 (2021).

CHAPTER 1

Introduction

Given the DiVincenzo criteria [DiV00] it becomes clear that if we are to use atomic ions as qubits in a register for quantum computing, we must create a system where we can initialize and read out the states of individual ions. In combination with laser cooling [WDW78] [NHT78], ion traps have been used to isolate individual ions for precise control of their electronic and motional states, finding critical use in high fidelity quantum operations [GTL16], atomic clocks [MCD25], high precision tests of the Standard Model [MSW23], and precision measurements of nuclear effects [WHD24], [HAC22].

Our ion of choice is ytterbium. Yb^+ has three low-lying metastable electronic states directly accessible from the ground state, each of which has found use in atomic clocks [HOL12], [RTG99], [ZSB21]. Yb^+ has not just been used as a workhorse for precision measurement, but also as the host of high fidelity qubits in trapped ion quantum processors [OYM07] [MBA23]. Throughout our work, we have aimed to increase the available tools for working with Yb^+ qubits, either by employing underutilized states in the ion [RRD21], or developing a “metastable” qubit in the long lived $^2\text{F}_{7/2}^o$ state [Rom21].

In this thesis, we will discuss several topics which I contributed to during my time at UCLA. This includes the work performed to increase the state preparation and measurement fidelity in the $^{171}\text{Yb}^+$ ground state qubit by highly manifold-selective electron shelving. In addition, I will cover the development of the metastable qubit in $^{171}\text{Yb}^+$, including its Heralded state preparation and manifold-selective readout. We also will go over our work with the often overlooked spinful isotope of ytterbium, $^{173}\text{Yb}^+$, which includes its trapping

and direct laser cooling, and spectroscopy of its metastable states.

Finally, I will discuss our work towards further expanding the capabilities available to the metastable qubit in Yb^+ , by investigating several narrow electric quadrupole transitions out of the $^2\text{F}_{7/2}^o$ state. These transitions can be used to impart a spin-dependent force on the metastable qubit. We will also discuss the possibilities of adding these new transitions to a multi-dimensional King Plot analysis in order to isolate and measure properties of the nucleus.

CHAPTER 2

Ion Traps

2.1 Basic Principle of Operation

Confining a charged particle with only static electric fields is impossible due to Earnshaw's theorem. To achieve confinement with electric fields, we need to look to dynamic fields. Classically, we know the motion of a charged massive particle in an electric field will depend on its charge-to-mass ratio. Filtering particles of a specific charge-to-mass ratio was a technique refined by Wolfgang Paul, after whom the “Paul trap” is named. The filter is comprised of four long parallel rods with dynamic fields applied to two opposite rods, and the other two grounded. The field created by this geometry is a quadrupole field, which resembles a rotating saddle potential. The basic goal is to create a field which (averaged over several periods of oscillation) creates a net restoring force for particles of a certain charge-to-mass ratio, whereas particles of different ratios will not see a net restoring force, sending them outside of the filter.

By making some variations on the design of this mass filter (i.e. plugging the ends with static fields), one can create a region where charged particles of a certain charge-to-mass ratio are “trapped” within the rods. The saddle-flipping time scale is typically fairly fast compared to the ion's orbital motion, and it is responsible for creating the time-averaged restoring force for the particle. There is also a slower motion which the particle experiences, whose amplitude is associated with the energy of the ion, and is proportional to the charge-to-mass ratio of the particle. The slower motion is called the “secular” motion, whereas the

fast motion is called the “micromotion”.

2.2 The Equations of Motion

For this section, we will assume the following are true: We are considering a 4 rod linear Paul trap, where the rods run along the $\hat{z}$ direction. Two opposite rods have an oscillating voltage applied to them: $\phi(t) = \phi_0 \cos(\Omega t)$, where the other two are set to ground. The distance from the $\hat{z}$ axis to the edge of any of the four electrodes is R' , with $R' \approx R$, and only equal if the rods are hyperbolic cylinders of infinite length. Additionally, at either end of the trap, there are “endcap” electrodes which have static potentials applied to them (ψ_0). Other trap geometries are possible, but they all will end up producing similar equations of motion, and the simplicity of this trap design is a good starting point for understanding the others.

The potential due to the four parallel rods near the $\hat{z}$ axis is:

$$\phi(x, y, t) = \frac{\phi_0 \eta}{2} \left(1 + \frac{x^2 - y^2}{R^2} \right) \cos(\Omega t) \quad (2.1)$$

Where ϕ_0 is the voltage applied to the two RF rods, η is a geometric scaling factor to account for the fact that the rods aren’t hyperbolic, Ω is the RF frequency, and R is the distance from the trap axis to the edge of the rods. This creates a saddle point on the $\hat{z}$ axis, flipping the sign of the saddle at the trap frequency.

The potential due to the endcaps is given by:

$$\psi(x, y, z) = \frac{\kappa \psi_0}{z_0^2} \left[z^2 - \frac{1}{2}(x^2 + y^2) \right] \quad (2.2)$$

Where κ is a geometrical factor which accounts for the fact that the endcaps are not hyperbolic, ϕ_0 is the voltage applied to the rods, and z_0 is the distance from an endcap electrode to the trap center. The full potential is then the sum of these two contributions. We then can find the electric field and write the equation of motion for each direction by using $Q\vec{E} = \vec{F} = m\vec{u}$.

Each coordinate u_i ($i = x, y, z$), has a contribution due to the static field, and the x and y dimensions have a contribution due to the dynamic field of the rods (we are assuming here that the variations due to the dynamic field in the z direction are negligible). Before writing the equations of motion, we associate parameters q_i and a_i for each coordinate.

$$q_x = \frac{2Q\phi_0\eta}{mR^2\Omega^2}, \quad q_y = -\frac{2Q\phi_0\eta}{mR^2\Omega^2}, \quad q_z = 0 \quad (2.3)$$

$$a_x = -\frac{4Q\psi_0\kappa}{mz_0^2\Omega^2}, \quad a_y = -\frac{4Q\psi_0\kappa}{mz_0^2\Omega^2}, \quad a_z = \frac{8Q\psi_0\kappa}{mz_0^2\Omega^2} \quad (2.4)$$

Defining these parameters seems unnatural at first, but they allow us to cast the equation of motion for each coordinate into the form of a Mathieu equation:

$$\ddot{u}_i + (a_i + 2q_i \cos(\Omega t)) \frac{\Omega^2}{4} u_i = 0 \quad (2.5)$$

There are several approaches to finding a first-order (in q) solution to the Mathieu equations. A relatively fast and simple way to find a first-order solution is to assume that the micro-motion and secular motion are decoupled ($u_i = u_{i,mm} + u_{i,sec}$), and then solve the resulting equations by noticing that $u_{i,mm}$ will change much faster than $u_{i,sec}$, but that the amplitude of $u_{i,sec}$ will be much larger than $u_{i,mm}$. Going through and solving in this manner gets us to a first order solution:

$$u_i(t) \approx A_i \cos(\omega_i t) \left(1 + \frac{q_i}{2} \cos(\Omega t) \right) \quad (2.6)$$

Where $\omega_i = \frac{\Omega}{2} \sqrt{a_i + \frac{1}{2}q_i^2}$ is the secular frequency, and A_i is the amplitude of motion in the i direction. The one dimensional motion of an ion is plotted in figure 2.1. It's important to note that this is only a first order solution, and higher-order contributions become more important the more an-harmonic the trap is (i.e. small ion-electrode distances) [HHJ11].

2.3 Oblate Ring Trap

The ion trap that was in use when I joined at UCLA was an oblate ring trap manufactured by Translume. Rather than creating linear crystals of ions, this trap produced 2D ion

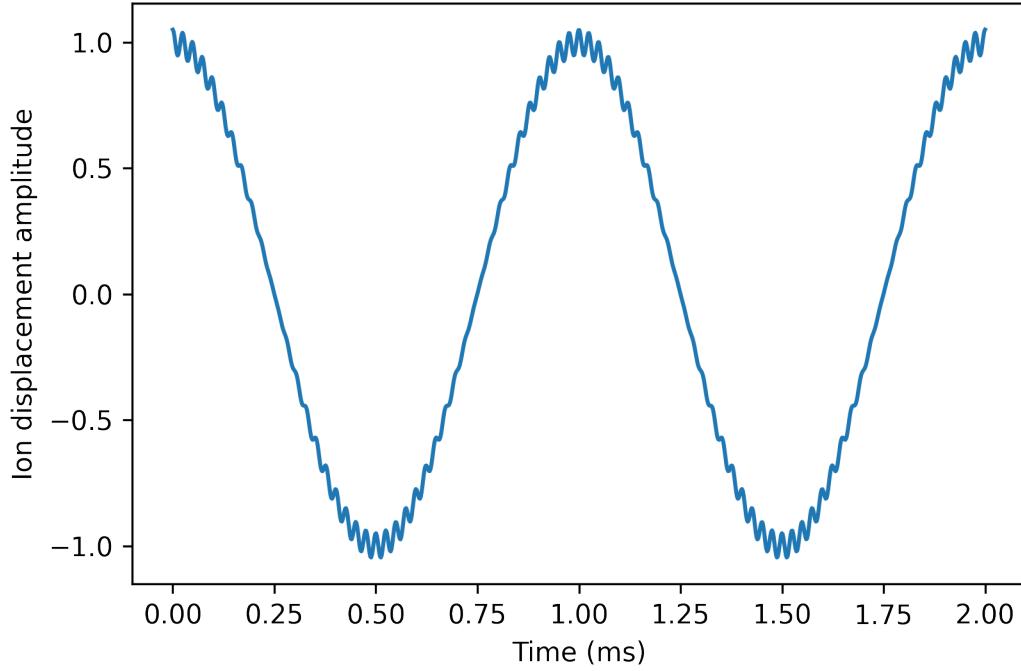


Figure 2.1: Motion of an ion in 1D. The long timescale motion is due to the ion’s “secular” motion, whereas the fast, small motion is the “micromotion”. This plot is made with $q = 0.1$, $w = 2\pi \times 1\text{MHz}$, and $\Omega = 2\pi \times 41\text{MHz}$

crystals. An image of the trap can be seen in figure 2.2. Details of this trap potential and its general function can be found in [YSD15], [Ran20], [Rom21], and [Del25]. The trap has $R_0 = 500\mu\text{m}$, and was driven at $\Omega = 2\pi \times 47\text{MHz}$ leading to typical secular frequencies of $(w_x, w_y, w_z) = 2\pi \times (540, 550, 1170)\text{MHz}$. Light delivery was achieved by matching the beam profile to tapered trenches in the sides of the 1mm thick substrate on which the electrodes were printed. Due to the trap potential, only one point in 3D space had excess micromotion minimization, so to avoid excess micromotion, only one ion could be loaded for experiments. Additionally, the micromotion null was rather difficult to find, and required frequent recalibration. We worked with this trap until mid 2022, when we began construction

of a new Yb^+ experiment, which will be discussed further in the next section. Ultimately, the ring trap experienced a critical electrode failure in 2023 which led to it being replaced.



Figure 2.2: Camera image of the oblate ring trap. The trap has a total of 8 pie-slice shaped DC compensation electrodes (4 above, and 4 below the trap), and RF is delivered to the central ring. Tapered trenches were used to couple light to the trap, and imaging was done from above and below.

2.4 New Ion Trap

Given the limitations of the ring trap when working with multiple ions, and the need for extensive recalibration of the DC voltages, we aimed to make a new ion trap. Our goal was to make a trap with a proven design that would allow for long trapping lifetimes, easier optical access, and allow for chains of ions to be trapped for future multi-qubit operations.

We decided on a 3D ion trap with needle endcap electrodes. It generally follows the design from [Olm09], with a few modifications. The trap rods and needles are made from Tungsten, and are held in place by alumina spacers. The spacers are then fixed by a stainless steel cage. The thickness of the rods is 480um, the spacing between adjacent rods is 510um, and the R_0 value is 460um. The needles are spaced by 1.5mm. A diagram of the trap can be seen in 2.3. A camera image of the trap in the vacuum chamber can be seen 2.4.

This system first trapped ions in July of 2023. We’ve been able to trap small crystals of ions for over 50 hours at a time. The limit on our ion lifetimes so far has been our laser locking instabilities.

2.4.1 Trap Electrode Polishing

We employed an electropolishing technique to achieve a good surface finish for the tungsten electrodes. We submerged the target tungsten electrode (anode) in the center of cylindrical symmetric stainless steel piece, all in a 2M NaOH solution. We placed a graphite plate in the solution as well, as our cathode for electropolishing. For our non-needle electrodes, we submerged ≈ 40 mm of tungsten in the solution, and set our power supply to a low voltage value (6V), and allowed it to polish for 5 minutes at a time, checking the diameter of the rod after each run. Once the target diameter of 0.48mm was achieved, and the surface appeared smooth under a microscope, we stopped polishing.

Our first attempts at making the needle electrodes followed the procedure in [KGM23]. However, we found more success when purchasing pre-made tungsten needles and electropolishing them down to size to fit in the spacers.

2.4.2 Vacuum Chamber and Construction

Despite the fact that ion traps are energetically “deep” (about 1eV), collisions with background gas molecules can still lead to ion loss, and cause major interruptions during exper-

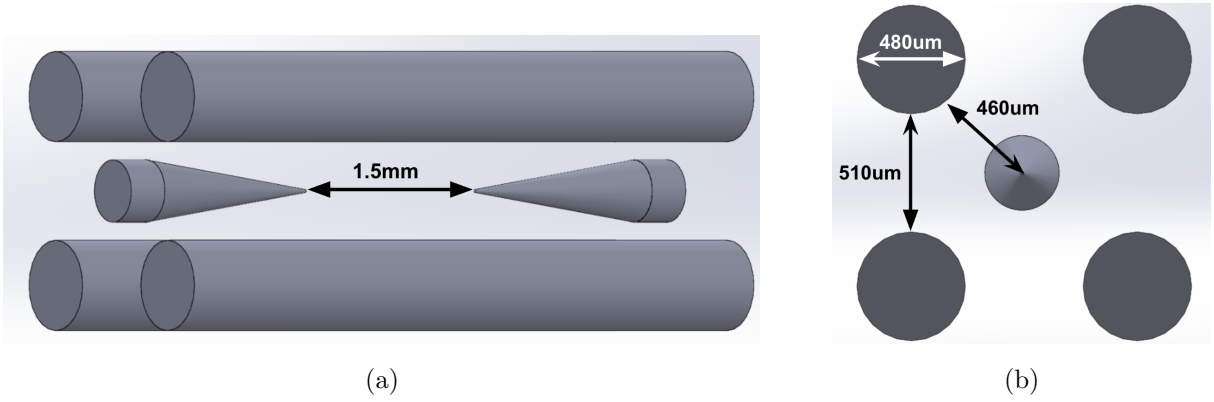


Figure 2.3: (a) A tilted side-on view of the ion trap. The spacing between the needles is 1.5mm. (b) Axial view of the trap. The rod width is 480um, the spacing between adjacent rods is 510um, and the distance from a rod to the center of the trap is 460um.

iments. To reduce the rate of background gas collisions, we must put the ion trap inside of an ultra high vacuum (UHV) chamber.

When designing the vacuum chamber, we opted for a 6" octagon with a high-conductance (4.5") pumping port (Kimball Physics MCF600-SphOct-F2E1C7). We aimed to maintain this high conductance throughout the vacuum chamber, so the majority of the UHV hardware was kept at a 4.5" diameter. We use three pumps in the system, a 20L/s ion pump (VacIon StarCell 9191145), a SAES Z400 Capacitron NEG (non-evaporable getter) pump, and a titanium sublimation (TiSub) pump (Gamma Vacuum G360819). We also include a UHV ion gauge (Agilent UHV 24: 9715007) in order to measure the pressure in the chamber. Two electrical feedthroughs are included, one 2-pin feedthrough for the trap RF, and one 15-pin feedthrough (pinout diagram in figure 2.5) for operating an atomic oven, and applying DC and microwave signals to the trap. An end-on diagram on the trap can be seen in figure 2.6, where the electrodes are labeled with the corresponding pins from the DSUB feedthrough.

The atomic oven is a small piece of Yb foil which was placed into a stainless steel tube. The source is heated by passing current through the tube. Before installation, the oven was tested by placing a microscope slide in front of the oven near the trap, and running current

(4.8A) through the oven for 30 minutes. After this time had passed, we investigated the slide and saw a ghostly blue hue on the slide. This gave us confidence that the oven was indeed creating Yb vapor.

In an attempt to reduce out-gassing, all of the stainless steel parts were taken to a local heat-treatment facility (Scarrott Metallurgical) to be heat treated. The process involved baking the pieces at 950C in vacuum for 2 hours, allowing them to cool fully, and then a bake at 400C in atmosphere for 2 hours.

Following heat treatment, each part of the vacuum hardware was cleaned in 5 steps, each step involved the parts being submerged in an ultra-sonic cleaner for 30 minutes in the following order:

- Alconox and deionized water mixture
- Deionized water (repeated if the parts were still visibly soapy)
- Isopropyl alcohol
- Acetone
- Methanol

After the cleaning, assembly of the vacuum chamber took place in a cleanroom tent, and great care was taken to avoid contamination of the vacuum parts. After construction, we performed a Helium leak check with a mobile Helium leak station (Agilent G8611B) to ensure all vacuum connections were sealed. An image of the completed vacuum chamber can be seen in figure 2.7.

2.4.3 Bakeout

In order to reduce out-gassing of water from the vacuum parts, the whole chamber undergoes a “bake”, where the system is heated above the boiling point of water, and pumped out with

a turbo pump. After the vacuum chamber was determined to be leak-free, we began the baking process. During the bake, we monitored the gas content of the system with an RGA (residual gas analyzer), and monitored the pressure of the chamber with a high-range vacuum gauge.

Our target bake temperature was 195C. We ramped the temperature at a rate of 2C per 5 minutes. Once at temperature, we began degassing the active vacuum components (TiSub, NEG, and ion gauge) to remove any residual gasses. These components were only roughly cleaned before the bake, and could not be sonicated, so heating them while gas is being removed with the turbo pump was necessary. After degassing, we fired the TiSub pump and activated the NEG to further reduce the pressure. We then began to use our ion gauge to monitor the pressure.

After about 1 week of baking, we decided to cool down the chamber to check the pressure. Once at room temperature, and after firing the TiSub again, the pressure was 5×10^{-11} Torr (according to the ion gauge). At this point we decided to end the baking process altogether, and moved the chamber to the optical table. Over the course of the next month, we fired the TiSub daily, eventually reaching a reading of 1.2×10^{-11} Torr on the ion gauge (this is below the lowest reading quoted on the gauge of 2×10^{-11} Torr).

2.4.4 Helical Resonator

In order to deliver clean, high voltage RF signals to an ion trap, we employ an impedance matched, quarter-wave helical resonant step-up transformer. An in-depth explanation to designing high quality factor resonators for ion traps can be found in [SSW12].

Our design follows that of the Monroe group from Duke, as well as taking lessons from other ion traps at UCLA from the $^{133}\text{Ba}^+$ and $^{43}\text{Ca}^+$ labs. It’s a bifilar resonator (meaning we have 2 main helical “pick-up” coils, each separately connected to a trap rod). The bifilar design gives us the ability to independently bias each rod to allow for easy excess micromotion

compensation.

The whole resonator is housed in a machined copper shield which is screwed to the trap chamber with a custom adapter. This forms a strong connection between the shield and the chamber, allowing for a well-defined ground reference. Inside the resonator, there are 2 large helical coils (8 turns each) which are held in place by 3D printed coil holders. These coils are attached to the electrical feedthrough pins to the trap, and the other ends are attached to SMA connectors on the shield. Both of the SMA connectors have bias tees attached to them so we can bias each coil separately. An image of the two pickup coils inside the shield and attached to the chamber can be seen in figure 2.8. The antenna coil's position relative to the two larger helical coils is variable and can be moved precisely by a high thread-count screw (1/4"-80). The position of the coil is adjusted to impedance match the RF signal to the trap. The antenna coil is connected to an SMA port for the input RF signal.

The two large helical coils were carefully wound to maintain a constant pitch and diameter. We carefully cleaned the shield and coils with Citronox prior to the final assembly to maintain surface cleanliness. When attached to the trap, we found the resonant frequency to be $2\pi \times 41.583\text{MHz}$, with a quality factor $Q \approx 130$.

2.5 Electronics for the Trap

To power the trap, we use a Rhode and Schwarz SMY 01 signal generator. For day-to-day operations as of writing this thesis, we operate with -6dBm output power from the signal generator, leading to a secular frequency of $2\pi \times 540\text{kHz}$. The RF signal gets amplified by a Mini-Circuits ZHL-5W-1+ amplifier which has a nominal gain of 40dB. The output of the amplifier is sent through a bi-directional coupler (Mini-Circuits ZFBD20-62HP-S+) to monitor the output signal, and the signal reflected back towards the amplifier from the trap. The input-coupled signal is used for triggering and monitoring on an oscilloscope, and the output-coupled signal is used for impedance matching. Moving the frequency of the

RF signal and the position of the antenna coil in the RF resonator will cause the output-coupled signal to change drastically near resonance. Because the output-coupled signal is representative of the rejected power from the RF resonator, when the output-coupled signal is a minimum, the forward power to the trap is maximized. The goal is to reduce the amplitude of the out-coupled signal on the oscilloscope to a minimum by changing the RF frequency and the position of the resonator’s antenna. Using this method, we can reduce the ratio of the amplitude of the rejected RF signal to the input signal to $\approx 1/20$.

The secular frequency of the ion depends on the RF voltage applied to the trap rods. In order to determine the secular frequency, we performed “tickle” scans of the ion by applying a small amplitude signal to the trap rods while varying the frequency of the signal. When the signal is resonant with the secular motion of the ion, the ion will experience heating due to being driven at its motional frequency, and the florescence from the ion will decrease. We performed these tickle scans at various values of the forward power to the trap. The results can be seen in figure 2.9.

The power supplies we use for DC voltages to the trap are made at the UCLA physics electronics shop by Peter Yu. The power supply for the needles has a range of $\pm 250\text{V}$, and the power supply for micromotion compensation electrodes is $\pm 10\text{V}$. All of the DC voltage signals are passed through low-pass filters to reduce pick-up of the trap RF signal.

The oven is heated by using a bench-top power supply (Keithley 2231A-30-3) in parallel mode to supply 5A of current to the oven for loading. We connect this supply to our experimental control system (see section 4.7) to turn it on and off when loading the trap.

2.6 Excess Micromotion Compensation

Micromotion is necessary for trapping ions, as it provides the small displacements needed to obtain a net restoring force for the ion in the trap. Even when the ion is perfectly on the trap axis, micromotion still is present (this is *intrinsic* micromotion). However, stray

electric fields can cause displacements of the ion's position from the trap's axis, leading to *excess* micromotion.

Excess micromotion leads to broadening of spectral lines, and can cause heating of ions leading to shorter trapping lifetimes. There are many methods of characterizing and detecting micromotion [BMB98]. Compensating for micromotion amounts to applying DC voltages to the trap electrodes in order to cancel any stray fields which push the ion off of the trap axis.

We perform micromotion compensation by measuring the line shape of our Doppler cooling (369nm) transition. Because our RF frequency is larger than the natural linewidth of the transition, we observe micromotion sidebands on the cooling transition. By scanning the voltage of a compensation electrode and performing scans, we can find the ion position that minimizes the excess micromotion. Good examples of micromotion compensation techniques and results from our group can be found in [Rom21], [Del25].

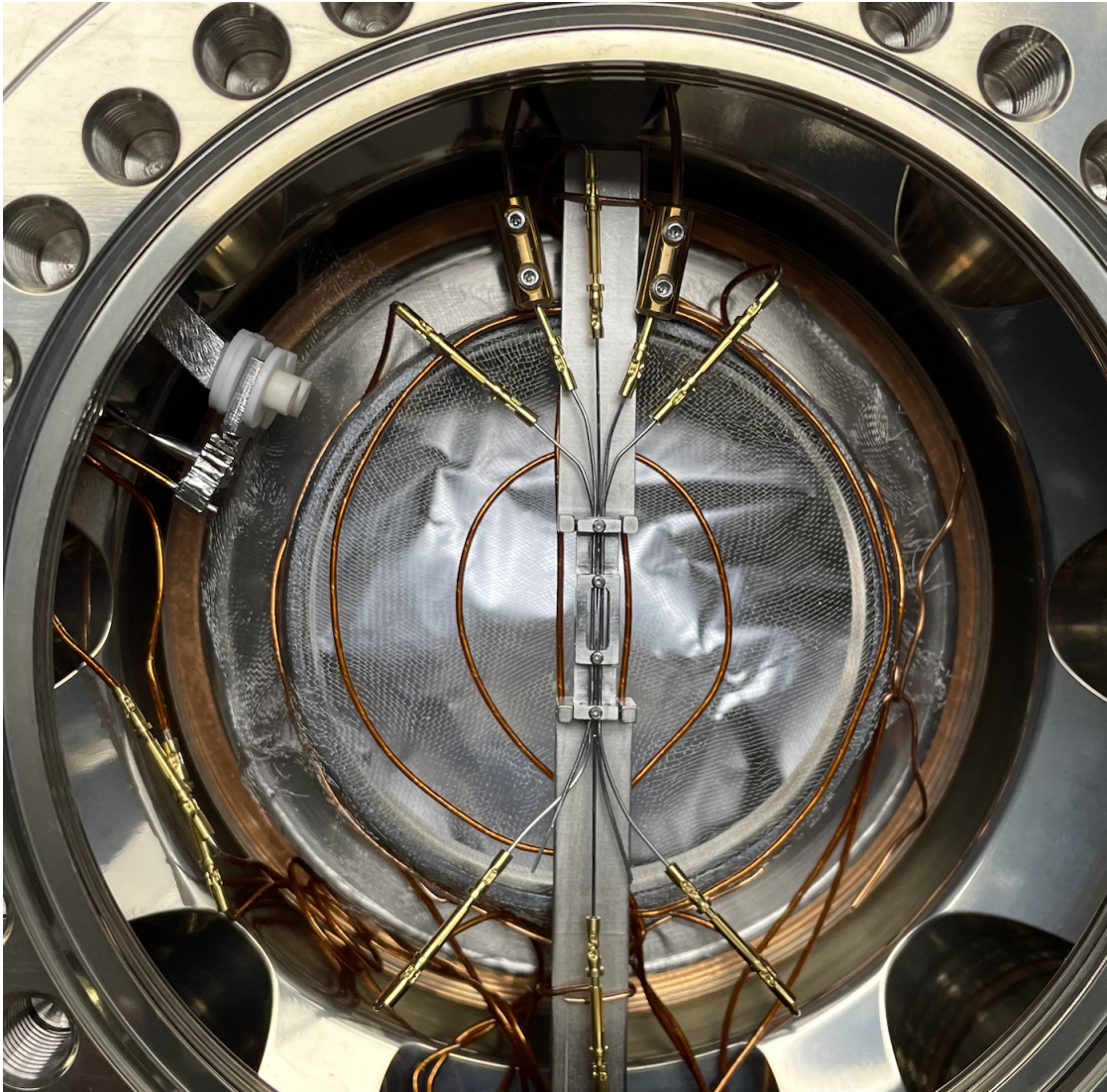


Figure 2.4: Camera image of the 4 rod trap in the vacuum chamber. All Tungsten rods are connected with T1 connectors. There are a total of 4 in-vacuum antennae. Two of the antennae are straight, un-terminated wires which run along the trap axis below the trap cage. The other two are circular, one about 1" in diameter, and the other is about 3" in diameter. On the left side of the image, the oven can be seen. The oven is electrically isolated from the chamber by including ceramic washers and screws between the oven holder and the groove-grabber. We placed a stainless steel mesh on both of the recessed viewports in an attempt to reduce the effects of charging from the windows.

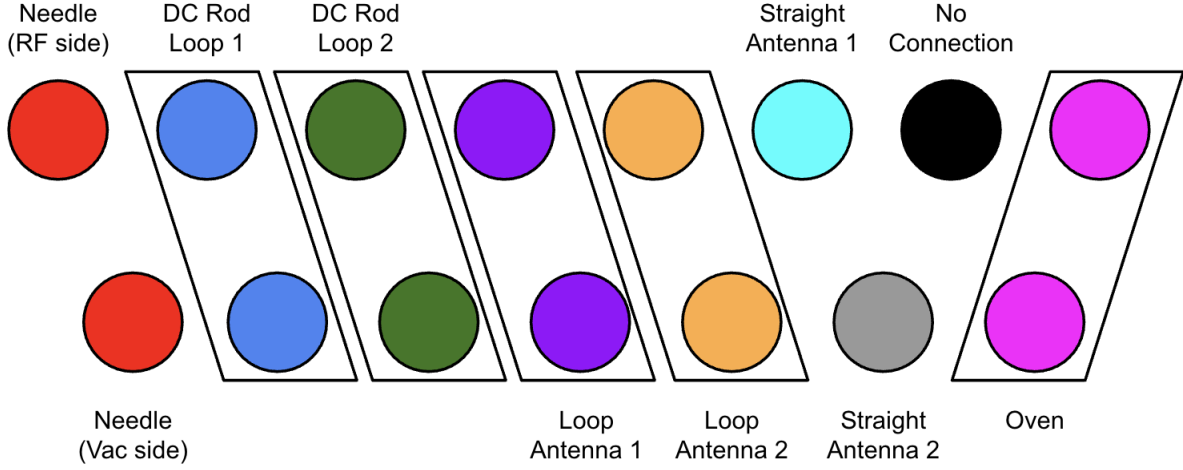


Figure 2.5: Pinout diagram of the 15-pin DSUB connector for our ion trap. Each pin is labeled on the diagram. The pins within a trapezoid are “through” connections, meaning they’re both connected to the same element in the chamber (oven, DC electrode, antenna).

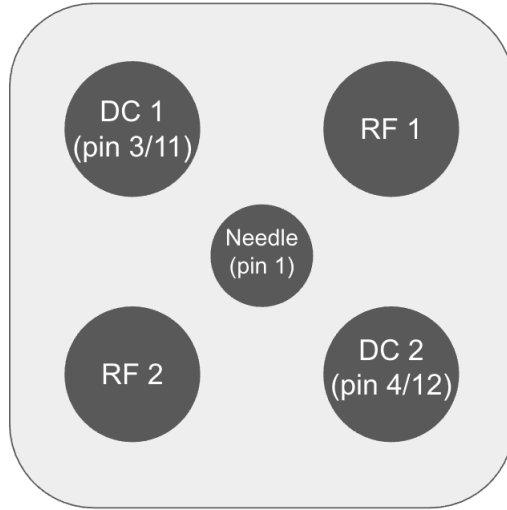


Figure 2.6: End-on view of the trap from the RF feedthrough side. The DC rods and needle electrodes are labeled with their corresponding pins on the DSUB connector from figure 2.5. The needle on the other side of the trap is not shown, however it is connected to pin 9. The RF 1 (2) rod is connected to the right (left) pin on the RF feedthrough.

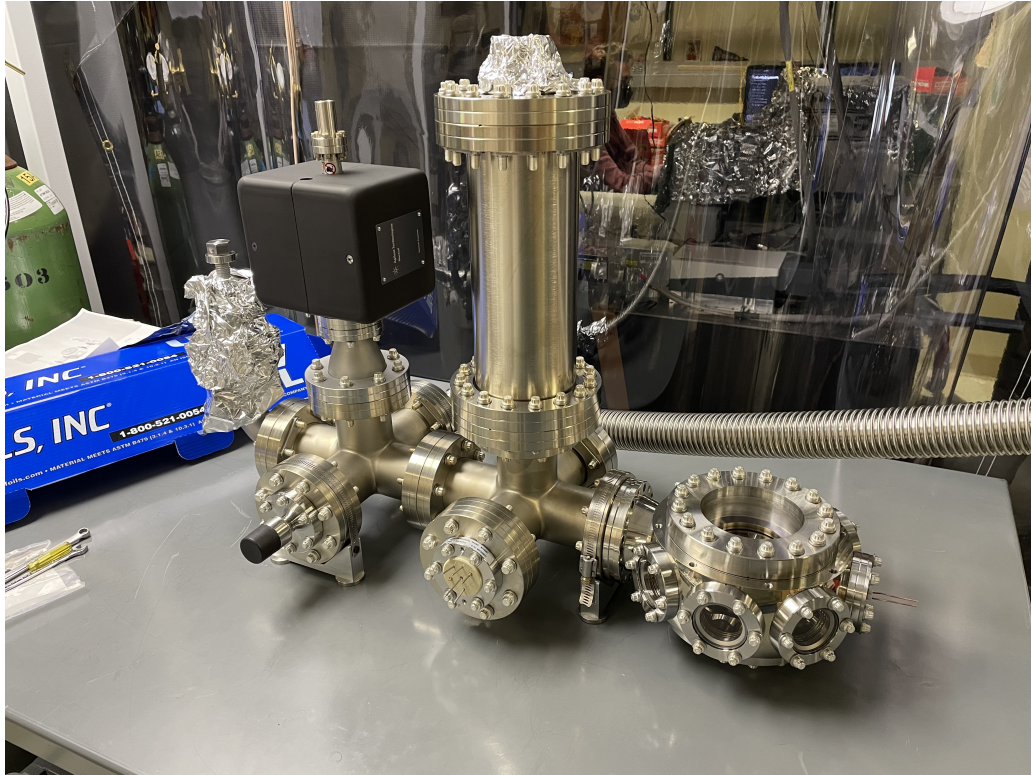


Figure 2.7: Assembled vacuum chamber before baking. The octagon can be seen on the right. There are 6 2.75" windows on the chamber, and 2 custom-made recessed viewports on the top and bottom of the octagon. There is a 2-pin feedthrough opposite from the pumping port. Our TiSub pump is situated in the large vacuum nipple, out of line-of-sight from any other vacuum component. In the same 5-way cross as the TiSub, we have our ion gauge and our 15-pin electrical feedthrough. On the other 5-way cross, the ion pump, NEG, and angle valve are connected. The last port is blanked.

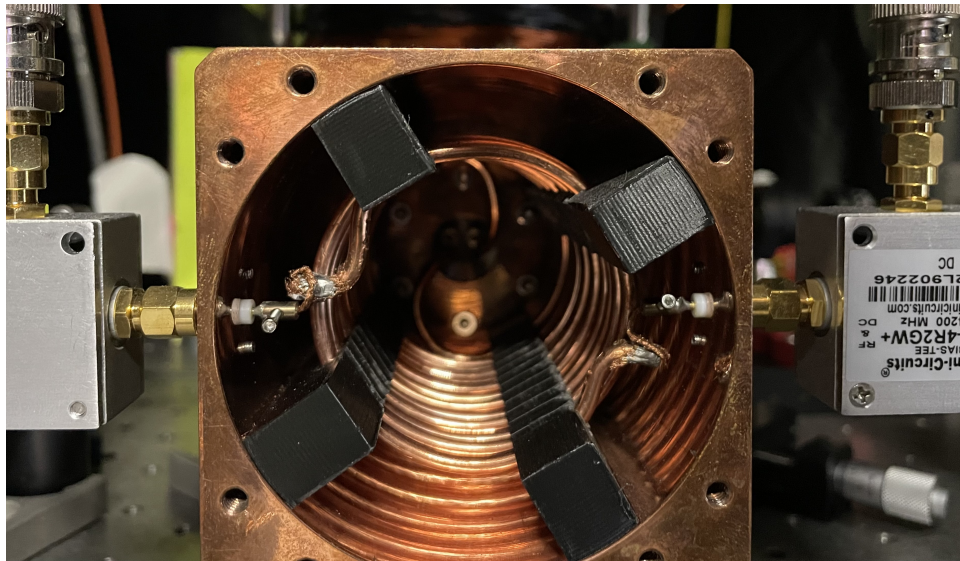


Figure 2.8: Camera image of the inside of the RF resonator. The two large helical “pick-up” coils are seen held in place by 3D printed coil holders. The ends of the coils are connected to SMA connectors which are connected to bias tees for applying small offset voltages.

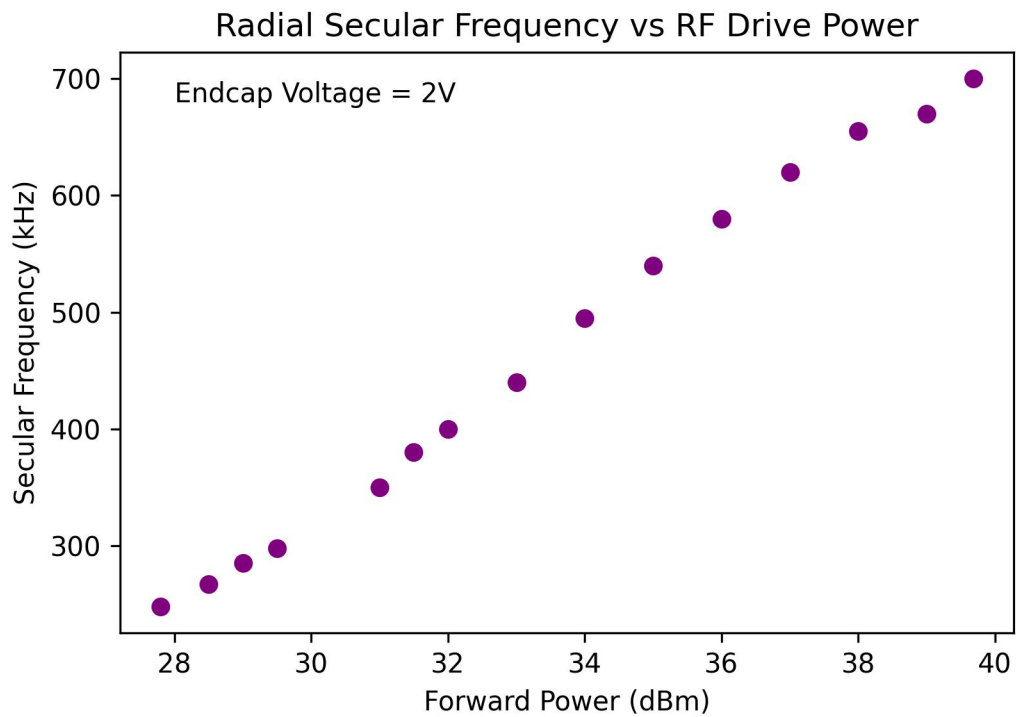


Figure 2.9: Plot of the measured radial secular frequency as a function of applied trap power. Power values are calculated by assuming a flat gain from the amplifier of +40dBm. Secular frequencies are measured by applying “tickle” scans to the ion via weak signals on the trap electrodes.

CHAPTER 3

Atoms and Light

Since our choice of physical qubit consists of energy levels in an atom, we must have an understanding of atomic structure and how to interact with these levels through electromagnetic radiation. The goal of this chapter is to discuss a few of the important aspects of atomic structure and transitions between atomic energy levels, mainly hyperfine structure, Zeeman structure, electric dipole transitions, electric quadrupole transitions, and magnetic dipole transitions.

3.1 Hyperfine Structure

The hyperfine structure comes from the interaction between the total angular momentum of the electron and the electric and magnetic moments of the nucleus. We will be concerned primarily with the magnetic dipole moment of the nucleus ($\vec{\mu}$) interacting with the angular momentum of the electron ($\vec{J}$). The effective Hamiltonian for the interaction is $H_{eff} = A\vec{I} \cdot \vec{J}$, where I is the nuclear spin, and A is the magnetic dipole hyperfine coefficient.

The hyperfine interaction causes energy levels to split into several different levels, ranging from $|I - J|$ to $I + J$ in integer steps. The general size of such splitting is typically $\approx 10 - 10,000\text{MHz}$, much smaller than optical transitions (≈ 100 's of THz). Because J is no longer a good quantum number, these manifolds are labeled with quantum number F , and the $2F + 1$ magnetic sublevels are labeled m_F .

3.1.1 Hamiltonian from the Vector Approach

The goal of this and the next section is to show that we can arrive at an effective Hamiltonian for the magnetic dipole hyperfine interaction: $H_{eff} = A\vec{I} \cdot \vec{J}$. We begin with the following Hamiltonian:

$$H = -\vec{\mu}_I \cdot \vec{B}_J(\vec{r} \approx 0) \quad (3.1)$$

Where $\vec{\mu}_I$ is the magnetic dipole moment of the nucleus, and $\vec{B}_J(\vec{r} \approx 0)$ is the magnetic field near the nucleus due to the total angular momentum of the electron. The magnetic dipole moment of the nucleus is proportional to the spin of the nucleus: $\hbar\vec{\mu}_I = g_I\mu_N\vec{I}$, where g_I is the g-factor for the nucleus, μ_N is the nuclear magneton, and $\vec{I}$ is the spin of the nucleus.

We then want to make the assumption that $\vec{L}$ and $\vec{S}$ precess quickly around $\vec{J}$. Thus we can estimate that the time-averaged total magnetic field due to $\vec{J}$ is the sum of the time averaged magnetic fields due to $\vec{S}$ and $\vec{L}$: $\langle \vec{B}_J \rangle = \langle \vec{B}_S \rangle + \langle \vec{B}_L \rangle$. In other words, we are going to ignore any terms which are non-diagonal in $\vec{J}$. So we can then re-write this time averaged sum.

$$\langle \vec{B}_J \rangle = \left(\left\langle \frac{\vec{B}_L \cdot \vec{J}}{\langle |\vec{J}| \rangle} \right\rangle + \left\langle \frac{\vec{B}_S \cdot \vec{J}}{\langle |\vec{J}| \rangle} \right\rangle \right) \frac{\langle \vec{J} \rangle}{\langle |\vec{J}| \rangle} \quad (3.2)$$

Then, we can incorporate this all into the Hamiltonian.

$$H_{hfi} = -\frac{g_I\mu_N}{\hbar} \vec{I} \cdot \langle \vec{J} \rangle \left\langle \frac{(\vec{B}_L + \vec{B}_S) \cdot \vec{J}}{J^2} \right\rangle \quad (3.3)$$

So now, we must deal with B_L and B_S . By Biot-Savart, we have

$$\vec{B}_L = -\frac{\mu_0 e}{4\pi} \frac{1}{|r_i|^3} \vec{v}_e \times \vec{r}' \quad (3.4)$$

Then, we know that $\vec{r}' = -\vec{r}_i$, since the position vector points inward to the nucleus, and the Angular momentum is defined as $\vec{L}_i = \vec{r}_i \times \vec{p}_i$. Using this, and the definition of the Bohr magneton, we may write

$$\vec{B}_L = -\frac{\mu_0\mu_B}{2\pi\hbar} \frac{1}{|r_i|^3} \vec{L}_i \quad (3.5)$$

Then for $\vec{B}_S$, we must use the vector potential.

$$\vec{A}_S = \frac{\mu_0}{4\pi} \frac{1}{|r'|^3} \vec{\mu}_S \times \vec{r}' \quad (3.6)$$

We then take the curl, and will assume that there is no overlap with $r = 0$, and that μ_s is constant as a function of position.

$$\vec{B}_S = \frac{\mu_0}{4\pi} \left(-(\vec{\mu}_S \cdot \vec{\nabla}_{r'}) \frac{\vec{r}'}{r'^3} \right) \quad (3.7)$$

With some vector math, this becomes:

$$\vec{B}_S = \frac{\mu_0}{4\pi} \left[-\frac{\vec{\mu}_S}{r^3} + \frac{3(\vec{r} \cdot \vec{\mu}_S)\vec{r}}{r^5} \right] \quad (3.8)$$

Then we can use $\hbar\vec{\mu}_S = -2\mu_B\vec{S}$, and put everything together to get:

$$H_{hfi} = \frac{g_I\mu_N\mu_0\mu_B}{2\pi\hbar^2\langle r^3 \rangle} \vec{I} \cdot \langle \vec{J} \rangle \left\langle \frac{[\vec{L} + (-\vec{S} + \frac{3(\vec{S} \cdot \vec{r})\vec{r}}{r^2})] \cdot \vec{J}}{\vec{J}^2} \right\rangle \quad (3.9)$$

Some more vector math leads to:

$$H_{hfi} = \frac{g_I\mu_N\mu_0\mu_B}{2\pi\hbar^2} \frac{\vec{I} \cdot \vec{J}}{\langle r^3 \rangle \vec{J}^2} \left\langle \vec{L}^2 - \vec{S}^2 + 3\left(\frac{\vec{S} \cdot \vec{r}}{r}\right)^2 \right\rangle \quad (3.10)$$

Then we can use the fact that we will be applying this in the $|LSJFM_F\rangle$ basis, and the fact that the electron spin is always $\pm 1/2$ to write:

$$H_{hfi} = \frac{g_I\mu_N\mu_0\mu_B}{2\pi\hbar^2} \frac{\vec{I} \cdot \vec{J}}{\langle r^3 \rangle J(J+1)} L(L+1) \quad (3.11)$$

We can write this as an effective Hamiltonian: $H_{eff,hfi} = A\vec{I} \cdot \vec{J}$, with

$$A = \frac{g_I\mu_N\mu_0\mu_B}{2\pi\hbar^2} \frac{L(L+1)}{\langle r^3 \rangle J(J+1)} \quad (3.12)$$

This only applies for $l \neq 0$ states, though, since we claimed that $r \neq 0$. For $l = 0$ states, we have $\vec{J} = \vec{S}$, and we can take the charge density of the electron near the origin to be constant and proportional to the square norm of the electronic wavefunction $|\phi(0)|^2$. The

magnetism due to this is: $\vec{M} = \vec{\mu}_S |\phi(0)|^2$. We can use the formula for the magnetic field due to a uniformly magnetized sphere to find:

$$\vec{B}_S = -\frac{2g_s\mu_B\mu_0}{3\hbar}\vec{S}|\phi(0)|^2 \quad (3.13)$$

Plugging this in with $\vec{B}_L = 0$, we find for $l = 0$ states:

$$A = \frac{2g_i g_s \mu_0 \mu_N \mu_B}{3\hbar^2} |\phi(0)|^2 \quad (3.14)$$

3.1.2 Hamiltonian from the Tensor Approach

Here we begin with the hyperfine interaction Hamiltonian. It is a sum over scalar products of irreducible spherical tensors.

$$H_{hfi} = \sum_{k,\mu} (-1)^\mu T_{k,\mu}^e T_{k,-\mu}^n \quad (3.15)$$

Where k indicates the rank of the tensors, μ is the coordinate, T^e is the tensor which acts on the electron space, and T^n acts on the nuclear space. This Hamiltonian acts on the $|IJFM_F\rangle$ basis. This is the conventional way to write this Hamiltonian, which in reality is simply the sum over the scalar products of all ranks k of the operators: $H_{hfi} = \sum_k \mathbf{T}_k^e \mathbf{T}_k^n$. We can find the first order energy shift due to this interaction Hamiltonian by simply taking the expectation value in the $|IJFM_F\rangle$ basis. By noticing that this is just the scalar product of these tensor operators, we can use this identity for the product of spherical tensors acting on a coupled basis:

$$\langle J'_1 J'_2 J' M' | T_k U_k | J_1 J_2 J M \rangle = \delta_{JJ'} \delta_{MM'} (-1)^{J_1+J_2+J} \begin{Bmatrix} J & J'_2 & J'_1 \\ k & J_1 & J_2 \end{Bmatrix} \langle J'_1 | T_k | J_1 \rangle \langle J'_2 | U_k | J_2 \rangle \quad (3.16)$$

Using our identity with our Hamiltonian:

$$\sum_k \langle IJFM_F | T_k^e T_k^n | IJFM_F \rangle = \sum_k (-1)^{I+J+F} \begin{Bmatrix} I & J & F \\ J & I & k \end{Bmatrix} \langle J | T_k^e | J \rangle \langle I | T_k^n | I \rangle \quad (3.17)$$

Then, we introduce an expression for the Wigner 6j symbol:

$$(-1)^{I+J+F} \begin{Bmatrix} I & J & F \\ J & I & k \end{Bmatrix} = \frac{(2I)!(2J)!\xi(IJ, Fk)}{\sqrt{(2I-k)!(2I+k+1)!(2J-k)!(2J+k+1)!}} \quad (3.18)$$

For this example, we are concerned only with the magnetic dipole interaction, so $k = 1$. However, we can use this expression to get higher multipole contributions ($k > 1$), as they become important with $I > 1$ nuclei (for instance ^{173}Yb). Expressions for the higher order contributions (E2 and M3) can be found in [Del25].

For $k = 1$, $\xi(IJ, F1) = \frac{Z}{2IJ}$, and $Z = F(F+1) - I(I+1) - J(J+1)$. Then we can use a convenient expression for the reduced matrix element:

$$\langle JJ | T_{k,0}^e | JJ \rangle = \frac{(2J)! \langle J | |T_k^e| | J \rangle}{\sqrt{(2J-k)!(2J+k+1)!}} \quad (3.19)$$

After substituting this in for T^e and T^n , we find:

$$E^{(1)} = \xi(IJ, F1) \langle JJ | T_{1,0}^e | JJ \rangle \langle II | T_{1,0}^n | II \rangle \quad (3.20)$$

In the nuclear shell model, the magnetic dipole moment of the nucleus is defined as the expectation value of this tensor in the stretched nuclear state: $\mu \equiv \langle II | T_{1,0}^n | II \rangle$. Furthermore, $\mu = g_I I \mu_N$. So our first order energy correction due to our interaction Hamiltonian is:

$$E^{(1)} = \frac{g_I I \mu_N}{2IJ} [F(F+1) - I(I+1) - J(J+1)] \langle JJ | T_{1,0}^e | JJ \rangle \quad (3.21)$$

If we then define $A \equiv \frac{g_I I \mu_N}{IJ} \langle JJ | T_{1,0}^e | JJ \rangle$, we see that $E^{(1)} = \frac{A}{2} [F(F+1) - I(I+1) - J(J+1)]$, which is exactly what we would get if we were to use the effective Hamiltonian: $H_{eff,hfi} = A \vec{I} \cdot \vec{J}$.

3.1.3 Quick Note on Clock States

In the context of hyperfine qubits, a “clock state” is one which is first-order insensitive to perturbations by weak magnetic fields. These states typically have a magnetic quantum number (m) equal to zero, meaning they have no projection due to “unpaired” spins. This is

well seen in the so-called “singlet” and “triplet” states which result from the addition of two spin $1/2$ particles. The singlet state is the antisymmetric combination of $|\uparrow, \downarrow\rangle$ and $|\downarrow, \uparrow\rangle$. The triplet state is effectively characterized by a spin 1 particle, containing $m = -1, 0, 1$. In the absence of an applied magnetic field, these three states are degenerate. But since the $m = 0$ state is the symmetric combination of $|\uparrow, \downarrow\rangle$ and $|\downarrow, \uparrow\rangle$, it has no “unpaired” spin which could project in the presence of a magnetic field.

Since to first order, the energy of each the singlet and the $m = 0$ state in the triplet do not shift when exposed to a magnetic field, the energy difference between the two is unaffected. Thus the splitting is robust to small fluctuations in the magnetic field. This makes these pairs of states particularly attractive for qubits, as the qubit frequency will remain static.

3.2 Zeeman Structure (Weak Field)

Each energy level with total angular momentum J will have $2J+1$ magnetic projection levels m_J . In the absence of a magnetic field, these levels are degenerate, however when exposed to a magnetic field, the magnetic moment of the electron will interact with the magnetic field through the Hamiltonian: $H_{eff} = -\vec{\mu} \cdot \vec{B}$. Where $\vec{\mu}$ is the magnetic dipole moment of the electron in the atom, which can be written as: $\vec{\mu} = \mu_B(g_L\vec{L} + g_s\vec{S})$, where μ_B is the Bohr magneton. To evaluate the first order perturbation due to this effective Hamiltonian, we take the expectation value.

This is typically done by separately evaluating the time-averaged values of L and S , and eventually arriving at a simple expression for the first-order perturbation:

$$\Delta E_B = \mu_B g_J m_J B \quad (3.22)$$

Where g_J is defined as (note that $g_L = 1$ and $g_s \approx 2$):

$$g_J = g_L \frac{J(J+1) - S(S+1) + L(L+1)}{2J(J+1)} + g_s \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \quad (3.23)$$

When including hyperfine structure, we arrive at a nearly identical expression for the

first order perturbation, only exchanging g_J for g_F , which is defined as:

$$g_F = g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} + g_I \frac{\mu_N}{\mu_B} \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)} \quad (3.24)$$

Where μ_N is the nuclear magneton, which is ≈ 3 orders of magnitude smaller than the Bohr magneton (μ_B), so typically the second term is neglected.

3.3 Electric Dipole Transitions

We interact with atoms via electromagnetic radiation. The dominant transitions between atomic states are “electric dipole” (E1) transitions, which we can examine by considering the perturbing Hamiltonian in the dipole approximation ($e^{i\vec{k}\cdot\vec{r}} \approx 1$): $H_{E1} = -\vec{d}\cdot\vec{E} = e\vec{r}\cdot\hat{\mathcal{E}}E_0e^{-i\omega t}$. Our job then becomes evaluating matrix elements of this perturbation between atomic states of interest.

Evaluating matrix elements will become much easier if we are able to rewrite $\vec{r}$ in terms of an irreducible spherical tensor $T_q^{(k)}$, as we can pull from a wealth of tensor identities to help us. Luckily, we can indeed do this by writing the position vector in spherical coordinates, plugging in the spherical harmonics, and then identifying the unit vectors as:

$$\hat{e}_0 = \hat{e}_z, \hat{e}_{\pm} = \frac{1}{\sqrt{2}}(\hat{e}_x \pm i\hat{e}_y) \quad (3.25)$$

Ultimately the position vector $\vec{r}$ can be written as:

$$\vec{r} = r\sqrt{\frac{4\pi}{3}}\left(\hat{e}_+Y_1^{-1} - \hat{e}_-Y_1^1 + \hat{e}_0Y_1^0\right) = T_q^{(1)} \quad (3.26)$$

We are typically interested in finding the Rabi frequency of the transition, which is defined as:

$$\Omega = \frac{\left|\langle\alpha, J, I, F, m_F| e\vec{r}\cdot\hat{\mathcal{E}}E_0 |\alpha', J', I', F', m'_F\rangle\right|}{2\hbar} \quad (3.27)$$

Where we’ve taken on the quantum numbers which are relevant in our experiments, J, I, F, m_F , and α representing any additional quantum numbers.

We can rewrite the Rabi frequency by taking the definition of the position vector, and noting that we can take some factors out of the matrix element.

$$\Omega = \frac{eE_0}{2\hbar} \sum_q \mathcal{E}_q |\langle \alpha, J, F, m_F | T_q^{(1)} | \alpha', J', F', m'_F \rangle| \quad (3.28)$$

We can apply the Wigner-Eckart theorem to remove the m_F dependence in the matrix element:

$$\langle j, m | T_q^{(k)} | j', m' \rangle = (-1)^{j-m} \begin{pmatrix} j & k & j' \\ -m & q & m' \end{pmatrix} \langle j || T^{(k)} || j' \rangle \quad (3.29)$$

And further we can remove the I dependence because we know $T_q^{(k)}$ doesn't act on the I subspace via the ‘‘Spectator Theorem’’:

$$\langle J_1, J_2, J || T^{(k)} || J'_1, J_2, J' \rangle = \quad (3.30)$$

$$(-1)^{J_1+J_2+J'+k} \sqrt{(2J+1)(2J'+1)} \begin{Bmatrix} J_1 & J & J_2 \\ J' & J_1 & k \end{Bmatrix} \langle J_1 || T^{(k)} || J'_1 \rangle \quad (3.31)$$

Finally, the result for our Rabi frequency is:

$$\Omega = \frac{eE_0}{2\hbar} \sum_q \mathcal{E}_q \left| (-1)^{F-m_F+J+I+F'+k} \sqrt{(2F+1)(2F'+1)} \right| \times \quad (3.32)$$

$$\left| \langle J || T^{(k)} || J' \rangle \begin{pmatrix} F & k & F' \\ -m_F & q & m'_F \end{pmatrix} \begin{Bmatrix} J & F & I \\ F' & J' & k \end{Bmatrix} \right| \quad (3.33)$$

What remains unknown here is the radial integral: $\langle J || T^{(1)} || J' \rangle$. To continue further, we'll have to rely either on atomic structure calculations, or experimental measurements for the reduced matrix element.

Fermi's Golden rule for E1 transitions is given by:

$$\frac{1}{\tau} = \frac{w^3 |\langle J || T^{(k)} || J' \rangle|^2}{3\pi\epsilon_0 \hbar c^3 (2J'+1)} \quad (3.34)$$

Using this we can plug in known values, and solve for the Rabi frequency. E1 transitions are typically ‘‘fast’’ meaning the excited state lifetimes are short (≈ 10 ns).

3.3.1 Doppler Cooling

Doppler cooling is a laser cooling technique which allows for fast, effective cooling of atoms and ions. In the lab frame, we set the laser light to be red-detuned from the atomic resonance. An atom counter-propagating in a laser beam ($-\hat{k}_{laser} = \hat{v}_{atom}$) will see the light as *blue-shifted* toward resonance in the atom frame. The atom absorbs the photon, receives a $-\hbar k$ momentum kick, and then emits the photon in a random direction. The net result over many cycles is the atom cooling down.

We want a large damping force in order to cool atoms quickly. The force will be proportional to the scattering rate of the transition used, so we want to choose transitions whose excited state lifetimes are short to achieve high scattering rates (typically E1 transitions are used). For the commonly trapped ions, the transition used is from the ground $^2S_{1/2}$ state to the short-lived $^2P_{1/2}^o$ or $^2P_{3/2}^o$ state, depending on the species. Typically the lifetime of the excited state is short ($\approx 10\text{ns}$), leading to a spontaneous emission rate of $A \approx 2\pi \times 20\text{MHz}$.

The scattering rate is given by the spontaneous emission rate (Γ) times the excited state population (ρ_{ee}):

$$\Gamma\rho_{ee} = \Gamma \frac{\Omega^2/4}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \quad (3.35)$$

Where Ω is the Rabi frequency, δ is the laser frequency's detuning from the atomic resonance, and $\Gamma = 1/\tau$, where τ is the lifetime of the atomic state. We can find the force on the atom as $\hbar k$ times the scattering rate. By balancing the cooling and heating rates, we can arrive at the Doppler limit, which is $T = \hbar\Gamma/(2k_B)$, where k_B is the Boltzmann constant. This limit can be achieved by setting the laser detuning to $\delta = -\Gamma/2$.

The Rabi frequency is related to the saturation parameter (s) by $s = I/I_s = 2\Omega^2/\Gamma^2$. Where I_s is the saturation intensity, and I is the laser intensity. The saturation intensity is defined as $I_s = \pi\hbar c/(3\lambda^3\tau)$. For Yb^+ , the saturation intensity is $I_s = 51.5\text{mW/cm}^2$. To achieve saturation with a beam with $1/e^2$ waist radius of $50\mu\text{m}$, we need only $\approx 80\text{nW}$.

3.4 Electric Quadrupole Transitions

States which do not couple together by an electric dipole moment can still couple by higher-order multipole moments and higher spatial derivatives of the electromagnetic waves applied to the atom. In our work, we are interested in state which are coupled by an electric quadrupole moment (E2). The E2 transition moments are typically much smaller than E1 transition moments, and when paired with narrow laser systems, they can be used to resolve motional modes of trapped ions.

The Rabi frequency of an electric quadrupole transition is given by [Jam98]:

$$\Omega = \frac{eE_0w}{2\hbar c} |\langle J, m_J | \hat{r}_i \hat{r}_j | J', m'_J \rangle \epsilon_i k_j| \quad (3.36)$$

where ϵ_i is the i th component of the unit polarization vector, and k_j is the j th component of the laser's unit wavevector. We are summing over repeated indices as well. We can rewrite the matrix element for $r_i r_j$ in terms of a spherical tensor operator and use the Wigner Eckart theorem to remove the dependence on m_J :

$$\langle J, m_J | \hat{r}_i \hat{r}_j | J', m'_J \rangle = \langle J | |r^2 C^{(2)}| | J' \rangle \sum_{q=-2}^2 \begin{pmatrix} J & 2 & J' \\ -m_J & q & m'_J \end{pmatrix} c_{ij}^{(q)} \epsilon_i k_j \quad (3.37)$$

where $c_{ij}^{(q)}$ is the ij component of a tensor which transforms the product $\epsilon_i k_j$ to the spherical basis. The term $c_{ij}^{(q)} \epsilon_i k_j$ is the source of the majority of (my) confusion when evaluating the Rabi frequency. We will go through the evaluation of this term explicitly in order to hopefully dispel confusion and make it easier for others to learn. These tensors ($c_{ij}^{(q)}$) are given by:

$$c_{ij}^{(q)} = \sqrt{\frac{10}{3}} (-1)^q \sum_{n_1, n_2=-1}^1 \begin{pmatrix} 1 & 1 & 2 \\ n_1 & n_2 & -q \end{pmatrix} b_i^{(n_1)} b_j^{(n_2)} \quad (3.38)$$

with $b^{(1)} = -1/\sqrt{2}(1, -i, 0)$, $b^{(0)} = (0, 0, 1)$, and $b^{(-1)} = 1/\sqrt{2}(1, i, 0)$. As an example, let's create the tensor $c_{ij}^{(2)}$.

$$c_{ij}^{(2)} = \sqrt{\frac{10}{3}} (-1)^2 \sum_{n_1, n_2=-1}^1 \begin{pmatrix} 1 & 1 & 2 \\ n_1 & n_2 & -2 \end{pmatrix} b_i^{(n_1)} b_j^{(n_2)} \quad (3.39)$$

There's only one combination of n_1 and n_2 (both must be $+1$) which results in a non-zero value of the Wigner $3j$ symbol, since the bottom row must sum to 0. Therefore this expression simplifies to

$$c_{ij}^{(2)} = \sqrt{\frac{10}{3}} \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & -2 \end{pmatrix} b_i^{(1)} b_j^{(1)} \quad (3.40)$$

The Wigner $3j$ symbol above is evaluated to $1/\sqrt{5}$, and we can take the outer product of $b_i^{(1)} b_j^{(1)}$ to obtain the explicit form of $c^{(2)}$.

$$c^{(2)} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & -i & 0 \\ -i & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.41)$$

Then, to evaluate the quantity of $c_{ij}^{(2)} \epsilon_i k_j$ it is rather straightforward if we recall that repeated indices are summed over. Since $i, j = \{1, 2, 3\}$, we can simply write out the sum.

$$\sum_{i,j} c_{ij}^{(2)} \epsilon_i k_j = c_{11}^{(2)} \epsilon_1 k_1 + c_{12}^{(2)} \epsilon_1 k_2 + c_{21}^{(2)} \epsilon_2 k_1 + c_{22}^{(2)} \epsilon_2 k_2 \quad (3.42)$$

Note that we've abused the fact that the matrix $c^{(2)}$ has only 4 non-zero values to truncate the sum. To go further with this example, we will need to write out explicit forms of the vectors $\vec{\epsilon}$ and $\vec{k}$, for which we will follow the reasoning presented in [Roo00].

We begin by defining our magnetic field along the $\hat{z}$ direction: $\vec{B} = B_0(0, 0, 1)$, and then allowing our laser's $\vec{k}$ vector to form an angle ϕ with the magnetic field. The $\hat{k}$ unit vector then may be written as $\vec{k} = (\sin(\phi), 0, \cos(\phi))$. By choosing this definition of $\vec{k}$, we've enforced that the laser and magnetic field lie in the $x - z$ plane. Then we define the "plane of incidence", which will be *perpendicular* to $\vec{k}$. This means the polarization $\vec{\epsilon}$ will lie in the plane of incidence. We then need to determine how to write $\vec{\epsilon}$, as it may for an arbitrary angle γ with respect to the magnetic field.

We can find an expression for $\vec{\epsilon}$ by considering the following. We can only rotate the polarization *about* the laser's $\vec{k}$, meaning whatever expression we come up with is essentially

going to be representative of a rotation of a unit vector about an arbitrary vector $\vec{k}$. This is precisely what Rodrigues rotation formula allows us to do. Rodrigues rotation formula lets us generate a rotation matrix $R_{\hat{k}}(\theta)$ for a rotation of angle θ about unit vector $\hat{k}$ by the following: $R_{\hat{k}}(\theta) = I + \tilde{w}\sin(\theta) + \tilde{w}^2(1 - \cos(\theta))$. Where $\tilde{w}$ is the antisymmetric matrix:

$$\tilde{w} = \begin{pmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{pmatrix} \quad (3.43)$$

So we can construct our rotation matrix for rotations θ about $\hat{k} = (\sin(\phi), 0, \cos(\phi))$:

$$R_{\hat{k}}(\theta) = \begin{pmatrix} \cos(\theta) & -\cos(\phi)\sin(\theta) & \frac{1}{2}\sin(2\phi)(1 - \cos(\theta)) \\ \cos(\phi)\sin(\theta) & \cos(\theta) & -\sin(\phi)\sin(\theta) \\ \frac{1}{2}\sin(2\phi)(1 - \cos(\theta)) & \sin(\phi)\sin(\theta) & \cos(\theta) + \cos^2(\phi)(1 - \cos(\theta)) \end{pmatrix} \quad (3.44)$$

Now, let's assume for simplicity that $\hat{e}$ lies purely in the $\hat{y}$ direction to begin with, so that $\hat{e} = (0, 1, 0)$. We can apply our rotation matrix to this vector to get an expression for the polarization unit vector as a function of angle ϕ (the angle between the laser's $\hat{k}$ vector and the projection of the magnetic field $\vec{B}$ into the plane of incidence), and an angle θ , which will be the angle between the $\hat{y}$ direction and the resultant vector $\hat{e}$.

$$\hat{e} = (-\cos(\phi)\sin(\theta), \cos(\theta), \sin(\phi)\sin(\theta)) \quad (3.45)$$

However, θ isn't as intuitive as using an angle γ which would be the angle between the magnetic field and $\hat{e}$. Since the magnetic field is defined along the $\hat{z}$, direction though, and we started our vector $\hat{e}$ along $\hat{y}$, we can simply identify $\gamma = \frac{\pi}{2} + \theta$. This results in:

$$\hat{e} = (\cos(\phi)\cos(\gamma), \sin(\gamma), -\sin(\phi)\cos(\gamma)) \quad (3.46)$$

Which lines up with the result in [Roo00]. Now, we can continue our example by plugging in expressions for $\hat{e}$ and $\hat{k}$ into equation 3.42:

$$c_{ij}^{(2)} \epsilon_i k_j = \frac{1}{\sqrt{6}} \left(\frac{1}{2} \cos(\gamma) \sin(2\phi) - i \sin(\gamma) \sin(\phi) \right) \quad (3.47)$$

We can then write down expressions for all $c_{ij}^{(q)} \epsilon_i k_j$ now that we have gone through an example.

$$c_{ij}^{(0)} \epsilon_i k_j = -\frac{1}{2} \left(\cos(\gamma) \sin(2\phi) \right) \quad (3.48)$$

$$c_{ij}^{(\pm 1)} \epsilon_i k_j = \frac{1}{\sqrt{6}} \left(\mp \cos(\gamma) \cos(2\phi) + i \sin(\gamma) \cos(\phi) \right) \quad (3.49)$$

$$c_{ij}^{(\pm 2)} \epsilon_i k_j = \frac{1}{\sqrt{6}} \left(\frac{1}{2} \cos(\gamma) \sin(2\phi) \mp i \sin(\gamma) \sin(\phi) \right) \quad (3.50)$$

Now that these geometrical factors are sorted, we can continue to calculate the Rabi frequency for an E2 transition. The spontaneous emission rate A ($1/\tau$) of an E2 transition is given by [Jam98].

$$A_{E2} = \frac{c\alpha k^5}{15(2J' + 1)} \left| \langle i || r^2 C^{(2)} || f \rangle \right|^2 \quad (3.51)$$

We can use this, along with equations 3.37 and 3.36 to write the Rabi frequency:

$$\Omega = \left| \frac{eE_0}{2\hbar} \sqrt{\frac{15(2J' + 1)A_{E2}}{c\alpha k^3}} \sum_{q=-2}^2 \begin{pmatrix} J & 2 & J' \\ -m_J & q & m_J' \end{pmatrix} c_{ij}^{(q)} \epsilon_i k_j \right| \quad (3.52)$$

The electric field amplitude E_0 of a beam with waist w_0 and power P is given by $E_0 = \frac{2}{w_0} \sqrt{\frac{P}{\epsilon_0 \pi c}}$, so we can plug in this expression to get Ω in only terms of parameters we use in the lab:

$$\Omega = \left| \frac{e}{w_0 c \hbar} \sqrt{\frac{15(2J' + 1)A_{E2}P}{\epsilon_0 \pi c \alpha k^3}} \sum_{q=-2}^2 \begin{pmatrix} J & 2 & J' \\ -m_J & q & m_J' \end{pmatrix} c_{ij}^{(q)} \epsilon_i k_j \right| \quad (3.53)$$

All of the geometric considerations are taken care of in the sum over q . Often in experiments we want to drive specific Δm_J transitions, and we can use this geometric factor to maximize the Rabi frequency of the specific transitions while suppressing others. The value $|c_{ij}^{(q)} \epsilon_i k_j|$ as a function of the angles γ and ϕ are plotted in figure 3.1. Two configurations are used in our work. The most common configuration is to select $\Delta m_J = \pm 2$ transitions and suppress the others by setting $\hat{k} \perp \hat{B}, \hat{\epsilon} \perp \hat{B}$ ($\gamma = \phi = \frac{\pi}{2}$). We also will want to drive $\Delta m_J = \pm 1$ transitions, so we can set $\hat{k} \perp \hat{B}, \hat{\epsilon} \parallel \hat{B}$ ($\gamma = 0, \phi = \frac{\pi}{2}$)

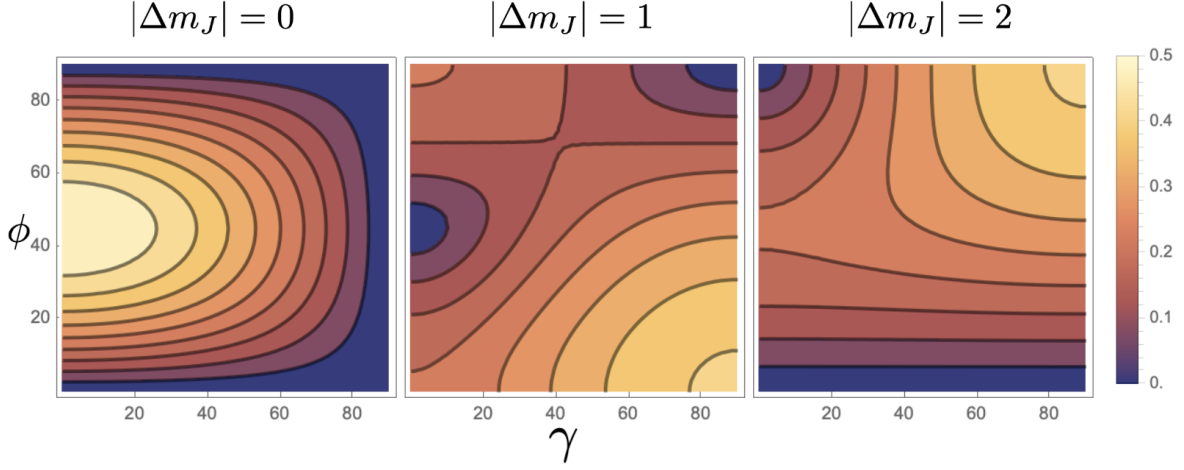


Figure 3.1: Plot of relative transition strengths for $|\Delta m_J| = 0, 1$, and 2 electric quadrupole transitions as a function of angle γ (the angle between the polarization vector $\hat{\epsilon}$ and the magnetic field $\vec{B}$), and ϕ (the angle between the magnetic field $\vec{B}$ and the laser's $\vec{k}$ vector).

3.5 Magnetic Dipole Transitions

For operating the hyperfine qubit in $^{171}\text{Yb}^+$, we rely on driving a magnetic dipole (M1) transition with an applied oscillating magnetic field. The perturbing Hamiltonian which couples the oscillating magnetic field with the magnetic dipole moment between the states is $H(t) = -\vec{\mu} \cdot \vec{B}(t, w)$, with $\vec{\mu}$ being the magnetic dipole moment of the transition, and $\vec{B}(t, w) = \vec{B} \cos(\vec{k} \cdot \vec{r} - wt + \phi)$ the applied magnetic field. We will assume $\vec{\mu} \cdot \vec{B} = \mu B$ for simplicity.

We are interested in investigating the time-dependence of this perturbing Hamiltonian on a two-level system with energy difference w_0 : $H_0 = -\frac{w_0}{2} |0\rangle \langle 0| + \frac{w_0}{2} |1\rangle \langle 1|$. We will take the dipole approximation, such that $\vec{k} \cdot \vec{r} \approx 0$, and additionally, we will preemptively take the rotating wave approximation. We can define the Rabi frequency as $\Omega = \mu B$, and set $\hbar = 1$ so we can write the full Hamiltonian as:

$$H = \begin{pmatrix} -\frac{w_0}{2} & -\frac{\Omega}{2} e^{-i(wt+\phi)} \\ -\frac{\Omega}{2} e^{i(wt+\phi)} & \frac{w_0}{2} \end{pmatrix} \quad (3.54)$$

We can go into the interaction picture with respect to the perturbing radiation by applying the transformation: $H_R = R^\dagger H R - iR^\dagger \frac{\partial R}{\partial t}$. The matrix R is defined as:

$$\begin{pmatrix} e^{i\omega t/2} & 0 \\ 0 & e^{-i\omega t/2} \end{pmatrix} \quad (3.55)$$

Applying this transformation to our Hamiltonian, defining a detuning parameter as $\delta = \omega_0 - \omega$, and factoring out a new parameter called the “generalized Rabi frequency” ($\Omega' = \sqrt{\delta^2 + \Omega^2}$), we arrive at our time-independent Hamiltonian:

$$H_R = \frac{\Omega'}{2} \begin{pmatrix} -\frac{\delta}{\Omega'} & -\frac{\Omega}{\Omega'} e^{-i\phi} \\ -\frac{\Omega}{\Omega'} e^{i\phi} & \frac{\delta}{\Omega'} \end{pmatrix} \quad (3.56)$$

What we want is the time evolution operator corresponding to this Hamiltonian: $U_R = e^{-iH_R t}$, as we want to be able to evolve any arbitrary initial state given any detuning and Rabi frequency. Luckily, we can decompose our Hamiltonian into a sum of Pauli matrices:

$$H_R = \frac{\Omega'}{2} \left(-\frac{\delta}{\Omega'} \hat{\sigma}_z - \frac{\Omega}{\Omega'} (\cos(\phi) \hat{\sigma}_x + \sin(\phi) \hat{\sigma}_y) \right) \quad (3.57)$$

This allows us to easily apply Euler’s formula for quaternions in order to find an expression for U_R : $e^{i\theta(\hat{n} \cdot \vec{\sigma})} = I \cos(\theta) + i(\hat{n} \cdot \vec{\sigma}) \sin(\theta)$. Applying this formula brings us to the time evolution operator for our two level system under interaction with radiation:

$$U_R(\Omega, \delta, \phi, t) = \begin{pmatrix} \cos(\frac{\Omega'}{2} t) + i \frac{\delta}{\Omega'} \sin(\frac{\Omega'}{2} t) & i \frac{\Omega}{\Omega'} e^{-i\phi} \sin(\frac{\Omega'}{2} t) \\ i \frac{\Omega}{\Omega'} e^{i\phi} \sin(\frac{\Omega'}{2} t) & \cos(\frac{\Omega'}{2} t) - i \frac{\delta}{\Omega'} \sin(\frac{\Omega'}{2} t) \end{pmatrix} \quad (3.58)$$

This is an incredibly powerful result as it lets us perform any rotation we wish as a function of Ω, δ, ϕ , and t . If we want to perform multiple pulses, we can simply concatenate multiple instances of this operator with different parameters per pulse.

3.6 Magnetic Dipole Decays

Fermi’s golden rule for M1 transitions is written as:

$$A_{M1} = \frac{\omega^3}{3\pi\epsilon_0\hbar c^5} \frac{|\langle n' J' L' S' | \vec{\mu} | n J L S \rangle|^2}{2J' + 1} \quad (3.59)$$

The primed quantities refer to the excited state. The magnetic dipole operator is: $\vec{\mu} = \mu_B(\vec{L} + g_S\vec{S})$. In the LS basis, evaluating this matrix element is rather straightforward. We will go through a quick example for the ${}^2D_{5/2}$ to ${}^2D_{3/2}$ transition in Yb^+ , as it will be relevant later. For this transition, we have $n' = n$, $L' = L$, and $S' = S$. The $\vec{L}$ operator only acts on the L basis, and $\vec{S}$ only on the S basis. So we have two matrix elements to evaluate:

$$\langle J'LS | \vec{L} | JLS \rangle = \sqrt{(2J+1)(2J'+1)}\sqrt{(2L+1)(L+1)}\bar{L}(-1)^J \begin{Bmatrix} L & J & S \\ J' & L & 1 \end{Bmatrix} \quad (3.60)$$

$$\langle J'LS | \vec{S} | JLS \rangle = \sqrt{(2J+1)(2J'+1)}\sqrt{(2S+1)(S+1)}\bar{S}(-1)^{J'} \begin{Bmatrix} S & J & L \\ J' & S & 1 \end{Bmatrix} \quad (3.61)$$

For the M1 decay from ${}^2D_{5/2}$ to ${}^2D_{3/2}$ in Yb^+ , we find:

$$|\langle n'J'L'S' | \vec{\mu} | nJLS \rangle|^2 = \frac{12}{5}\mu_B^2(g_S - 1)^2 \quad (3.62)$$

We arrive at the expression for the A coefficient for the ${}^2D_{5/2}$ to ${}^2D_{3/2}$ transition:

$$A_{M1} = \frac{2\omega^3\mu_B^2(g_S - 1)^2}{12\pi\epsilon_0\hbar c^5} \quad (3.63)$$

and plugging in the value for the energy between the states, we get: $A_{M1} = 2\pi \times 4.5\text{mHz}$.

It also it important to consider M1 decays between states of different coupling schemes, a discussion of which is given in section 8.2.

CHAPTER 4

Ytterbium Ions and Our Experiment

Ytterbium is element 70, making it one of the heaviest of the trapped ions. There are seven stable isotopes of ytterbium: ^{168}Yb , ^{170}Yb , ^{171}Yb , ^{172}Yb , ^{173}Yb , ^{174}Yb , and ^{176}Yb . Many modern atomic physics experiments use ytterbium as their atom of choice, including neutral atom arrays [MBL22], searches for new physics and precision measurement [HAC22] [LPH21], and trapped ion quantum computing [MBA23].

4.1 Photoionization of Neutral Yb

Our atomic oven produces natural abundance neutral Yb, but the goal is to trap Yb^+ , so we must remove one electron from neutral Yb in the trapping region. In order to perform photoionization with a single laser, we would need light at 198nm since the ionization threshold is at 50443 cm^{-1} [JBE11]. This would be a vacuum ultra-violet laser, so instead we choose to perform a $1 + 1$ resonance-enhanced multiphoton ionization (REMPI) scheme. This is diagrammatically shown in figure 4.1. We first excite neutral Yb atoms with the strong $^1\text{S}_0 \rightarrow ^1\text{P}_1^o$ transition ($\tau_{^1\text{P}_1^o} = 5.4\text{ns}$) [TKH04]. To promote the electron to the continuum, we require a second photon with wavelength less than 394nm. We don't use a 394nm laser in our lab, but for cooling Yb^+ we use a 369nm laser, so we can complete the photoionization with our Doppler cooling light [JBE11]. This means our second ionization step is not resonant, so our scheme is less efficient than if we used a 394nm laser, but still sufficient for loading single ions quickly (≈ 3 minutes).

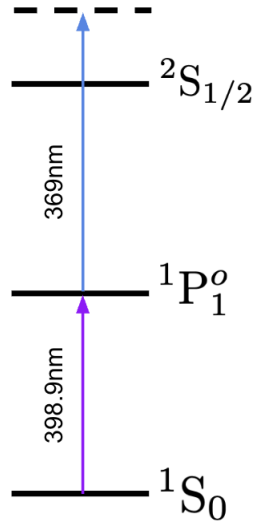


Figure 4.1: Our 1+1 REMPI scheme for Yb. The first step is exciting neutral Yb atoms with 399nm light on the strong $1S_0 \rightarrow 1P_1^o$ transition. Then light at 369nm is used to promote the excited electron to the continuum, leaving behind a Yb ion in the $2S_{1/2}$ state.

Whenever we want to load ions into our trap, we have a specific target isotope. The atomic oven is natural abundance, so all isotopes are present in the atomic beam. Luckily, the first step of our 1+1 REMPI process is resonant, so we can select which isotope we want to load by tuning the frequency of our 399nm laser to the target frequency. However, our atomic oven (see section 2.4.2) produces a beam of hot Yb atoms aimed at the trapping region, so there is considerable Doppler broadening in the 399nm spectrum if the laser has a projection onto the velocity of the atoms. To reduce the Doppler broadening of the 399nm transition, we take $\vec{k}_{399nm} \perp \vec{v}_{Yb}$, giving us the ability to load specific isotopes. For measurements of the 399nm spectrum in neutral Yb, see [KGB16], and for the natural abundances of the isotopes of Yb, see table 4.1 ([LBB03]).

Isotope	Mole Fraction	Nuclear Spin
^{168}Yb	0.0013(1)	0
^{170}Yb	0.0304(15)	0
^{171}Yb	0.1428(57)	1/2
^{172}Yb	0.2183(67)	0
^{173}Yb	0.1613(27)	5/2
^{174}Yb	0.3183(92)	0
^{176}Yb	0.1276(41)	0

Table 4.1: Mole Fractions and nuclear spin of each stable isotope of Yb [LBB03].

4.2 Structure of Yb^+

The level structure of the spin 0 isotopes of Yb^+ can be seen in figure 4.2. The Yb^+ ground state electron configuration is $4f^{14}6s\ ^2\text{S}_{1/2}$, meaning there is a single electron in the valence $6s$ shell, above a filled $4f$ shell. Largely one can ignore the electrons in the $4f$ shell, as Doppler cooling, state preparation, and readout are all mostly accomplished by just moving the $6s$ electron. However, it is relatively easy to cause one of the $4f$ electrons to become excited, making states with 2 excited electrons. An additional difficulty comes from the fact that when an electron leaves the $4f$ shell, an angular momentum “hole” is left behind, interacting with the 2 excited electrons. A table of the relevant states, their lifetimes, and their branching ratios to different states is provided in table 4.2.

One special state to make note of is the first excited state in Yb^+ , the $4f^{13}6s^2\ ^2\text{F}_{7/2}^o$ state. The transition between the ground $^2\text{S}_{1/2}$ and this $^2\text{F}_{7/2}^o$ state is an electric octupole (E3) transition, as it involves a change in orbital angular momentum $\Delta L = 3$. This state is incredibly long lived (1.5 years) [LPH21], making it essentially the “second ground state” of the ion. This state is occasionally populated while Doppler cooling (via collisions [SDN09]), necessitating a method to rescue population from this state. Many methods of depopulating

State	E (cm ⁻¹)	Lifetime	Decay Path (Branching Ratio)
$^2F_{7/2}^o$	21418.75	1.58(8)yr [LPH21]	$^2S_{1/2}(1)$
$^2D_{3/2}$	22960.80	54.83(18)ms [SYM23]	$^2S_{1/2}(1)$
$^2D_{5/2}$	24332.69	7.1(4)ms [TEM21]	$^2F_{7/2}^o(0.83)$, $^2S_{1/2}(0.17)$ [TEM21]
$^2P_{1/2}^o$	27061.82	8.12(2)ns [OHM09]	$^2S_{1/2}(0.995)$, $^2D_{3/2}(0.005)$ [OYM07]
$^3[3/2]_{1/2}^o$	33653.86	32.02ns*	$^2S_{1/2}(0.982)^*$, $^2D_{3/2}(0.018)^*$
$^1[3/2]_{3/2}^o$	34575.37	29.31ns*	$^2S_{1/2}(0.974)^*$, $^2D_{3/2}(0.004)^*$, $^2D_{5/2}(0.022)^*$

Table 4.2: Summary of relevant information (energy levels, total radiative lifetimes, and the decay pathways with branching ratios) for the commonly used states in our work. Only E1 decays are considered for decay pathways. Values with an asterisk (*) refer to calculated values from the DREAM database [QP20]. Energy levels are from measurements made in [Meg67].

this state have been explored ([Sug99]). However given it's long lifetime, the $^2F_{7/2}^o$ state is a haven to store quantum information, and can be an excellent subject for new spectroscopy from the state.

	J_1K Coupling	J_1J_2 Coupling
First Step	$\vec{J}_c = \vec{L}_c + \vec{S}_c$	$\vec{J}_c = \vec{L}_c + \vec{S}_c$
Second Step	$\vec{K} = \vec{J}_c + \vec{L}_o$	$\vec{J}_o = \vec{L}_o + \vec{S}_o$
Final J	$\vec{J} = \vec{K} + \vec{S}_o$	$\vec{J} = \vec{J}_c + \vec{J}_o$
Term	$^{2s_o+1}[K]_J$	$(J_c, J_o)_J$

Table 4.3: Coupling orders and term symbols for J_1K and J_1J_2 coupling schemes.

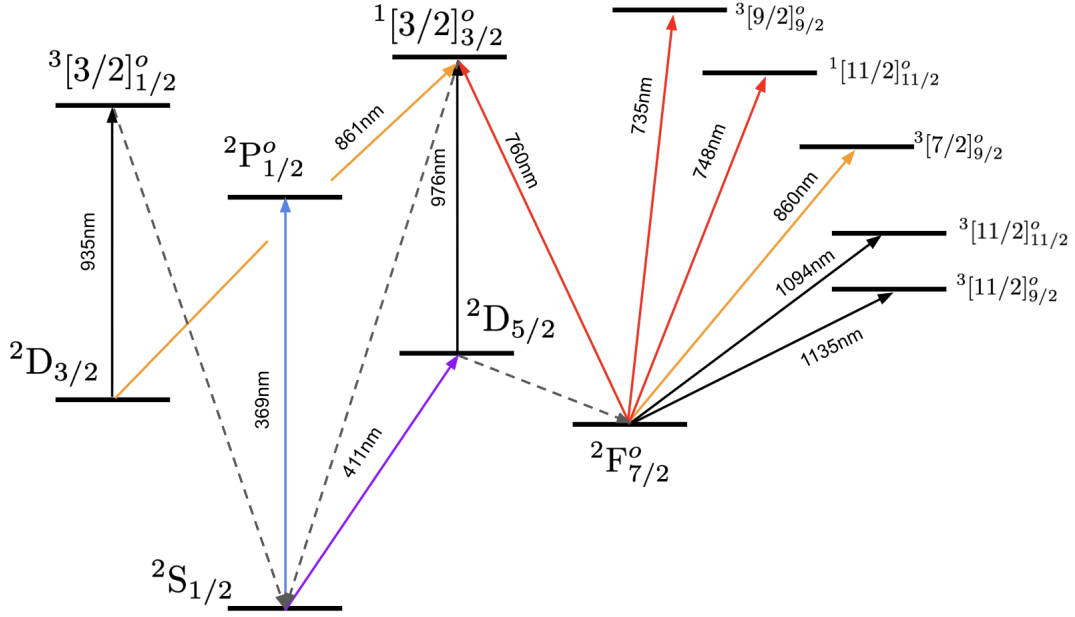


Figure 4.2: Energy level diagram of relevant states in Yb^+ . Laser driven transitions are shown as solid lines between states, and select E1 decays are shown as dashed lines.

4.2.1 J_1K and J_1J_2 Coupled States in Yb^+

In Yb^+ , many states have two excited electrons, which causes the normal Russell-Saunders (LS) coupling scheme to be ineffective. In order to find effective term symbols to use, a different coupling scheme is used. The two schemes used are the J_1K coupling scheme, and the J_1J_2 coupling scheme. The J_1K scheme leads to states with term symbols that look like: $^{2S_2+1}[K]_J$, and the J_1J_2 scheme leads to terms $(J_1, J_2)_J$. For the J_1K scheme, the angular momenta of the electrons are coupled in the orders shown in table 4.3

Where for both schemes the subscript c denotes the core electrons, and o denotes the orbital electrons. Note that the “1” subscript is used sometimes to refer to the core electrons, and “2” used for the outer electrons. Transitions between the typical LS states and the other coupling schemes are possible, and commonly used for several operations in Yb^+ .

A good rule of thumb when evaluating transitions between these coupling schemes is to consider the electron configurations of the initial and final states, and keep in mind that

J and parity are still good quantum numbers for both coupling schemes. For example, we can consider an electric dipole (E1) transition between the $4f^{14}5d\ ^2D_{3/2}$ and the $4f^{13}5d6s\ ^3[3/2]_{1/2}^o$ state. We can see that $\Delta J = 1$, and parity changes, so generally this transition will behave similarly to a standard E1 transition. However, its transition moment is much weaker than that of a standard E1 transition, because of the large angular momentum change that the individual electrons undergo. In addition, we can consider other types of transitions, such as the $4f^{13}6s^2\ (^2F_{7/2}^o)$ state to the $4f^{13}5d6s\ (^1[3/2]_{3/2}^o)$ state. We can see that there is no parity change, and $\Delta J = 2$, making this an electric quadrupole (E2) transition.

4.3 Hyperfine Structure in $^{171}\text{Yb}^+$

As discussed in section 3.1, the presence of nuclear spin will cause a splitting of energy levels. In $^{171}\text{Yb}^+$, $I = 1/2$, meaning each state will split into two different hyperfine manifolds $F = |J - 1/2|, |J + 1/2|$. Since $I = 1/2$, we are only concerned with the magnetic dipole hyperfine interaction, meaning our perturbing Hamiltonian can be expressed simply by equation 3.21, only requiring the hyperfine A coefficient to describe the hyperfine splitting. A collection of hyperfine A coefficients for relevant states of $^{171}\text{Yb}^+$ is given in table 4.4. A discussion of the hyperfine structure of $^{173}\text{Yb}^+$ is given in chapter 5.

4.4 Cooling Yb^+

The primary transition used in Doppler cooling Yb^+ is the $^2S_{1/2} \rightarrow ^2P_{1/2}^o$ E1 transition at 369.5nm. The structure of Yb^+ contains some low-lying metastable ^2D_J states however (see table 4.2), and E1 selection rules allow transitions between the excited $^2P_{1/2}^o$ state to the $^2D_{3/2}$ state, with a branching ratio of 0.005 [OYM07]. The long lifetime (54.83(18)ms) [SYM23] of this $^2D_{3/2}$ states necessitates a “repump” laser in order to maintain the high photon scattering rate that is provided on the $^2S_{1/2} \rightarrow ^2P_{1/2}^o$ transition for optimal cooling

State	Hyperfine A (MHz)
$^2S_{1/2}$	12642.8121184690(8) [HXZ24]
$^2F_{7/2}^o$	905.1 [Rom21]
$^2D_{3/2}$	430(10) [OYM07]
$^2D_{5/2}$	-63.368(1) [TEM21]
$^2P_{1/2}^o$	2104.9(13) [MGH94]
$^3[3/2]_{1/2}^o$	-2209.5(11) [OYM07]
$^1[3/2]_{3/2}^o$	4431(2) [Rom21]

Table 4.4: Hyperfine A coefficients for relevant states in $^{171}\text{Yb}^+$.

and state detection.

The transition at 935nm from the $^2D_{3/2}$ state to the short-lived excited $^3[3/2]_{1/2}^o$ state is what we use to return population to the cooling cycle (see table 4.2 for branching ratios and lifetime). With the 369nm and 935nm light, we can continuously, rapidly cycle population on the $^2S_{1/2} \rightarrow ^2P_{1/2}^o$ transition.

An alternative repump scheme can be used, but it requires more laser tones. This repump scheme uses light at 861nm, coupling the $^2D_{5/2}$ state to the excited $^1[3/2]_{3/2}^o$ state. This state (as seen in table 4.2), decays to the ground state and the other metastable D_J state, the $^2D_{5/2}$. The $^2D_{5/2}$ has a lifetime of 7.1(4)ms [TEM21], meaning we also want to pump population out of this state. We can do this at 976nm, coupling it also to the $^1[3/2]_{3/2}^o$ state. However, population can decay from this excited state via an E2 process to the $^2F_{7/2}^o$ state, which lives for 1.5 years [LPH21], so finally repump light at 760nm coupling the $^2F_{7/2}^o$ state to the $^1[3/2]_{3/2}^o$ state must be used in order to fully cycle population to allow for cooling when using this scheme.

Other repump schemes may be used, specifically thanks to the work of [DRM24], in which several visible transitions from these metastable D_J states have been investigated.

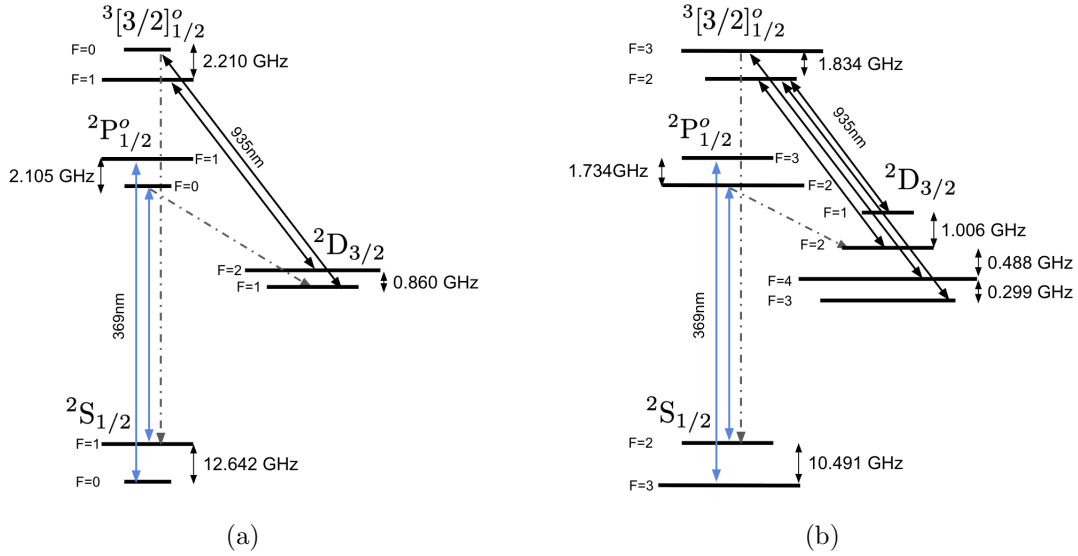


Figure 4.3: (a) Energy level diagram for Doppler cooling $^{171}\text{Yb}^+$ References for the hyperfine splittings may be found in table 4.4. (b) Energy level diagram for Doppler cooling $^{173}\text{Yb}^+$.

4.4.1 $^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$

Cooling both $^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$ (figure 4.3) is slightly more complicated due to the nuclear spin ($I = 1/2$ and $I = 5/2$ respectively). The nuclear spin couples to the angular momentum of the electrons causing a hyperfine splitting in each electronic level. If we simply tried to ignore the hyperfine splitting, population would become trapped in one of the unaddressed hyperfine levels, causing the ion to go dark, as the splittings lie far outside the natural linewidths of the E1 transitions that are commonly used for cooling. So in practice, this requires us to have additional laser tones. One approach is to incorporate a set of fully separate laser systems, but this would be costly and difficult to manage.

We employ resonant and broadband electro-optic modulators (EOMs) to generate the additional tones that we require. These devices use a birefringent crystal to which a time-dependent electric field is applied. The birefringence will change with the applied electric field, generating a time-dependent phase shift at the frequency of the applied field. This makes “sidebands” on the laser light at the frequency of the electric field. With a resonant

EOM, the applied frequency is static, whereas with broadband EOMs, the frequency can be highly variable, allowing for great tunability of the generated sidebands.

Two EOMs are required to cool $^{171}\text{Yb}^+$, one to cover the hyperfine splitting in the ground $^2\text{S}_{1/2}$ state, and one to cover the hyperfine splitting in the $^2\text{D}_{3/2}$ state. For $^{173}\text{Yb}^+$, an additional 935nm laser is used to increase the overall scattering rate, but strategically placed EOM tones can overcome the need for a second laser in order to trap and cool it. We will discuss $^{173}\text{Yb}^+$ further in chapter 5.

For cooling $^{171}\text{Yb}^+$, the Doppler cooling laser is tuned to address the $^2\text{S}_{1/2}, F = 1 \rightarrow ^2\text{P}_{1/2}^o, F = 0$ transition. This transition is chosen in order to prevent population from accumulating in the $^2\text{S}_{1/2}, F = 0$ state, as the $^2\text{P}_{1/2}^o, F = 0 \rightarrow ^2\text{S}_{1/2}, F = 0$ transition is forbidden by electric dipole selection rules when the hyperfine structure is considered. Due to the finite hyperfine splitting of the $^2\text{P}_{1/2}^o$ state however, the $^2\text{P}_{1/2}^o, F = 1$ manifold is populated off-resonantly by 369nm light, and the decay from the $^2\text{P}_{1/2}^o, F = 1$ manifold to the $^2\text{S}_{1/2}, F = 0$ manifold is allowed, thus pumping population dark. Our EOM is tuned to the sum of the hyperfine splittings of the $^2\text{S}_{1/2}$ and $^2\text{P}_{1/2}^o$ states (14.746GHz) in order to drive population back into the cooling cycle.

4.5 State Preparation and Measurement of $^{171}\text{Yb}^+$

State preparation and measurement (SPAM) of atomic qubit systems is rather species-specific, as the atomic structure determines the schemes used and the ultimate fidelity limits. We will discuss the SPAM process in $^{171}\text{Yb}^+$ in the “standard” scheme, where we do not employ electron shelving; that technique will be discussed in chapter 6.

The qubit commonly defined in $^{171}\text{Yb}^+$ is on the zero-field hyperfine clock states in the $^2\text{S}_{1/2}$ state. We define our “0” qubit state as $|0\rangle = |^2\text{S}_{1/2}, F = 0, m_F = 0\rangle$, and our “1” qubit state as $|1\rangle = |^2\text{S}_{1/2}, F = 1, m_F = 0\rangle$. Of course, since our $|1\rangle$ state is hosted in a hyperfine manifold with $F = 1$, this means there are two other Zeeman states in this manifold with

$m_F = \pm 1$. In order to lift the degeneracy of these levels with our chosen qubit state, we apply a small magnetic field of 4G to split these levels by $\approx 5\text{MHz}$. This does mean that there is a magnetic field sensitivity introduced on the “clock” qubit in this instance, but it is small, and we do not experience adverse effects in our experiments.

The two qubit states are coupled together by a magnetic dipole moment. We can perform rotations between the two states by applying an oscillating magnetic field at a frequency equal to the energy difference between the levels (the hyperfine splitting of the $^2\text{S}_{1/2}$ state, given in table 4.4).

4.5.1 State Preparation

When working with the ground state qubit in $^{171}\text{Yb}^+$, the first step of any operation is to prepare a single quantum state. Luckily, atomic species with a strong transitions (such as those used for Doppler cooling), a $^2\text{S}_{1/2}$, and a spin $I = 1/2$ nucleus have an easy avenue for state preparation in the form of “optical pumping”. This scheme leverages the fact that the $^2\text{S}_{1/2}, F = 0$ manifold is well separated from the $F = 1$ manifold, meaning if we turn “off” the light needed to rescue population from the $F = 0$ manifold, population will accumulate in it. However, as discussed in section 4.4.1, this process is off-resonant, meaning it will be very slow.

Instead, we apply another laser tone with an EOM to purposefully drive the $^2\text{S}_{1/2}, F = 1$ manifold to the $^2\text{P}_{1/2}^o, F = 1$ manifold, while keeping off the light needed to rescue population from the $^2\text{S}_{1/2}, F = 0$ state. This will cause population to rapidly accumulate in the $^2\text{S}_{1/2}, F = 0$ state, our qubit $|0\rangle$ state. This EOM tone has a frequency equal to that of the $^2\text{P}_{1/2}^o$ hyperfine splitting (2.105GHz). During this process, we keep the 935nm light on in order to prevent any population becoming trapped in the $^2\text{D}_{3/2}$ state. The polarization of the the 369nm and 935nm lasers are set to transfer population from each available sublevel, as to not strand population in the $^2\text{S}_{1/2}, F = 1$ manifold, or in any level of the $^2\text{D}_{3/2}$ state. A diagram of optical pumping can be seen in figure 4.4a. This process can be very fast, taking

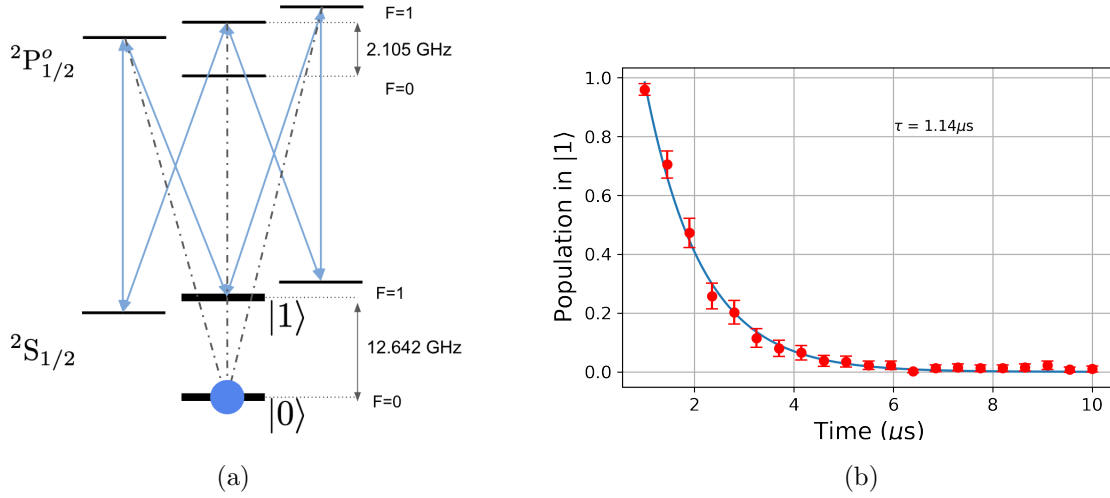


Figure 4.4: (a) Diagram for optical pumping of the ground state hyperfine qubit in $^{171}\text{Yb}^+$. References for the hyperfine splittings may be found in table 4.4. Light is applied resonant to the $2S_{1/2}, F = 1$ manifold to the $2P_{1/2}^o, F = 1$ manifold transition in order to “pump” population to the qubit $|0\rangle$ state. The presence of the $2D_{3/2}$ state and its repump scheme are omitted for simplicity. (b) Example optical pumping rate plot. Population in the $|1\rangle$ qubit state is measured after a variable time of optical pumping light. The time constant τ is extracted from a simple $e^{-t/\tau}$ fit model.

only a few microseconds. A plot of the population remaining in the $2S_{1/2}, F = 1$ manifold as a function of time of the applied optical pumping light can be seen in figure 4.4b.

4.5.2 Single Qubit Rotations

Single qubit rotations are performed with a microwave horn outside of the vacuum chamber pointed at the trap through the window. We use the applied microwaves to couple to the magnetic dipole moment of the transition between the qubit $|0\rangle$ and $|1\rangle$ states. The theory behind the coupling and the dynamics can be found in chapter 3. In summary, we may use the time evolution operator that we derived to perform arbitrary rotations on the Bloch sphere by applying radiation of varying time and phase (equation 3.58). In order to perform

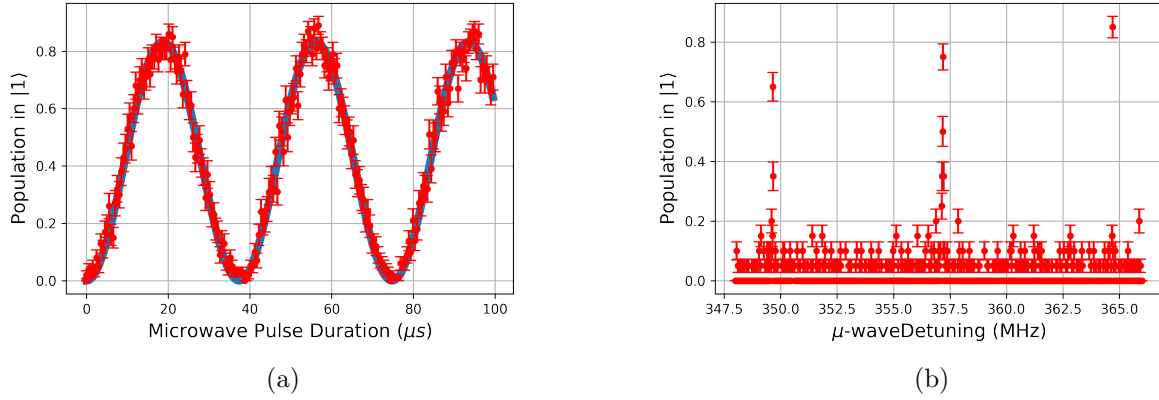


Figure 4.5: (a) Example of Rabi flopping on the ground state hyperfine qubit prior to optimization of experimental parameters. (b) Full range microwave linescan of the $|0\rangle$ qubit state to all states in the $^2\text{S}_{1/2}, F = 1$ manifold. From this scan, we can see the separation between the Zeeman sensitive transitions and the “clock” transition is about 7MHz. In practice, to better determine the qubit transition we perform narrower scans about the qubit transition.

a full rotation on the Bloch sphere (i.e. move all population from $|0\rangle$ to $|1\rangle$), we need to apply resonant radiation ($\delta = 0$, such that $\Omega' = \Omega$) of the same phase for a time $T_\pi = \pi/\Omega$. This is what’s referred to as the “ π -time” for the transition.

Due to experimental drifts, it’s important to calibrate the π -time in order to achieve good transfer probabilities. One can do this by performing a “Rabi flopping” experiment, where the applied microwave pulse is of static frequency and phase, but with a variable time. An example of Rabi flopping can be seen in figure 4.5. It’s also incredibly important to apply radiation of the correct frequency. This quantity can be calibrated by performing a “linescan” of the transition by applying fixed time and phase pulses, while varying the frequency of the pulse. An example of this can be seen in figure 4.5. The theory of prediction each of these processes can be achieved by applying the time evolution operator in equation 3.58.

More precise characterization of the qubit frequency may be achieved by performing

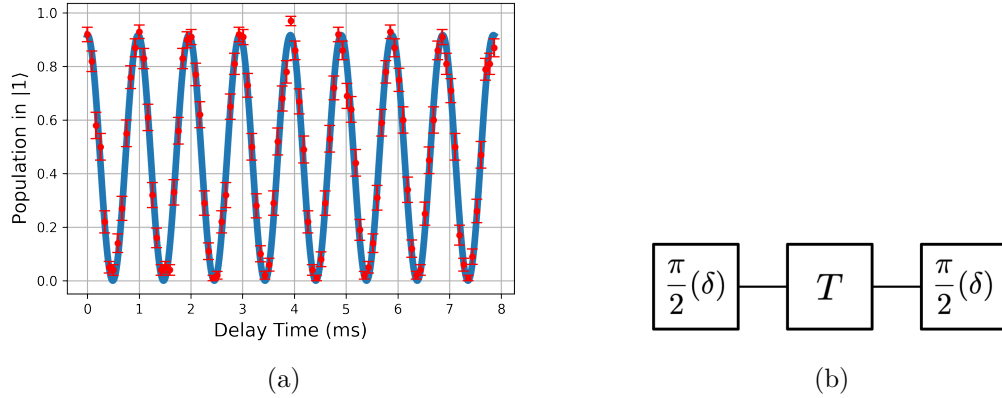


Figure 4.6: (a) Example Ramsey experiment. The fringe traces out an oscillation at the frequency of the detuning used in the $\pi/2$ pulses. (b) Simplified pulse sequence for Ramsey experiment in our lab. A frequency-detuned $\pi/2$ pulse is followed by a variable wait time, and an identical $\pi/2$ pulse is applied after the wait time.

a “Ramsey experiment”. There are many variations of the Ramsey experiment, but I’ll describe only the one we commonly use in the lab. We begin by performing a fixed phase, slightly off-resonant (for instance, $\delta = 1\text{kHz}$) $\pi/2$ pulse (meaning we end up near the equator of the Bloch sphere). Then, a delay time is introduced, where the qubit is allowed to freely evolve on the Bloch sphere for a variable time T . Then, an identical $\pi/2$ pulse is applied. As T changes, a fringe at the frequency of the detuning (δ) is traced out, allowing us to determine the frequency of the detuning precisely, thus giving us a precise measurement of the qubit frequency. A plot of this process is seen in figure 4.6a, along with a diagram of the pulse sequence for clarity.

Because the generalized Rabi frequency depends on the detuning, it’s important to have a good handle on the value of the qubit frequency in order to perform single qubit rotations well. In addition, knowing the Rabi frequency is incredibly important, as errors in timing will cause errors in the single qubit rotations. The frequency and timing are linked, often requiring multiple calibrations to iterate on one another to “tune up” the single qubit rota-

tions. In practice, this will entail performing a microwave linescan (figure 4.5b) to determine the frequency, then a Rabi experiment (figure 4.5a) to determine the optimal π -time, and then performing another microwave linescan or Ramsey experiment (figure 4.6a) to better determine the qubit frequency.

4.5.3 Measurement

We measure the qubit state by laser-induced fluorescence. We leverage the fact that the $^2P_{1/2}^o, F = 0$ state cannot decay to the qubit $|0\rangle$ state due to selection rules, and apply light resonant to the $^2S_{1/2}, F = 1$ manifold to $^2P_{1/2}^o, F = 0$ state and collect photons from the ion. As discussed in section 4.4.1, off-resonant coupling the the $^2P_{1/2}^o, F = 1$ manifold will cause decays to the $|0\rangle$ state, resulting in a “mixing” error, meaning we will misidentify the qubit as being in the $|0\rangle$ state when having been prepared in the $|1\rangle$ state. This process ultimately limits the fidelity of SPAM in the $^{171}\text{Yb}^+$ ground state qubit. Much effort has gone into optimizing timing, power, detuning, and polarization for the highest SPAM fidelity in $^{171}\text{Yb}^+$ possible, some good resources for optimizing the readout fidelity can be found in [ETH10], [NVG13]. The highest SPAM fidelity to date using this technique is given in [CCV19].

The 935nm laser still plays an important part in readout. Population still may be trapped in the $^2D_{3/2}$ state, causing a misidentification of the qubit state. Since photons are being scattered off of the $^2P_{1/2}^o, F = 0$ manifold, only the $^2D_{3/2}, F = 1$ manifold should become populated due to selection rules. This means our placement of the 935nm repump tone to the $^3[3/2]_{1/2}^o$ state is important. The selection rules for E1 transitions still apply to the bracket states, so if we drive the $^2D_{3/2}, F = 1 \rightarrow ^3[3/2]_{1/2}^o, F = 0$ transition, we can ensure that population will only return to the $^2S_{1/2}, F = 1$ manifold, which allows for further photons to be scattered without causing a qubit mixing error.

A diagram of the readout process is given in figure 4.7, along with a histogram showing the distribution of counts received from the ion after preparing the qubit $|0\rangle$ and $|1\rangle$ states.

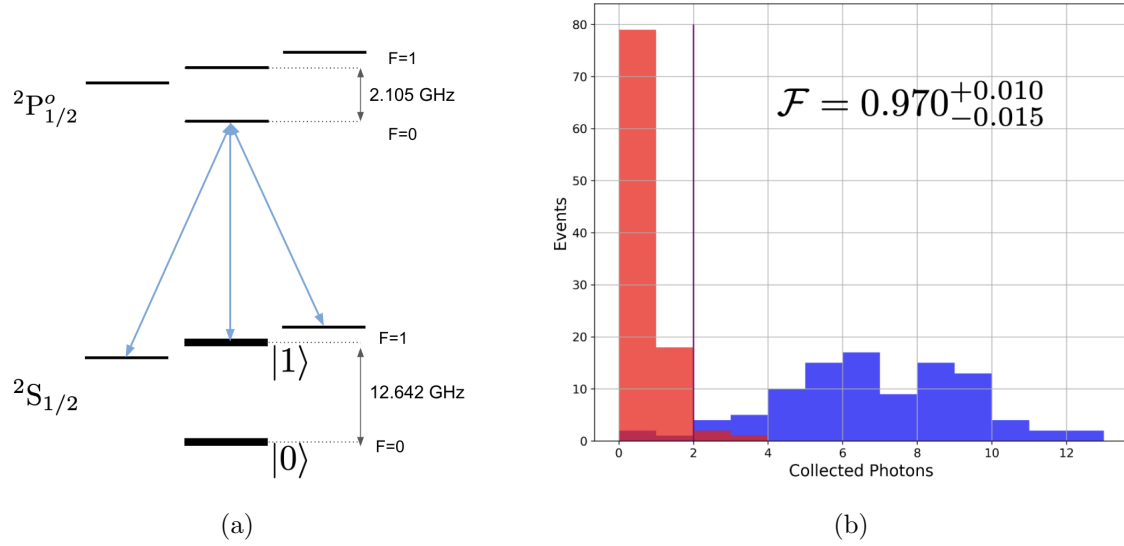


Figure 4.7: (a) Readout diagram for measuring the qubit state in $^{171}\text{Yb}^+$. Photons are scattered by the ion if the qubit is in state $|1\rangle$, whereas no photons will be scattered if the qubit was prepared in state $|0\rangle$. The presence of the $^2D_{3/2}$ state and its repump scheme are omitted for simplicity. (b) Histogram showing the number of photons collected in each experiment after preparing both the qubit $|0\rangle$ state (red), and $|1\rangle$ state (blue). The fidelity is presented with a 1σ Wilson confidence interval, discussed in section 4.5.3.1

4.5.3.1 Collection Statistics

When performing experiments, we need to metaphorically “draw a line” to determine if we count the ion as “bright” or “dark”, which we call the “discriminator”. The discriminator is simply the number of photons which will let us distinguish between a bright and dark ion. If we intended to prepare the ion in the bright state, but the counts fall below the discriminator, this is an error. Likewise, if we prepare the ion in the dark state and we retrieve more counts than the discriminator, it is an error.

One method of assigning error bars is a straightforward application of Gaussian statistics which only require the number of total experiments performed (N_t), and the number of “successful” experiments (N_s) (ones where the identification of the qubit state is correct). We can define the population as $p = N_s/N_t$, and then our confidence in the measurement of the population can be given as:

$$p \pm \sqrt{\frac{p(1-p)}{N_t}} \quad (4.1)$$

However, it becomes clear that when our population is near 0 or 1, this will yield errorbars which are unphysical (greater(less) than 1(0)). This is where the Wilson confidence interval is useful, as it gives a better result for confidence intervals near the extremes. We take the number of total attempts again (N_t), and the number of failures ($N_f = N_t - N_s$), and a desired number of standard deviations from the mean (z_α), and the interval is:

$$p_{\pm} = \frac{N_s + \frac{z_\alpha^2}{2}}{N_t + z_\alpha^2} \pm \frac{z_\alpha}{N_t + z_\alpha^2} \sqrt{\frac{N_s N_f}{N_t} + \frac{z_\alpha^2}{4}} \quad (4.2)$$

Values with these type of errors are reported as $\bar{p}_{\bar{p}-p_-}^{\bar{p}-p_+}$, such as in figure 4.7b, where $\bar{p}$ is the mean value of p .

4.6 Lasers and Light Collection

All but two of our lasers are located on a three-level optical table structure we refer to as “Duckberg”. This structure is fed temperature controlled air from vents in the building, and has great temperature stability. The top layer of Duckberg contains our 4 repump lasers: 935nm, 976nm, and two 760nm lasers. The middle layer holds our 369nm and 411nm lasers, along with a Rubidium vapor cell setup for wavemeter calibration. And the bottom layer holds a temperature stabilized optical cavity. All of the laser systems contained on Duckberg are built on (mostly) self-contained aluminum breadboards for modularity. Although the temperature stability on Duckberg is excellent, one main issue is its sensitivity to vibrations, due to the supports which hold each table. If more space is available in the future, I recommend relocating these lasers to a standard optical table to reduce sensitivity of the laser locks to vibrations.

4.6.1 Quad Cavity

The cavity spacer (ZERODUR) contains four through holes which each contain an optical cavity. Two of the mirrors are coated for red light, and two for blue light, as shown in figure 4.8. Each cavity is equipped with two piezos for frequency adjustment based on the length of the cavity. The cavity spacer is housed in a vacuum chamber and then placed inside two layers of shielding for temperature stability. The FSR of each cavity is $\approx 1.5\text{GHz}$. The cavities are all hemispherical, and the radius of curvature of each curved mirror is $r = 500\text{mm}$. For further temperature stabilization, Peltier coolers are used in conjunction with a temperature controller (Arroyo 5305) to maintain the temperature of the cavity at 18.5C . We have found the drift rate of the cavity to be $\approx 10\text{MHz}$ per day with temperature stabilization active. This means we need to do regular (2-3 times per day) recalibration of the laser frequencies. The design is strongly based off of designs from [Kum15] and [Ben08].

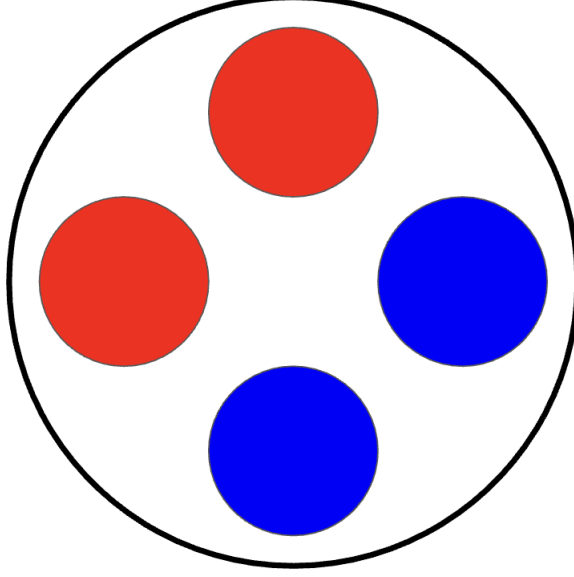


Figure 4.8: Layout of the quad cavity mirrors as viewed from the side *opposite* of the electronics (TEC, thermistors, piezo, and ion pump cable) output.

4.6.2 369nm laser

Our main 369nm light source is a MogLabs ECDL system. It is a fiber-out laser which directly goes to a double-pass AOM setup. The AOM in the double pass is a Gooch & Housego AOM tuned to a 200MHz center frequency. Double-passed light is directed toward a fiber-coupled EOM from AdvR to provide sidebands for qubit operations and cooling. We incorporate a double-pass AOM system in order to finely tune the frequency of this beam in software during our experiments. The non-double-passed light is directed toward an AOM from Brimrose whose center frequency is 110MHz. This light is used as a far red-detuned beam referred to as the “protection beam” to cool particularly hot ions when loading or after an experiment. A free-space output from the laser is used to couple light to a wavemeter for determining the laser frequency, and to the quad cavity for stabilization. This laser is used to address the $^2S_{1/2} \rightarrow ^2P_{1/2}^o$ transition at 369.5nm for Doppler cooling, state preparation,

and state detection.

Before the vacuum chamber, the double-passed light exits from a fiber and is focused into a pinhole to further clean up the spatial mode, and then allowed to expand before encountering a final lens, being focused into the trap at a waist ($1/e^2$ radius) of $\approx 50\mu m$.

4.6.3 411nm laser

The 411nm laser system is an ECDL from MogLabs. Just like the 369nm laser system, the light is directed toward a double-pass AOM setup. The double-pass AOM setup is identical to that of the 369nm system, with a 200 MHz Gooch & Housego AOM. We stabilize this laser to the quad cavity using the free-space output, and additionally monitor it's frequency on the wavemeter. This light is used to address the $^2S_{1/2} \rightarrow ^2D_{5/2}$ transition for shutting population to the metastable $^2F_{7/2}^o$ state.

4.6.4 935nm and 976nm lasers

Our 935nm and 976nm repump lasers are both fiber-coupled butterfly mounted DBR lasers from Thorlabs. Both systems are nearly identical. The light from each laser is sent through their respective AOMs. The 935nm light is sent through a 200MHz AOM, while the 976nm light is sent through a 320MHz AOM. Some of the light from each laser is sent to a wavemeter for monitoring and stabilization. The beams are combined on a PBS (polarizing beam-splitter), and then coupled into a single-mode fiber which goes to the trap.

Light at 935nm is used to address the $^2D_{3/2} \rightarrow ^3[3/2]_{1/2}^o$ transition in order to clear out stranded population while laser cooling. Light at 976nm is used to address the $^2D_{5/2} \rightarrow ^1[3/2]_{3/2}^o$ transition, for use in fast clear-out of the $^2F_{7/2}^o$ state, or employed in an alternative Doppler cooling scheme. This light at 976nm may also be used when considering narrow-band state preparation with 411nm light as well.

4.6.5 760nm lasers

We use two free space output 760nm DBR lasers from Photodigm for repumping population from the $^2F_{7/2}^o$ state. Both systems are identical. We use two optical isolators in each beam-path to protect from back-reflections. The light from each laser is sent through a 160MHz Brimrose AOM and then fiber-coupled to the experiment separately. We purposefully chose to have separate beampaths for polarization control over each beam at the trap. Some of the light from each laser is sent to a wavemeter for monitoring and stabilization.

4.6.6 399nm laser

We share a 399nm laser with the other trapped Yb^+ experiment in the group. It is a Toptica ECDL which is used for the first step of the photo-ionization process to excite neutral Yb atoms on the $^1S_0 \rightarrow ^1P_1^o$ transition. We use a simple shutter constructed from an old hard-drive to extinguish the light after an ion is detected in the trap, as to not photoionize more atoms than needed for our experiments.

4.6.7 Ti:Sapph laser

For the spectroscopy of the metastable states, we employ a SpectraPhysics Matisse Titanium Sapphire laser. We pump the Ti:Sapph cavity with a 15W SpectraPhysics Millennia laser. We have a 320MHz single-pass AOM system in order to incorporate power stabilization in the future. We send the first-order diffracted light into an 80MHz double-pass AOM setup. The light is then coupled into a fiber and brought to the experiment. Some of the Matisse light is sent through a free-space resonant EOM and then coupled into a fiber which brings light to the quad cavity for stabilization. This laser is fully controlled through the Matisse Commander software. We can lock this laser either to the wavemeter, or with the quad cavity. This laser is tuned to several wavelengths for general tune-up of our day-to-day operation. We have operated this laser at 976nm, 935nm, and 861nm for aligning the beam well to the

ion, as well as using it at 860nm, 748nm, and 735nm for spectroscopy of metastable states.

4.6.8 1108nm laser

We have used a tripled 1108nm light source to cool, trap, and manipulate our ions. This laser is a MogLabs cateye laser, which is fiber-coupled out. We send the light into a Cybel fiber amplifier where we can achieve a high power output. This light is then sent into a 3 axis Thorlabs fiber bench, where light is split and coupled into two different fiber-coupled harmonic generation modules from AdvR. The first module is a second harmonic generation (SHG) module, where 554nm light is created. This light is split and sent to an optical cavity and a saturated absorption spectroscopy setup. The optical cavity is stabilized to a transition in molecular Iodine by the SatAbs lock. The laser is then locked to the cavity. The second module is a combination second harmonic generator and a sum frequency generator (“third harmonic generator” or THG), where the 554nm light and the 1108nm light combine to achieve 369nm light. This light is then sent to a 369nm double-pass setup and subsequently to the trap to perform Doppler cooling of ions.

4.6.9 Imaging Systems

We employ 2 imaging systems for our ion trap. We have a platform above the chamber which holds a Hamamatsu (H10682-01) PMT, and below we have a camera imaging setup with a Thorlabs CC215MU camera. Both begin with Mitutoyo (0.4NA, f=10mm) infinitely corrected objectives (378-817-8), and are mounted onto 3-axis translation stages for adjustability. For the camera system, we place a 100mm lens about the focal length away from the camera. This leads to a magnification of 10x. We can visualize the trapped ions through the ThorCam software.

For the PMT imaging system, we use a 200mm lens after the infinitely corrected objective. We place an iris at the focal distance from the lens to remove as much background as possible,

and then place the PMT after the iris. Using scattering rate analysis, we find that we're able to collect 0.2% of the total photons emitted from a single $^{174}\text{Yb}^+$. The PMT output signal is sent to our ARTIQ experimental control system.

4.6.10 Wavemeter Calibration

We perform Doppler-free spectroscopy on a room-temperature Rubidium vapor cell in order to calibrate our High Finesse WS-U wavemeter periodically. By having counter-propagating pump and probe beams overlapped in the vapor cell, we address a single velocity class of atoms, eliminating the Doppler broadening of the atoms in the cell. This allows us to resolve hyperfine transitions, to which we can stabilize a laser. The transition frequencies of Rb are very well known in the literature, allowing us to accurately identify which transitions we lock the laser to. Once locked, we can use the light to calibrate the wavemeter through the wavemeter software.

The pump beam in our system is sent through a double-pass AOM operated at a frequency of 169.2MHz, with an added dither frequency for error-signal generation. The probe beam is unshifted, but will pick up the dither when interacting with the atoms which are pumped to the excited that by the pump beam. When we mix the probe beam signal with the dither frequency, we can generate the error signal. Because the two beams have an overall frequency difference between them, we are not addressing the zero velocity class of atoms in the cell. We are in fact addressing the class of atoms which have velocity -169.2MHz relative to the probe beam's $\hat{k}$. This means in our determination of the frequency, we must subtract this AOM shift from the literature frequency.

The line we lock to is the transitions from the $4p^65s$ ($^2\text{S}_{1/2}$) $F = 2$ manifold to the $4p^65p$ ($^2\text{P}_{3/2}$) $F = 3$ manifold in ^{87}Rb . The frequency separation of these states is 384.2281152THz [BSS99], [YSJ96]. Taking into account our AOM shift, we find the laser frequency which we calibrate the wavemeter is 384.227946THz. In day-to-day operation, we recalibrate the wavemeter every day in the morning.

4.7 Experimental Control

When working on the oblate ring trap experiment (section 2.3), we used an experimental control device called the “Pulser”, whose details can be found [PK15]. We interfaced with the Pulser and other devices through LabRAD, an asynchronous server/client system developed at UC Santa Barbara. Although this system worked well for us, we wanted a more “plug-and-play” solution for the new experiment, so when designing the experiment, we decided to use an ARTIQ (Advanced Real-Time Infrastructure for Quantum physics) control system.

4.7.1 Our ARTIQ Crate

ARTIQ hardware is largely modular, so when initially designing our crate, we opted for cards that would allow us to begin experiments quickly. With this in mind, we chose to include the cards shown in figure 4.9. The hardware we selected is as follows:

- 1125 Carrier Kasli-SOC
- 2118 BNC TTL (2 sets of 4 TTL channels, one set for input, one for output)
- 3x 4410 DDS Urukul (4 DDS channels, 1-400 MHz)
- 4457 HF Synthesizer Mirny + Almazny (4 channels, 53-12000 MHz)
- 5432 DAC Zotino (32 channels, $\pm 10V$)

Our TTL card is used for reading in PMT counts for monitoring the ion’s florescence and for qubit state readout, and also is used as output signals for activating a shutter for loading ions, and also as a trigger signal for a high-power microwave switch for qubit operations.

The DDS cards are used primarily for driving AOMs, but a channel is also used as a tunable frequency source for applying microwaves for qubit operations. The Mirny+Almazny card is essentially a high-frequency DDS card, and we use it for applying EOM tones to

produce sidebands on our laser light for Doppler cooling and state preparation for $^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$. Currently, the DAC card is not in use.

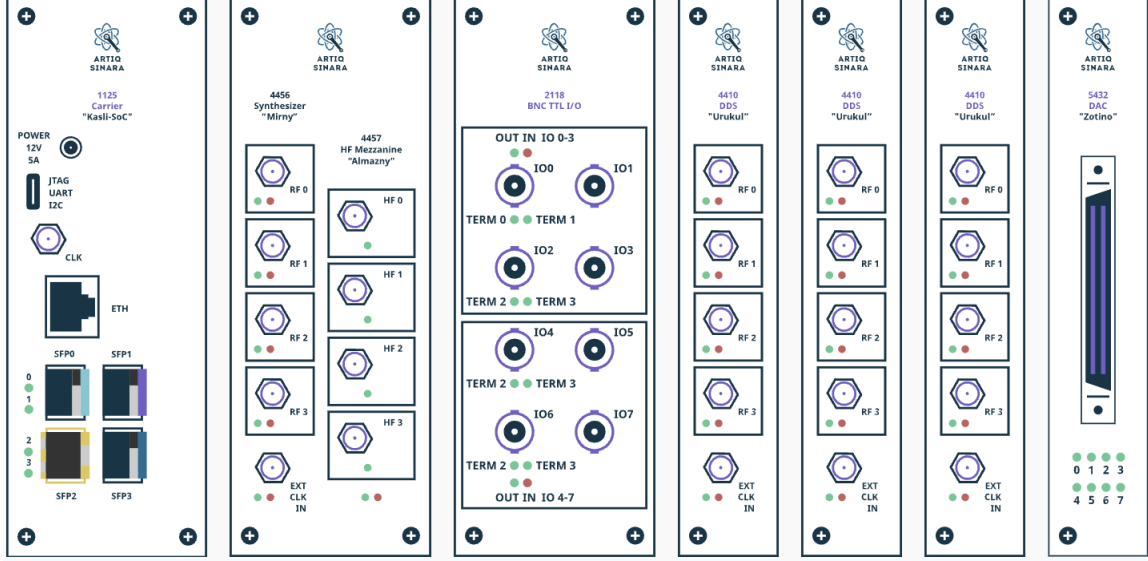


Figure 4.9: Layout of our ARTIQ crate. We opted to include 3 DDS cards for driving AOMs, a microwave generator for our EOM tones, a TTL card for sending/receiving signals, and a DAC card for application of DC voltages.

4.8 Microwave System

We perform single qubit operations in $^{171}\text{Yb}^+$ using microwaves at 12.6 GHz. Our microwave system is copied from the successful system used in the oblate ring trap. We begin with an HP signal generator operated at 13GHz (8672A). The output is sent through a high-power microwave switch (Kratos F192A), which can be activated by our ARTIQ TTL signal. We use this to ensure that there are no off-resonant microwaves at the ion when we want free-qubit evolution. We then amplify the signal going into a simple level 7 mixer, where we mix on a $\approx 400\text{MHz}$ tone to be resonant with the hyperfine splitting in the $^2\text{S}_{1/2}$ state in $^{171}\text{Yb}^+$. The mixed signal is then sent to a high-pass filter (HP 08672-60093), and a 12.2-12.7GHz band-pass filter (Pasternack PE87FL1004). We then amplify the microwave signal

with a high-power amplifier (MiniCircuits ZVE-3W-183+), whose output is sent through a directional coupler to monitor the frequency is needed (Krytar 1824). Finally, the signal is sent to a microwave horn waveguide (Pasternack PE9855B/SF-15) and directed at the trap through a vacuum chamber window.

CHAPTER 5

Spectroscopy of $^{173}\text{Yb}^+$

$^{173}\text{Yb}^+$ is the often overlooked isotope of naturally abundant Yb isotopes. The relatively high nuclear spin ($I = 5/2$) of $^{173}\text{Yb}^+$ causes cooling and state preparation to be more difficult (see section 4.4.1), thus limiting its use as a qubit ion. However, in recent years, $I > 1/2$ ions have been shown to have fast and efficient state preparation. $^{137}\text{Ba}^+$ ($I = 3/2$) has been used to achieve high fidelity SPAM via a “cabinet shelving” scheme [ARS22]. In addition to its potential as a qubit ion, the nucleus of ^{173}Yb is the subject of a rather large measurement discrepancy. This discrepancy can be resolved by measuring the hyperfine structure in the $^2\text{F}_{7/2}^o$ state in $^{173}\text{Yb}^+$ [XLC20]. And even further, the octupole transition at 467nm in $^{173}\text{Yb}^+$ is attractive for operating an optical clock, as the nuclear spin will cause the $^2\text{F}_{7/2}^o$ state to mix with the $^2\text{P}_{3/2}^o$ state, leading to lifetime quenching to lifetimes on the order of days, rather than years for the other isotopes of Yb^+ [DF16].

5.1 Trapping and Cooling $^{173}\text{Yb}^+$

Ultimately, we want to perform single-ion spectroscopy of transitions in $^{173}\text{Yb}^+$. In order to do this we need to load and directly laser cool $^{173}\text{Yb}^+$, which prior to our work had not been done. However, the hyperfine splittings of the $^2\text{S}_{1/2}$ and the $^2\text{P}_{1/2}^o$ states were determined by other experiments ([MBG87] and [MGH94] respectively). This simplifies our task considerably, as we can strategically place laser tones with an EOM to match the hyperfine splittings on the main cooling transition.

Additionally, theoretical estimates for the hyperfine structure of the $^2D_{3/2}$, $^2D_{5/2}$, and the $^2F_{7/2}^o$ states have been made ([Ita06], [XLC20]), and in addition, isotope shift spectroscopy was available [MGH94] for the centroid of the 369nm cooling transition, meaning the absolute frequencies of the required tones could be estimated.

5.1.1 Initial Scheme: Sidebands Only

Initially, we operated in a “sidebands only” regime for cooling and trapping, where we set our 369nm laser frequency to be detuned by $+1/2$ of the hyperfine splitting of the $^2S_{1/2}$ ground state with respect to the $^2S_{1/2}, F = 2 \rightarrow ^2P_{1/2}^o, F = 3$ transition. An EOM was used to provide sidebands on the laser which were at $1/2$ the $^2S_{1/2}$ hyperfine splitting, operated at a power that maximized the power in the first sidebands. This means we have two primary laser tones, resonant with the $^2S_{1/2}, F = 2 \rightarrow ^2P_{1/2}^o, F = 3$ and $^2S_{1/2}, F = 3 \rightarrow ^2P_{1/2}^o, F = 3$ transitions. This means both ground-state hyperfine manifolds were addressed, so optical cycling on the $^2S_{1/2} \rightarrow ^2P_{1/2}^o$ can occur.

The next challenge however is the required repump light at 935nm. The $^2P_{1/2}^o, F = 3$ manifold may decay to three manifolds in the $^2D_{3/2}$ state (the $F = 2$, $F = 3$, and $F = 4$ manifolds). These manifolds are calculated to be closest in energy to one another, so it will be easier to cool in this configuration with less knowledge of the hyperfine structure in the $^2D_{3/2}$ state. This necessitates some knowledge of the hyperfine splitting in the $^2D_{3/2}$ state *and* in the excited $^3[3/2]_{1/2}^o$ state. There is a theoretical estimate for the hyperfine structure in the $^2D_{3/2}$ state from [Ita06], but to get an estimate of the hyperfine splitting in the $^3[3/2]_{1/2}^o$ state we will have to be a bit clever.

Since the $J = 1/2$ state has no contribution from the hyperfine B or higher coefficients, we can use existing measured hyperfine A coefficients to estimate the magnetic dipole hyperfine coefficient (A) of the $^3[3/2]_{1/2}^o$ state. We can take the ratio:

$$A^{173}(^3[3/2]_{1/2}^o) = \frac{A^{173}(^2S_{1/2}) \times A^{171}(^3[3/2]_{1/2}^o)}{A^{171}(^2S_{1/2})} \quad (5.1)$$

Applying this equation, we find an estimate of the hyperfine coefficient for the $^3[3/2]_{1/2}^0$ state to be $A^{173}(^3[3/2]_{1/2}^0) = 608\text{MHz}$. This leads to an overall hyperfine splitting of 1.82GHz . We can see a diagram of this scheme in figure 5.1.

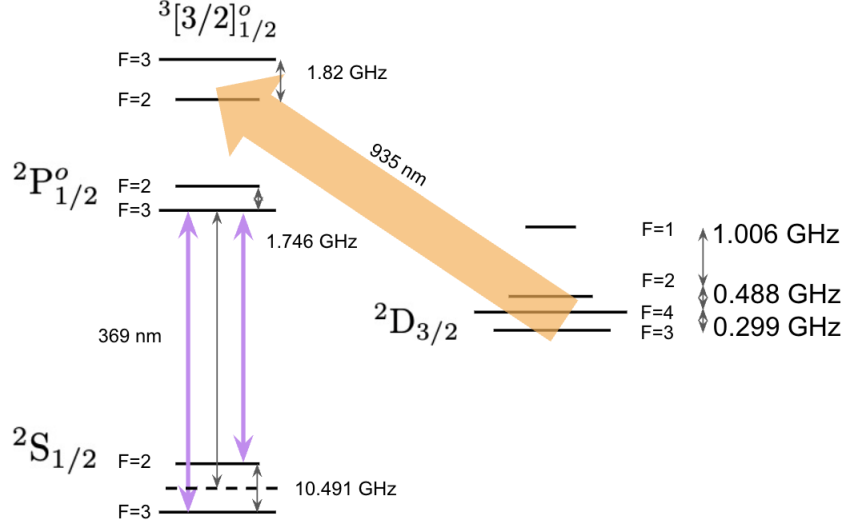


Figure 5.1: Diagram of applied laser tones for our initial trapping of $^{173}\text{Yb}^+$. We apply symmetric sidebands at $1/2$ the hyperfine splitting of the ground state with the laser detuned symmetrically between the hyperfine levels in the ground state, addressing the $F = 3$ manifold in the $^2P_{1/2}^o$ state. A broad 935nm laser tone is applied to address the majority of the hyperfine manifolds in the $^2D_{3/2}$ state.

5.1.2 Current Scheme: Carrier and Sideband

After performing some additional spectroscopy on the $^2D_{3/2} \rightarrow ^3[3/2]_{1/2}^0$ transition (discussed below in section 5.2.1), we decided to swap cooling schemes to allow for attempts at manifold preparation in an effort to begin microwave spectroscopy in the ground $^2S_{1/2}$ state.

This scheme is rather simple, we just move the laser carrier to be resonant with the $^2S_{1/2}, F = 3 \rightarrow ^2P_{1/2}^o, F = 3$. Then, an EOM tone is applied at the hyperfine splitting of the ground state to address the $^2S_{1/2}, F = 2 \rightarrow ^2P_{1/2}^o, F = 3$ transition. When we want to

prepare a single manifold, we then turn off the EOM tone, and population will be pumped into the $^2S_{1/2}, F = 2$ manifold.

Of course, if we wanted to prepare the $F = 3$ manifold in the $^2S_{1/2}$ state, we could move the laser center frequency to address the $F = 2$ manifold in the ground state, and use the EOM tone to repump from the $F = 3$ manifold instead. Either way, this scheme will allow us to prepare a manifold in the $^2S_{1/2}$ state. We chose to prepare the $^2S_{1/2}, F = 2$ manifold as it has fewer Zeeman sublevels.

5.2 Spectroscopy Results

We have performed initial spectroscopy of several transitions in the $^{173}\text{Yb}^+$. The results allow us to trap and cool the ion well, and allow us to cycle population to and from some of the hyperfine manifolds in the $^2F_{7/2}^o$ state. For the preliminary results of the hyperfine coefficients, please see the great analysis performed in [Del25]. The measured coefficients will not be presented here.

5.2.1 $^2D_{3/2} \rightarrow ^3[3/2]_{1/2}^o$ Spectroscopy

Our first spectroscopic task when trapping $^{173}\text{Yb}^+$ was to determine the transition frequencies of the 935nm transition from the $^2D_{3/2}$ state to the $^3[3/2]_{1/2}^o$ state. Spectroscopy of the 935nm transition is very “friendly” in the sense that the ion fluorescence is highly dependent of the frequency of the 935nm light, because population will get optically pumped into unaddressed $^2D_{3/2}$ hyperfine manifolds. This means we can purposefully pump population into a “target” manifold by repumping all non-target $^2D_{3/2}$ manifolds, with the intent of re-pumping population from that hyperfine manifold with a separate laser.

For instance, if we are cooling with the 369nm laser addressing the $^2P_{1/2}^o, F = 2$ manifold, we will mainly populate the $^2D_{3/2}, F = 1, 2, 3$ manifolds. If we wish to perform spectroscopy out of the $^2D_{3/2}, F = 1$ manifold specifically, we can tune our 935nm laser and it’s EOM

tones to repump population from the $F = 2$ and $F = 3$ manifolds only, while *not* repumping population from the $F = 1$ manifold with that laser. We then take a second 935nm laser, and sweep the frequency over the resonance. As the frequency crosses the resonance, the ion will become bright as all population is able to be optically cycled. Scans of all of the 935nm transitions in $^{173}\text{Yb}^+$ can be seen in figure 5.2.

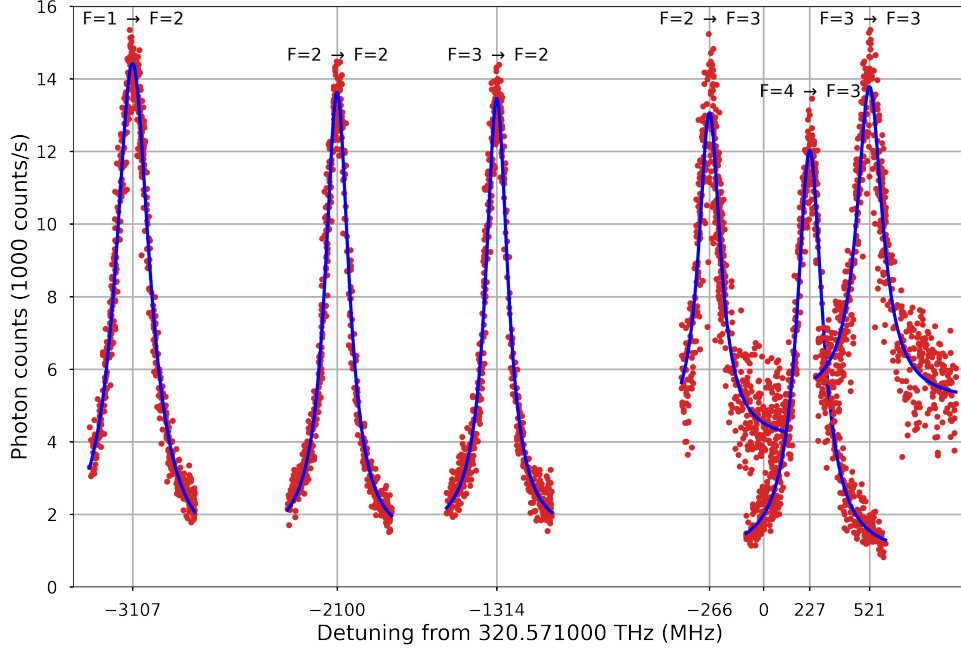


Figure 5.2: Linescans of the 935nm transitions in $^{173}\text{Yb}^+$. Each scan was fit to a separate Lorentzian lineshape function. Each scan is labeled as $F \rightarrow \mathcal{F}$, where F represents the hyperfine manifold in the $^2\text{D}_{3/2}$ state, and $\mathcal{F}$ represents the hyperfine manifold of the excited $^3[3/2]_o^o_{1/2}$ state. The 935nm laser had its frequency shifted upward by a 320MHz AOM for all of these measurements, but all measurements were taken with an uncalibrated wavemeter.

5.2.2 $^2S_{1/2} \rightarrow ^2D_{5/2}$ Spectroscopy

Performing spectroscopy on the 411nm transition in $^{173}\text{Yb}^+$ requires us to shuttle population into the very leaky $^2D_{5/2}$ state. Without an appropriate repump scheme from the $^2F_{7/2}^o$ state, a huge portion of observed quantum jumps to the $^2D_{5/2}$ state leads to population becoming stuck in the $^2F_{7/2}^o$ state.

For our initial spectroscopy, we used these quantum jumps to measure the frequency of the 411nm transitions. This led to a process that involved sending many ions to the $^2F_{7/2}^o$ state, only for them to never return to the cooling cycle and requiring the trap to be re-loaded. We used this technique to find all transitions from the $^2S_{1/2}, F = 2$ manifold to the $^2D_{5/2}, F = 0, 1, 2, 3, 4$ manifolds. Our results are shown in figure 5.3. We assigned hyperfine F levels to these transitions by comparing to theory given in [Ita06], and the $F = 0$ and $F = 1$ manifold assignments were confirmed when we were able to recover population from the $^2F_{7/2}^o$ state with 760nm light. We were ultimately not able to distinguish between driving to the $F = 3$ and the $F = 4$ manifolds, as our laser frequency resolution was not high enough.

5.2.3 $^2F_{7/2}^o \rightarrow ^1[3/2]_{3/2}^o$ Spectroscopy

To begin spectroscopy on the 760nm transition from the $^2F_{7/2}^o$ state to the $^1[3/2]_{3/2}^o$ state in $^{173}\text{Yb}^+$, we aimed to populate a single hyperfine manifold in the $^2F_{7/2}^o$ state. This is possible when driving population on the $^2S_{1/2}, F = 2 \rightarrow ^2D_{5/2}, F = 0$ transition, since decays from the $^2D_{5/2}, F = 0$ manifold to the $^2F_{7/2}^o$ state can only populate the $F = 1$ manifold. Luckily, since our results from the 411nm transition lined up well with theory, we had some confidence that we were able to drive specifically the $^2S_{1/2}, F = 2 \rightarrow ^2D_{5/2}, F = 0$ transition, meaning we can indeed populate the $^2F_{7/2}^o, F = 1$ manifold only.

From that point, we began to perform 760nm spectroscopy by a binary swept search method. This involved setting a frequency range $\mathcal{R}$ to sweep the laser frequency over, and divide that range into two pieces ($\mathcal{R}_1$ and $\mathcal{R}_2$). We sweep though $\mathcal{R}_1$, looking for ion

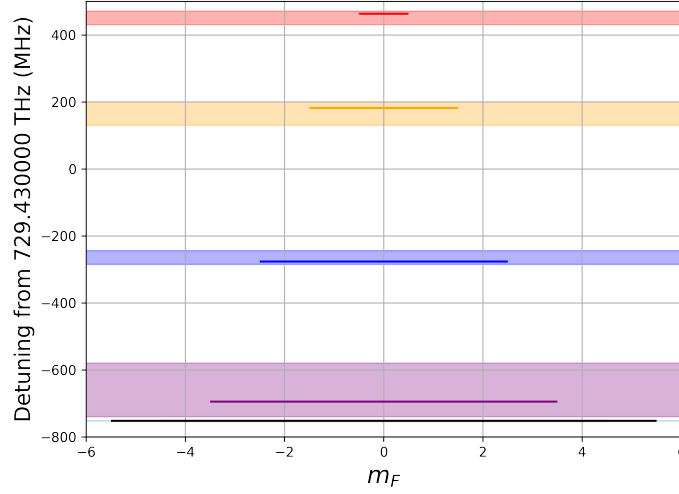


Figure 5.3: Results from quantum jump spectroscopy of the $^2S_{1/2} \rightarrow ^2D_{5/2}$ transition in $^{173}\text{Yb}^+$. Theoretical predictions are given by the solid, shorter lines, while the measured frequency interval is given by the lighter bars spanning the x-axis. The x-axis represents the number of m_F sublevels in the corresponding hyperfine manifold.

recoveries out of the $^2F_{7/2}^o$ state. If ion recoveries are found within $\mathcal{R}_1$, we further split $\mathcal{R}_1$ into $\mathcal{R}_{11}$ and $\mathcal{R}_{12}$, and continue in this manner until the frequency is found to be within error on our wavemeter.

With this method we were able to determine the frequency of the $^2F_{7/2}^o, F = 1 \rightarrow ^1[3/2]_{3/2}^o, F = 1, 2, 3$ transitions. Once these transitions were found, we began to iterate on this process, populating two hyperfine manifolds in the $^2F_{7/2}^o$ state ($F = 1$ and $F = 2$), while repumping out of the $F = 1$ well using our results from the first round of spectroscopy. We used the same binary swept search to identify the $^2F_{7/2}^o, F = 2 \rightarrow ^1[3/2]_{3/2}^o, F = 1, 2, 3$ transitions. We were not able to locate the $^2F_{7/2}^o, F = 2 \rightarrow ^1[3/2]_{3/2}^o, F = 4$ transition. This may have been due to the very small relative transition strength of that specific transition, due to the Wigner6J symbol associated with the coupling of the angular momenta. Overall, we were able to locate 6 of the 17 total 760nm transitions.

5.3 Future Spectroscopy

To resolve the nuclear magnetic octupole moment measurement discrepancy in $^{173}\text{Yb}^+$, we must have precise determinations of the hyperfine splittings in the $^2\text{F}_{7/2}^o$ state. A fantastic review of potential systematic error contributions for such measurements is given in [Del25]. Here, we will discuss only the measurement protocol for determining the hyperfine splittings, and the close ties between this protocol and operation of a metastable qubit in $^{173}\text{Yb}^+$.

5.3.1 More Transitions Out of the $^2\text{F}_{7/2}^o$

To progress further in our spectroscopy, our first objective is to find more 760nm transitions from the higher F manifolds in the $^2\text{F}_{7/2}^o$ state. This will do toward helping us refine our understanding of the coarse hyperfine splittings in the $^2\text{F}_{7/2}^o$ state, and the measurements of the hyperfine splittings in the $^1[3/2]_{3/2}^o$ state as well. To do this however, we need to incorporate at least *one* additional 760nm laser tone, either with the use of a tunable EOM, or a wholly separate laser system. Our lab only has used two 760nm lasers thus far, and the power loss through our tunable EOM has so far been unacceptable for performing spectroscopy. We have exhausted the use of $^2\text{D}_{3/2}$ manifolds which only populate only 1 or 2 hyperfine manifolds in the $^2\text{F}_{7/2}^o$ state, and therefore we cannot continue the search for additional 760nm clear-out transitions without the use of 1 additional 760nm tone.

We do have options in the near future however. The first option is to simply use our current EOM and work toward reducing our beam waist at the ion to counter-act the insertion loss. Additionally, we have access to a tunable Ti:Sapph laser which can be used to provide the third tone required, or be set at a known value to allow the other two 760 lasers to perform the spectroscopy.

5.3.2 Refining the $^2D_{5/2}$ Measurements

Results presented in section 5.2.2 were all taken with quantum jump spectroscopy, and the collection of statistics was largely limited due to the rate of ion loss, as recovering population from the $^2F_{7/2}^o$ was slow, if not impossible, at the time. With the 760nm transitions found out of two of the manifolds in the $^2F_{7/2}^o$ state, we may begin to refine our results for at least the $^2S_{1/2}, F = 2 \rightarrow ^2D_{5/2}, F = 0$ and $^2S_{1/2}, F = 2 \rightarrow ^2D_{5/2}, F = 1$ transitions with more precise scans of the resonances found in our initial search.

If we are to achieve our goal of finding the additional required 760nm transitions out of the higher F hyperfine manifolds in the $^2F_{7/2}^o$ state, then we can proceed to perform additional spectroscopy of the other 411nm transitions, allowing us to better measure the hyperfine coefficient of the $^2D_{5/2}$ state. This will also give us better control for populating specific hyperfine manifolds in the $^2F_{7/2}^o$ state, making the required microwave spectroscopy more consistent.

5.3.3 Toward Microwave Spectroscopy in the $^2F_{7/2}^o$

The long lifetime of the $^2F_{7/2}^o$ state (even considering the theoretically estimated lifetime quenching [DF16]) means that spontaneous emission from the $^2F_{7/2}^o$ state is experimentally a non-issue. Therefore it is wise to consider avenues for precise spectroscopy, and one of these avenues is using direct microwaves to perform M1 transitions between the zero-field hyperfine clock states in each hyperfine manifold.

We can begin our attempts at microwave spectroscopy by populating a single manifold in the $^2F_{7/2}^o$ state. By driving the $^2S_{1/2}, F = 2 \rightarrow ^2D_{5/2}, F = 0$ transition, we will populate *only* the $^2F_{7/2}^o, F = 1$ manifold by selection rules. This means we can then perform a single microwave π -pulse from the $^2F_{7/2}^o, F = 1, m_F = 0$ state to the $^2F_{7/2}^o, F = 2, m_F = 0$ state, and then clear-out remaining population in the $^2F_{7/2}^o, F = 1$ manifold with the highly manifold-selective 760nm E2 transition. We may then perform simple Doppler cooling to

detect if population has returned to the ground state, and if it has *not* returned, then we have projected the population into the ${}^2F_{7/2}^o, F = 2, m_F = 0$ state. This process would be very similar to the heralded metastable qubit preparation in ${}^{171}\text{Yb}^+$ that will be discussed in chapter 7.

Once this state preparation has been completed, we may begin microwave Ramsey spectroscopy on the ${}^2F_{7/2}^o, F = 2, m_F = 0 \rightarrow {}^2F_{7/2}^o, F = 1, m_F = 0$ transition, allowing us to precisely determine the hyperfine splitting between the manifolds. We can readout the state of this process by again using the manifold-selective 760nm E2 transition, which will only depopulate one of these hyperfine manifolds. From here, it is easy to imagine iteratively continuing this cycle, where we populate one manifold in the ${}^2F_{7/2}^o$ state, and perform sequential microwave π -pulses to populate the $m_F = 0$ state in any manifold in the ${}^2F_{7/2}^o$ state, allowing for precise determination of *all* hyperfine manifold splittings.

Consequentially, this is effectively establishing “metastable qubit” single qubit capabilities in the ${}^2F_{7/2}^o$ state, where we will need state preparation, single qubit gates, and a measurement protocol in order to perform this spectroscopy.

5.4 Towards SPAM in ${}^{173}\text{Yb}^+$

Fast, efficient state preparation of the hyperfine clock states in the ground state of ions and atoms with nuclear spin $> 1/2$ has been shown [ARS22], [TPL21]. These techniques can be applied to ${}^{173}\text{Yb}^+$ for ground state qubit state preparation.

5.4.1 Microwave Assisted Optical Pumping

Microwave assisted optical pumping (MAOP) can be used to to prepare a single atomic state in the ground state of ${}^{173}\text{Yb}^+$. The general idea is to use light to pump population into a single hyperfine manifold, and then use resonant microwave tones to perform sequential π -pulses to remove population from unwanted Zeeman levels in the prepared manifold. A

diagram of this process can be seen in figure 5.4. In $^{173}\text{Yb}^+$ according to the cooling scheme presented in section 5.1.2, this would involve:

- Turning off the EOM tone that repumps population from the $F = 2$ manifold in the $^2\text{S}_{1/2}$ state
- Allowing light resonant with the $^2\text{S}_{1/2}, F = 3 \rightarrow ^2\text{P}_{1/2}^o, F = 3$ transitions to pump population into the $^2\text{S}_{1/2}, F = 2$ manifold
- Performing microwave π -pulses on the $m_F \neq 0$ states in the $F = 2$ manifold to corresponding states in the $F = 3$ manifold
- Perform the above processes until population has accumulated in the $^2\text{S}_{1/2}, F = 2, m_F = 0$ ($|0\rangle$) state

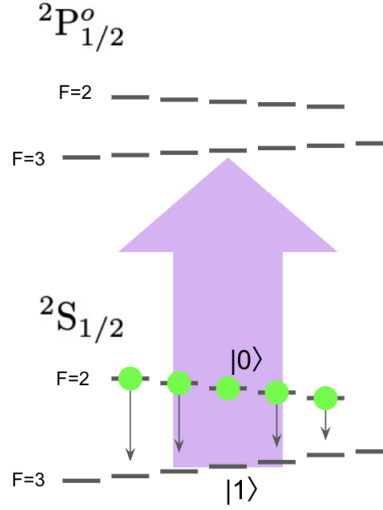


Figure 5.4: Diagram of microwave assisted optical pumping in $^{173}\text{Yb}^+$. We apply light resonant with the $^2\text{S}_{1/2}, F = 3 \rightarrow ^2\text{P}_{1/2}^o, F = 3$ transition, pumping population into the $^2\text{S}_{1/2}, F = 2$ manifold. Then microwave pulses clear out all $m_F \neq 0$ sublevels, and the process is repeated.

5.4.2 Narrowband Optical Pumping

An incredibly similar process can be performed which relies on a narrow, highly selective transition to remove population from the non-target state. This process is called “narrow-band optical pumping” (NBOP), and can be performed with a narrow E2 transition which is Zeeman-selective.

In $^{173}\text{Yb}^+$, this can be achieved with the 411nm transition from the $^2\text{S}_{1/2}$ state to the $^2\text{D}_{5/2}$ state, or alternatively the 435nm transition from the $^2\text{S}_{1/2}$ state to the $^2\text{D}_{3/2}$ state. In our lab, we use 411nm light, so it would be a more natural choice at this time. A diagram is shown in 5.5. Our sequence would be:

- Turning off the EOM tone that repumps population from the $F = 2$ manifold in the $^2\text{S}_{1/2}$ state
- Allowing light resonant with the $^2\text{S}_{1/2}, F = 3 \rightarrow ^2\text{P}_{1/2}^o, F = 3$ transitions to pump population in to the $^2\text{S}_{1/2}, F = 2$ manifold
- Performing optical π -pulses on the $m_F \neq 0$ states in the $F = 2$ manifold to corresponding states in the $F = 2$ manifold in the $^2\text{D}_{5/2}$ state
- Repump population from the $^2\text{D}_{5/2}$ state with light at 976nm to the $^1[3/2]_{3/2}^o, F = 1$ manifold (this will only decay to the $^2\text{S}_{1/2}, F = 2$ manifold for another attempt at populating the $|0\rangle$ state)
- Perform the above processes until population has accumulated in the $^2\text{S}_{1/2}, F = 2, m_F = 0$ ($|0\rangle$) state

5.4.3 Readout of the Ground Qubit in $^{173}\text{Yb}^+$

One downside of operating a hyperfine qubit in an ion with nuclear spin $I > 1/2$ is that there is no selection rule which allows one to scatter photons on a single manifold without

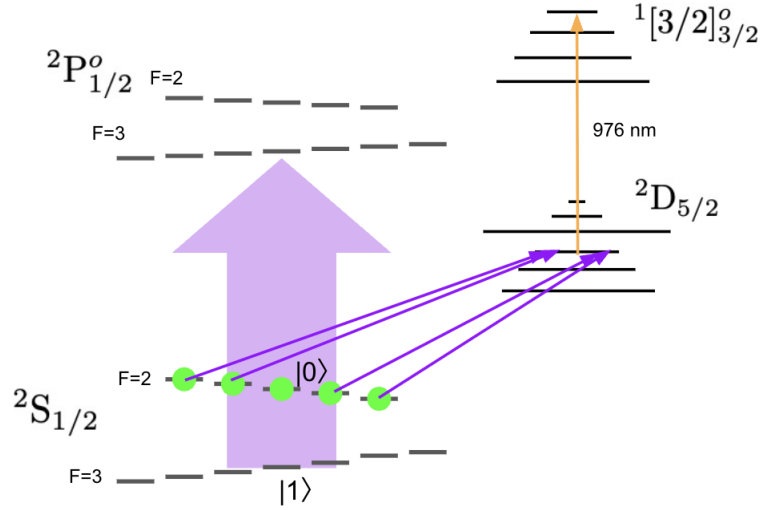


Figure 5.5: Diagram of narrow-band optical pumping in $^{173}\text{Yb}^+$. We apply light resonant with the $^2\text{S}_{1/2}, F=3 \rightarrow ^2\text{P}_{1/2}^o, F=3$ transition, pumping population into the $^2\text{S}_{1/2}, F=2$ manifold. Then 411nm pulses clear out all $m_F \neq 0$ sublevels to the $^2\text{D}_{5/2}, F=2$ manifold, to be repumped into the $^1[3/2]_o^o, F=1$ manifold, which decays back to the $^2\text{S}_{1/2}, F=2$ manifold, for the process to be repeated.

causing mixing into the other manifold. This means that one must employ *shelving* readout for detecting the qubit state. This will be further explained in chapter 6, but in essence we must move one of the qubit states to a relatively long-lived state in order to scatter photons for state detection.

In $^{173}\text{Yb}^+$, we can use the 411nm transition from the $^2\text{S}_{1/2}, F=2$ manifold to the $^2\text{D}_{5/2}, F=0$ manifold to move population into the $^2\text{F}_{7/2}^o, F=1$ manifold. This state will live for times much longer than we need to state detect on the $^2\text{S}_{1/2} \rightarrow ^2\text{P}_{1/2}^o$ transition, so we can discriminate between the qubit $|0\rangle$ and $|1\rangle$ states in this manner.

This comes with the added benefit that the $^2\text{D}_{5/2}, F=0$ manifold cannot decay into the $^2\text{S}_{1/2}, F=3$ manifold, so this decay will not mix our qubit state.

CHAPTER 6

High Fidelity State Preparation and Measurement

As discussed in section 4.5.3, it’s clear that there are limitations to the achievable SPAM fidelity when performing standard operations in $I = 1/2$ nuclear spin ions. This limitation is due to off-resonant scattering causing qubit mixing. To obtain higher-fidelity SPAM results, we will need to mitigate this off-resonant scattering in some way. The way that we get around this in our work with $^{171}\text{Yb}^+$ is through a process called “electron shelving”.

6.1 Electron Shelving

Electron shelving is a powerful technique used by many types of quantum systems including tweezer arrays [FTS24], trapped ions [BHR04], and also super-conducting circuits [CXN21]. The general idea behind the technique is to move population into a relatively long-lived state separate from the optical detection cycle in order to collect light without affecting the “shelved” population. This effectively allow you to collect photons from the detection cycle longer before mixing errors become a limitation, so long as the population transfer fidelity is high, and the shelving process is highly state-selective.

6.1.1 Shelving in $^{171}\text{Yb}^+$

The Yb ion is equipped with several long-lived states which are accessible from the ground state. In other trapped-ion systems (such as Ca^+), the natural choice of state is the $^2\text{D}_{5/2}$. For instance, the lifetime of the $^2\text{D}_{5/2}$ state in $^{40}\text{Ca}^+$ is 1.168(7)s [BDL00], giving ample time

to detect the quantum state of the ion through fluorescence. However, the $^2D_{5/2}$ state in Yb^+ is only 7.1(4)ms [TEM21], meaning there is significantly less time to detect the state of the qubit. This short lifetime is due to the available E1 transition to the $^2F_{7/2}^o$ state. Another option for Yb^+ is the $^2D_{3/2}$ state, but again it's lifetime (54.83(18)ms [SYM23]) is not nearly as long as the metastable state available to Ca^+ . The $^2D_{3/2}$ state in $^{171}Yb^+$ has been used to perform shelved state detection of the hyperfine qubit, with an achieved SPAM fidelity of 99.88(2)% [MZL24].

The longest available state that Yb^+ has access to for shelving is the $^2F_{7/2}^o$ state. This state lives for 1.58(8)yr [LPH21], and only decays back to the $^2S_{1/2}$ state. However, it's exceedingly long lifetime means that populating this state is incredibly difficult. This transition has been directly driven with lasers [HOL12], but it requires an excellent optical cavity and laser-locking electronics to do so.

Instead, we can consider populating the $^2F_{7/2}^o$ state in an incoherent manner, through a state which "leaks" population into it. A pathway directly accessible from the ground state would be preferred, and thus we look to the $^2D_{5/2}$ state. Further, the $^2D_{5/2}$ state has a favorable branching ratio (0.83 [TEM21]) into the $^2F_{7/2}^o$ state by emitting a $3.4\mu m$ photon. The big downside with using the $^2D_{5/2}$ state is it's long lifetime compared to most of the qubit operations performed. It will take many multiples of the lifetime to get low population transfer infidelities. However, once population is transferred to the $^2F_{7/2}^o$ state, the long lifetime will allow for effectively unlimited state detection time, and there are no light sources near the transitions which could bring population back to the ground $^2S_{1/2}$ state active during state detection. An additional benefit of using the 411nm transition to the $^2D_{5/2}$ state is that since it's an E2 transition, it is inherently highly manifold-selective, meaning off-resonant excitation of the non-shelved qubit state is unlikely.

6.1.2 Shelving Scheme

To increase our SPAM fidelity, we are going to shelve one of our qubit states to the ${}^2F_{7/2}^o$ state via the leaky ${}^2D_{5/2}$ state. At first glance, we have two options. We can either shelve the $|0\rangle$ state by coupling to the ${}^2D_{5/2}, F = 2$ manifold, or we can shelve the $|1\rangle$ state by coupling to either the ${}^2D_{5/2}, F = 2$ or $F = 3$ manifolds. However, when we recall that we are performing this process incoherently, it's evident that only one of these options will work. We rely on the decay from the ${}^2D_{5/2}$ state into the ${}^2F_{7/2}^o$ state, and the branching ratio *back* to the ${}^2S_{1/2}$ state is not negligible (0.17 [TEM21]). Because of this, any excitation to the ${}^2D_{5/2}, F = 2$ manifold will have a significant probability of decaying to the opposite qubit state, causing a mixing error.

This means our only real option is to shelve our qubit $|1\rangle$ state by driving the 411nm transition between the ${}^2S_{1/2}, F = 1$ manifold and the ${}^2D_{5/2}, F = 3$ manifold. Because we are relying on decays from the ${}^2D_{5/2}$ state, we will also need to address every Zeeman sub-level in the ${}^2S_{1/2}, F = 1$ manifold. Luckily, we can calculate the Lande-g factors for both hyperfine manifolds, and we see that they are nearly identical ($g_F \approx 1$). This means transitions with the same value of Δm_F will be degenerate, so we can clear out all Zeeman sublevels with a single laser frequency.

Then we must decide on which Δm_F transitions we would like to drive. Again, the Lande-g factors come into play. The Lande-g factor of the ${}^2D_{5/2}, F = 2$ manifold is larger by about 40% than that of the ${}^2D_{5/2}, F = 3$ manifold ($g_F({}^2D_{5/2}, F = 2) = 1.4$), meaning larger m_F levels in the $F = 3$ manifold will be farther separated from the sublevels in the $F = 2$ manifold. Thus, we want to drive transitions with a positive value of Δm_F , and further, since this is an E2 transition, we can drive the $\Delta m_F = +2$ transitions to be maximally separated from the $F = 2$ manifold, lessening the chances of off-resonant excitations to the $F = 2$ manifold leading to qubit mixing errors. A diagram of this process is presented in figure 6.1. As discussed in section 3.4, we can maximize the $\Delta m_F = \pm 2$ transitions by setting

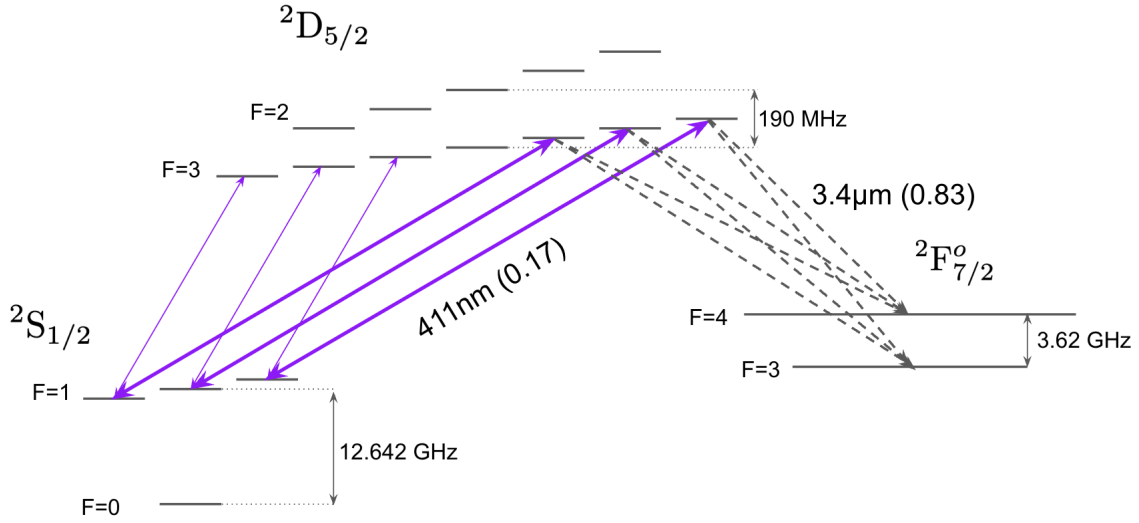


Figure 6.1: Diagram of the shelving scheme used for ground state hyperfine qubit SPAM. Thick purple lines represent the 411nm transitions used, which are degenerate due to the near-identical Lande-g factors of the $^2S_{1/2}, F = 1$ and $^2D_{5/2}, F = 3$ manifolds. Light purple lines show the other allowed (off-resonant) $|\Delta m_F| = 2$ transitions from the ground state. Dashed black lines indicate decays from the upper states to the $^2F_{7/2}^o$ state. Zeeman structure of the $^2F_{7/2}^o$ state has been omitted for simplicity, along with the $^2D_{3/2}$ state and M1 decay pathway.

our magnetic field ($\vec{B}$), our laser's $\vec{k}$ vector, and the laser's polarization ($\vec{\epsilon}$) to be mutually perpendicular ($\vec{\epsilon} \perp \vec{B} \perp \vec{k}$).

Before attempting to perform high-fidelity experiments, we should ensure that shelving does work for moving population to the $^2F_{7/2}^o$ state. We can perform a simple experiment in a spinless isotope (for instance $^{174}\text{Yb}^+$) where we illuminate the ion with 411nm light for increasing times and detect the remaining population in the $^2S_{1/2}$. The results can be seen in figure 6.2, and more detailed analysis can be found in section 6.2. It's clear that population does exponentially decrease in the $^2S_{1/2}$ manifold, so we need to determine how long we need to transfer population before an appropriate infidelity is obtained.

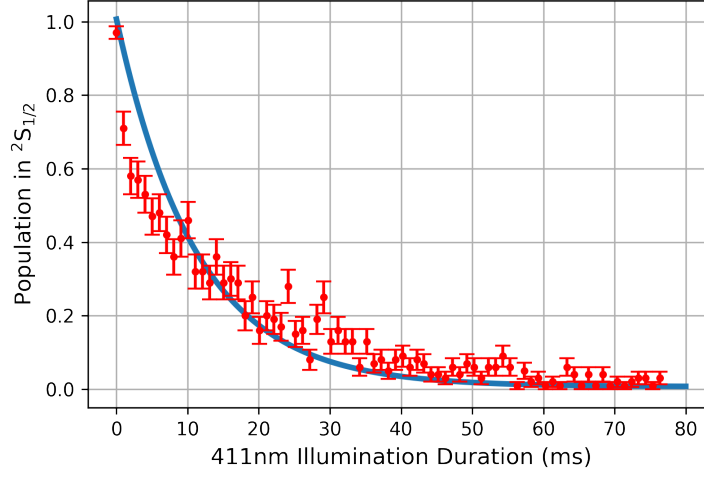


Figure 6.2: Population remaining in the $^2S_{1/2}$ manifold in $^{174}\text{Yb}^+$ as a function of 411nm illumination time. The data are fit to a simple exponential decay function, yielding a time constant $\tau = \frac{2}{\Gamma_D \mathcal{B}_{DF}} \approx 17\text{ms}$, where Γ_D is inverse of the lifetime of the $^2D_{5/2}$ state, and $\mathcal{B}_{DF}$ is the branching ratio from the $^2D_{5/2}$ to the $^2F_{7/2}^o$ state.

Of course, because of the incredibly long lifetime of the $^2F_{7/2}^o$ state, in order to use the same ion to perform many shelving experiments, we need a way to bring the population back to the cooling cycle to restart. This means we also need to *deshelve* population, which we can do with the best-known repump scheme (760nm light exciting to the $^1[3/2]_{3/2}^o$ state) that was first driven in $^{174}\text{Yb}^+$ in [Sug99]. A plot of population returning to the ground state due to illumination by 760nm light can be seen in figure 6.3. Note this example experiment is very slow, but much higher deshelling rates have been achieved in our lab ($\tau \approx 350\mu\text{s}$).

Finally, it's important to note that there are 3 total states lower in energy than the $^2D_{5/2}$ state. Those are the $^2S_{1/2}$, $^2F_{7/2}^o$, and $^2D_{3/2}$ states. The majority of the branching ratio (nearly unity) from the $^2D_{5/2}$ state go to the $^2S_{1/2}$ and $^2F_{7/2}^o$ states, but there is an allowed M1 decay to the $^2D_{3/2}$ state at $7.289\mu\text{m}$. The theory for this is presented in section 3.6. The A_{M1} coefficient for this process is calculated to be $A_{M1} = 2\pi \times 4.5\text{mHz}$. Despite its small transition moment, when performing many experiments with high fidelity, this decay becomes

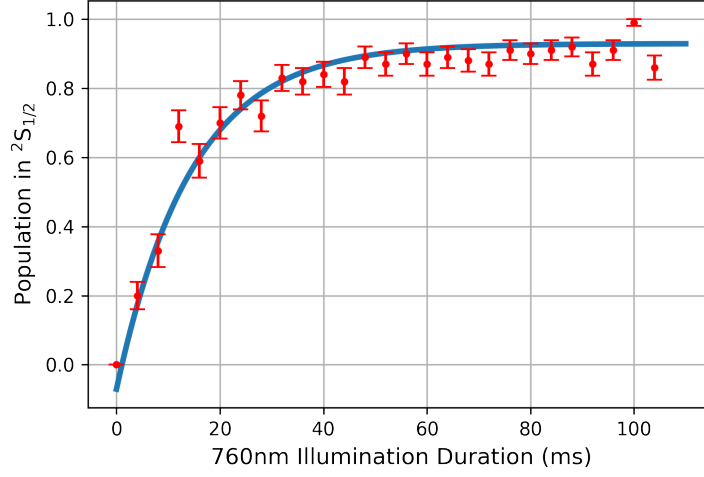


Figure 6.3: Population returning to the $^2S_{1/2}$ manifold from the $^2F_{7/2}^o$ state in $^{174}\text{Yb}^+$ as a function of 760nm illumination time. Our rate back to the ground state can be improved with more 760nm laser intensity and employing to a better laser lock scheme.

very important when considering long shelving times. A diagram that better describes the full shelving and deshelling process can be seen in figure 6.4.

6.2 Shelving Duration and Infidelity

Excellent analyses of the achievable infidelities as a function of shelving time by using the 411nm transition to shelve the $|1\rangle$ qubit state in $^{171}\text{Yb}^+$ are given in [Ran20], [Rom21] and [Del25], with the latter providing some additional context with hyperfine structure. Here we will present a simple rate model which is used to predict the shelving infidelity with and without considering the M1 decay.

We can write down simple rate equations for shelving as a function of time by inspection. For this first pass, we will not consider the effect of M1 decays to the $^2D_{3/2}$ state. The population of the $^2S_{1/2}$, $^2D_{5/2}$ and $^2F_{7/2}^o$ states ($\mathcal{P}_S$, $\mathcal{P}_D$, and $\mathcal{P}_F$ respectively), can be written as:

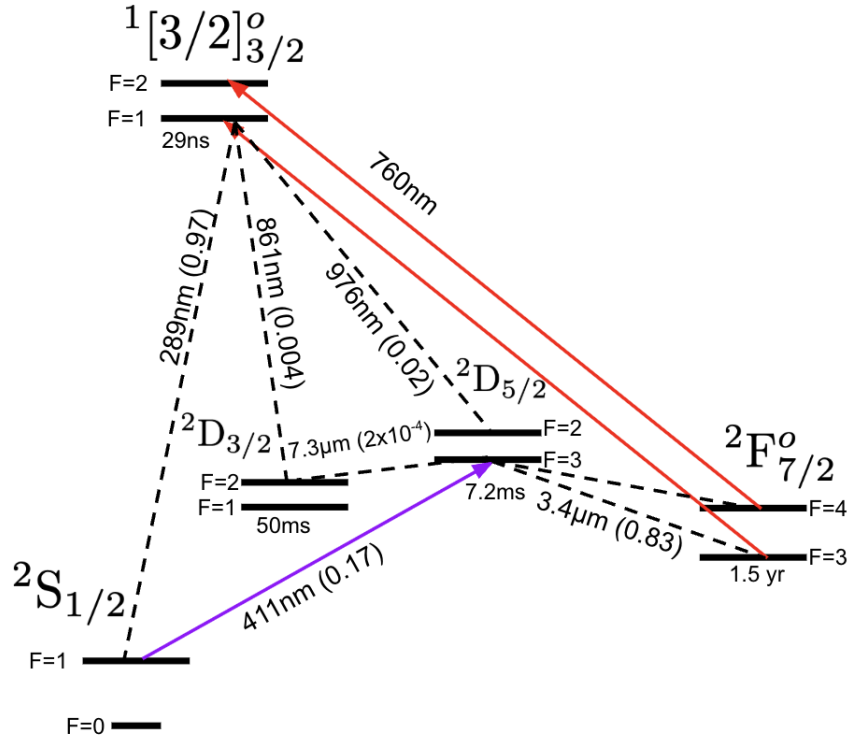


Figure 6.4: Full diagram of the shelving and deshelling scheme used for ground state hyperfine qubit SPAM. Hyperfine splittings are omitted but can be seen in table 4.4. Effects on the branching ratios due to hyperfine selection rules are also omitted.

$$\dot{\mathcal{P}}_S(t) = \Gamma_F \mathcal{P}_F(t) + \Gamma_D \mathcal{B}_{BS} \mathcal{P}_D(t) + \Omega \mathcal{P}_D(t) - \Omega \mathcal{P}_S(t) \quad (6.1)$$

$$\dot{\mathcal{P}}_D(t) = -\Gamma_D \mathcal{P}_D(t) - \Omega \mathcal{P}_D(t) + \Omega \mathcal{P}_S(t) \quad (6.2)$$

$$\dot{\mathcal{P}}_F(t) = \Gamma_D \mathcal{B}_{DF} \mathcal{P}_D(t) - \Gamma_F \mathcal{P}_F(t) \quad (6.3)$$

Where Γ_i is the spontaneous emission rate ($1/\tau_i$) of level i , $\mathcal{B}_{ij}$ is the branching ratio for decays from level i to level j , and Ω is the Rabi frequency of the transition between the $^2\text{S}_{1/2}$ and $^2\text{D}_{5/2}$ states.

Including the effect of the M1 decay from the $^2\text{D}_{5/2}$ to the $^2\text{D}_{3/2}$ state is a little more complicated. We will introduce a new population $\mathcal{P}_{|0\rangle}$ to describe population in the unshelved

state ($^2S_{1/2}, F = 0, m_F = 0$ state), and we will rewrite the population in the shelved manifold as $\mathcal{P}_{S1}$. Instead of modelling the population in the $^2D_{3/2}$ state and including its lifetime, we will introduce the 935nm light into our experiments. This will clear out any population in the $^2D_{3/2}, F = 2$ manifold by exciting it to the $^3[3/2]_{1/2}, F = 1$ manifold. From here, population can decay to both ground $^2S_{1/2}$ manifolds, but each process has a different amplitude due to hyperfine angular momentum addition. The probability of decaying to the $^2S_{1/2}, F = 1$ manifold from the $^3[3/2]_{1/2}, F = 1$ manifold is $2/3$, whereas the decay to the $|0\rangle$ state has probability $1/3$. We can then add this process into our rate equations:

$$\dot{\mathcal{P}}_{S1}(t) = \Gamma_F \mathcal{P}_F(t) + \Gamma_D \mathcal{B}_{BS} \mathcal{P}_D(t) + \Omega \mathcal{P}_D(t) - \Omega \mathcal{P}_{S1}(t) + \frac{2}{3} \Gamma_D \mathcal{B}_{DD3} \mathcal{P}_D(t) \quad (6.4)$$

$$\dot{\mathcal{P}}_{|0\rangle}(t) = \frac{1}{3} \Gamma_D \mathcal{B}_{DD3} \mathcal{P}_D(t) \quad (6.5)$$

$$\dot{\mathcal{P}}_D(t) = -\Gamma_D \mathcal{P}_D(t) - \Omega \mathcal{P}_D(t) + \Omega \mathcal{P}_{S1}(t) \quad (6.6)$$

$$\dot{\mathcal{P}}_F(t) = \Gamma_D \mathcal{B}_{DF} \mathcal{P}_D(t) - \Gamma_F \mathcal{P}_F(t) \quad (6.7)$$

Where we've introduced $\mathcal{B}_{DD3}$ as the branching ratio of the M1 decay. Both sets of equations can be solved numerically for the population remaining in the $^2S_{1/2}$ state. This will allow us to calculate the infidelity of this measurement process, as any population in the $^2S_{1/2}$ state will fluoresce when detection light is applied, causing an error. The infidelity as a function of shelving time for the situations where we ignore the M1 decay, and include the M1 decay are presented in figure 6.5 along with data.

6.3 Measurement of M1 Decay

In order to measure the M1 transition rate, we need to be able to compare the results where we do not correct for the decay errors versus those where it is corrected. We can correct the error by applying light at 861nm.

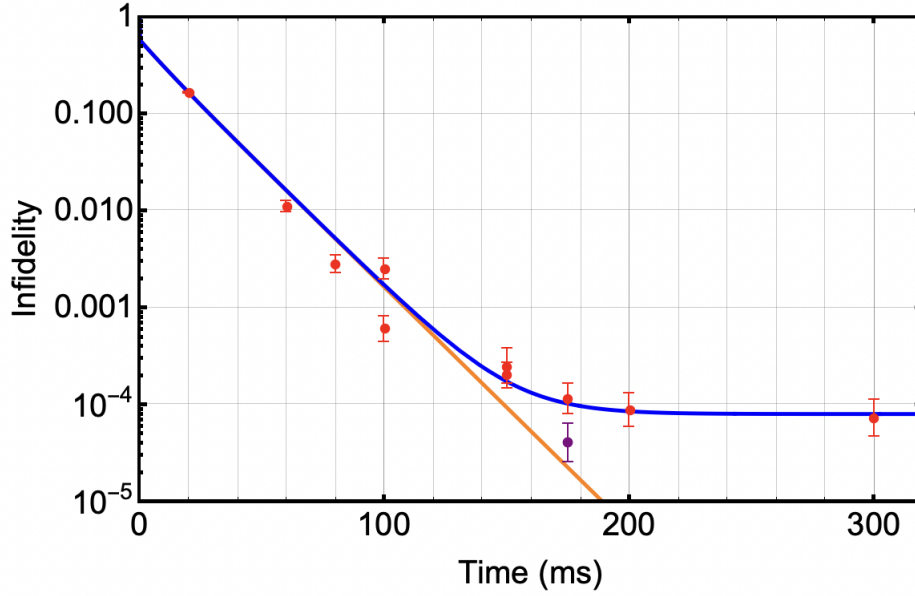


Figure 6.5: Plot of measured SPAM infidelity as a function of shelving time. Solid lines represent the theoretical rate models. The blue line includes the M1 decay from the $^2D_{5/2}$ state to the $^2D_{3/2}$ state during shelving, where the orange line ignores this effect. Red datapoints are taken without correcting the M1 error, while the purple datapoint is taken when correcting for the M1 decay with an additional repump laser at 861nm (discussed in section 6.3).

6.3.1 861nm Repump Scheme

During shelving, we are populating the $^2D_{5/2}, F = 3$ manifold. The M1 decay will populate the $^2D_{3/2}, F = 2$ manifold. Population in this state can decay directly to the $^2S_{1/2}, F = 0$ state (our qubit $|0\rangle$ state), causing the ion to be bright when we've prepared it dark. In the previous section (6.2), we discussed partially correcting this with the 935nm laser. However this only partially corrects the issue, as we are driving to the $^3[3/2]_{1/2}^o, F = 1$ state, which can decay to the qubit $|0\rangle$ state still. So we need to find a state that has a high branching ratio to the $^2S_{1/2}, F = 1$ state only.

Luckily, we have access to such a state, and we already populate it when we want to

deshelve population. It's the $^1[3/2]_{3/2}^o, F = 2$ manifold. This will not decay into the qubit $|0\rangle$ state. We can couple the $^2D_{3/2}, F = 2$ manifold to the $^1[3/2]_{3/2}^o, F = 2$ manifold with light at 861nm, an easily accessible color either with a direct diode or a TiSaphh laser.

It is important to note of course that the branching ratio of the decay back to the ground state is 0.974, according to the DREAM database [QP20]. So this means we will only directly correct for 97% of the M1 decays. The remaining population can decay to:

- 2.2% to the $^2D_{5/2}$ state
- 0.4% to the $^2D_{3/2}$ state

Population that decays to the $^2D_{3/2}, F = 2$ manifold will get re-excited for another chance at a decay to the $^2S_{1/2}, F = 1$ manifold, or will decay to the $^2D_{3/2}, F = 1$ manifold. If light at 935nm is applied for *only* clearing out the $^2D_{3/2}, F = 1$ manifold, it can be addressed as usual on the $^2D_{3/2}, F = 1 \rightarrow ^3[3/2]_{1/2}^o, F = 0$ transition, allowing only decays to the $^2S_{1/2}, F = 1$ manifold. This would just involve shutting off the EOM tone on the 935nm during shelving.

Population that ends up in the $^2D_{5/2}, F = 3$ manifold will either be shelved, or decay back to the $^2S_{1/2}, F = 1$ manifold, which is not an issue. However, population that decays to the $^2D_{5/2}, F = 2$ manifold from the $^1[3/2]_{3/2}^o, F = 2$ manifold can cause an issue. This state can decay to the $^2S_{1/2}, F = 0$ state, causing a qubit mixing error. This may be able to be circumvented by applying a single tone at 976nm addressing the $^2D_{5/2}, F = 2 \rightarrow ^1[3/2]_{3/2}^o, F = 2$ transition. However, some analysis must be performed to ensure off-resonant scattering of the 976nm laser doesn't depopulate the $^2D_{5/2}, F = 3$ manifold, as this may slow down the shelving process. Off-resonant excitation is more likely in this case due to the small hyperfine splitting of the $^2D_{5/2}$ state (190MHz [TEM21]). However, the upper state ($^1[3/2]_{3/2}^o$) has a large hyperfine splitting (8.86GHz [Rom21]), and since we would be coupling to the $^1[3/2]_{3/2}^o, F = 2$ manifold with the 976nm light, population can only end

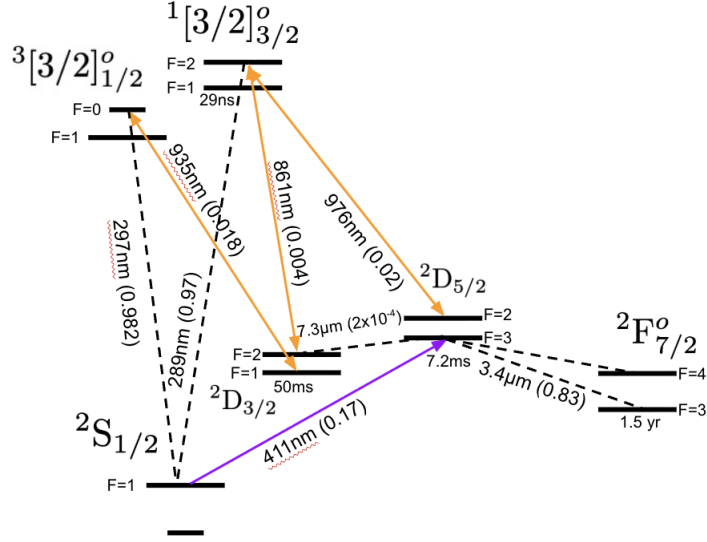


Figure 6.6: Full repump scheme to correct for M1 decays during shelving. This scheme includes 3 repump tones. The first addresses the $^2D_{3/2}, F=2 \rightarrow ^1[3/2]_{3/2}^o, F=2$ transition at 861nm. The second addresses the $^2D_{3/2}, F=1 \rightarrow ^3[3/2]_{1/2}^o, F=0$ transition at 935nm. And finally we apply 976nm light to address the $^2D_{5/2}, F=2 \rightarrow ^1[3/2]_{3/2}^o, F=2$ transition.

up in the $^2S_{1/2}, F=1$ manifold, for another attempt at shelving the correct qubit state. So the only errors possible for this repump scheme is the far off-resonant process coupling $^2D_{5/2}, F=3$ to $^1[3/2]_{3/2}^o, F=1$, detuned by nearly 9GHz. The full M1 repump scheme can be seen in figure 6.6.

The full repump scheme has not been implemented. Instead, we only applied light at 861nm tuned to the $^2D_{3/2}, F=2$ manifold to the $^1[3/2]_{3/2}^o, F=2$ manifold. The result of this measurement can be seen as the purple datapoint in figure 6.5. We can see clearly the presence of the repump light does in fact help reduce our infidelity. For the shown experiments, we employ a slightly modified version of the SPAM experiment, discussed in section 6.4. We perform 3 microwave π -pulses, one resonant with each of the Zeeman transitions from the $|0\rangle \rightarrow ^2S_{1/2}, F=1, m_F = \{-1, 0, 1\}$ transitions to better depopulate the $|0\rangle$ state.

The difference between the infidelities of the two 175ms shelving time experiments lets us measure the M1 decay rate. We measure this value to be $A_{M1} = 2\pi \times 4.1^{+2.3}_{-1.5}$ mHz, which is consistent with the calculated value for this decay we found in section 3.6.

6.4 SPAM Experiment

Ultimately what we want to show is that the shelving readout method helps to improve the SPAM fidelity of the ground state hyperfine qubit in $^{171}\text{Yb}^+$. To do this, we need to perform a SPAM experiment. Here we will discuss the SPAM experiment that we performed to (at the time) break the SPAM fidelity record.

6.4.1 SPAM Sequence

To perform a single shot of the SPAM experiment, we will adhere to a strict protocol. With regular re-calibrations of important experimental parameters. We perform a blinded analysis to set our thresholds for the full dataset.

6.4.1.1 Doppler Cooling

We begin each experiment with Doppler cooling the ion. This serves to ensure the ion is cool prior to beginning an experiment. While cooling, we collect photons to diagnose potential issues before or after an experiment. If Doppler cooling photon counts are low, it indicates the ion is hot or not present in the trap. The immediate run after one of these events occurs is flagged to be removed, as we did not start the experiment in a suitable state. In addition, the run immediately prior to the identification of low Doppler counts is flagged as a *storage* error, meaning we lost the ion during the run. This will contribute to the *infidelity* of the measurement, as it is an event which reflects on the inefficiency of our system. However, it will not contribute to the *inaccuracy* of our measurement, because it is

clearly a system-dependent error, which can be reduced by technological improvements.

We perform a scan of the Doppler cooling frequency every 2000 experiments in order to maintain the scattering rate of photons from the ion. Drifts in the frequency occur due to the cavity laser lock, as the cavity spacing changes slightly with room temperature changes.

6.4.1.2 $|0\rangle$ State Preparation

We perform optical pumping by applying a tone resonant to the $^2S_{1/2}, F = 1 \rightarrow ^2P_{1/2}^o, F = 1$ transition as described in section 4.5.1. We apply optical pumping light for $750\mu s$ to achieve preparation infidelities as low as 2×10^{-6} .

An additional optical pumping step may be used to further decrease the state preparation infidelity. By using a narrow, manifold selective transition along with a strategic repump tone, we can further out-couple population from the $^2S_{1/2}, F = 1$ manifold and trap it in the $|0\rangle$ state without fear of off-resonant out-coupling. We can apply light at 411nm addressing the $^2S_{1/2}, F = 1 \rightarrow ^2D_{5/2}, F = 2$ transition. Light at 976nm is also applied addressing the $^2D_{5/2}, F = 2 \rightarrow ^1[3/2]_{3/2}^o, F = 1$ transition. This excited $^1[3/2]_{3/2}^o, F = 1$ manifold will decay to both ground state manifolds, however over many cycles population will accumulate into the $|0\rangle$ state. Because the first step of this process is highly manifold selective given the narrow transition linewidth, the probability of accidentally exciting population is less than that of the situation when using just 369nm light to optically pump. This process was used in some early SPAM experiments, but not used in the final SPAM measurement.

6.4.1.3 Microwave Rotations

We apply resonant microwaves to perform single qubit rotations from $|0\rangle$ to $|1\rangle$. For theory on this see section 3.5. We calibrate the qubit frequency and π -time every 2000 experiments to account for drifts in the oscillator's frequency or applied microwave power. The majority of our error sources come from our ability to transfer population coherently with applied

microwaves. We need to characterize the fidelity of our single qubit rotations in order to evaluate the contribution of the error to the total SPAM result. We do this via randomized benchmarking.

6.4.1.4 Single Qubit Randomized Benchmarking

Randomized benchmarking is a procedure which was proposed in [KLR08]. This procedure is used to evaluate the error per gate by performing randomized pulses which rotates the qubit state around the Bloch sphere. We pick rotations from the Clifford group ($e^{\pm i\pi\sigma/4}$), which are essentially $\pi/2$ -pulses for the σ_x and σ_y operators. We also employ Pauli pulses (π -pulses) as well to depolarize the errors. Generating sequences for randomized benchmarking follows the protocol:

- Start by picking a set of sequence lengths: $L_1 < L_2 < \dots < L_N$
 - For every sequence length L , randomly pick $L-1$ Clifford gates ($e^{\pm i\pi\sigma/4}$) N_C times (σ_x and σ_y)
 - * For every Clifford gate sequence, randomly pick $L+1$ Pauli gates ($e^{\pm i\sigma\pi/2}$) N_P times
 - * For each sequence, determine the final state when $(P_L C_{L-1} P_{L-1} \dots P_2 C_1 P_1)$ is applied to the qubit
 - * Pick a final Clifford gate randomly from Clifford gates which leaves you in an eigenstate of σ_z
 - * Then determine the final expected state, and record it. $(P_{L+1} C_L P_L \dots P_2 C_1 P_1)$
 - Perform the gate sequence N_e times

This process generates statistics for gate sequences over $LN_C N_P N_e$ experiments. We can then plot the fidelity of the sequences, and fit the data to the following equation as a function

of sequence length N :

$$\mathcal{F}(N) = \frac{1}{2} + \frac{1}{2}(1 - 2\epsilon_{SPAM})(1 - 2\epsilon_g)^N \quad (6.8)$$

Where ϵ_{SPAM} is the SPAM error, and ϵ_g is the error per single qubit gate. For our experiment, we performed randomized benchmarking using *standard* qubit detection techniques, meaning our ϵ_{SPAM} is representative of the un-shelved SPAM fidelity. This should not however effect the achieved ϵ_g . We performed sequences of lengths $l = \{2, 3, 5, 8, 13, 21, 34, 55, 89, 144\}$, with $N_C = 10$, $N_P = 10$, and $N_e = 150$. The results are plotted in figure 6.7. We find a “SPAM intercept” $\epsilon_{SPAM} = 9.8(6) \times 10^{-3}$, and an error per single qubit gate of $7.4(1.0) \times 10^{-5}$.

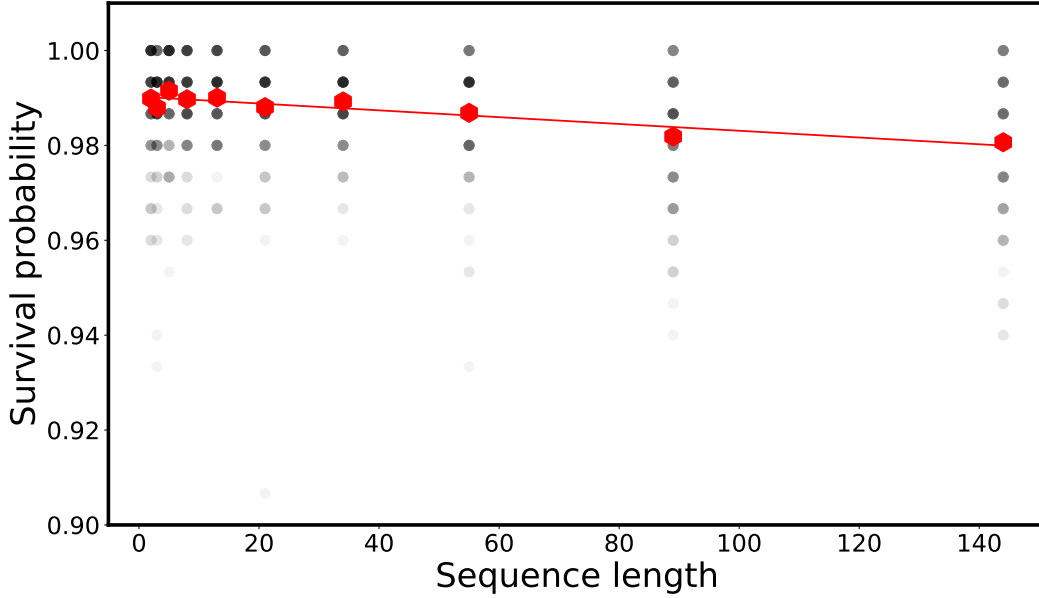


Figure 6.7: Results of randomized benchmarking of single qubit gates on the hyperfine qubit in $^{171}\text{Yb}^+$. The achieved SPAM error is $\epsilon_{SPAM} = 9.8(6) \times 10^{-3}$, and the achieved error per single qubit gate is $7.4(1.0) \times 10^{-5}$. Red hexagonal datapoints indicate the average result per sequence length, where the black/gray points indicate the results of each Pauli and Clifford randomization of that sequence length.

6.4.1.5 Shelving and State Detection

After we have prepared the qubit either in the $|0\rangle$ or $|1\rangle$ state, we have finished with the “state preparation” portion of our SPAM experiment. What follows is all the “measurement” scheme, beginning with 200ms of 411nm light resonant with the $^2S_{1/2}, F = 1 \rightarrow ^2D_{5/2}, F = 3$ transition. We apply light at 935nm during this process for the SPAM result, *not* fully correcting for M1 decays. The laser parameters follow what was discussed in section 6.1.2. This transfers population prepared in the $^2S_{1/2}$ manifold to the $^2F_{7/2}^o$ state, where the ion will not fluoresce.

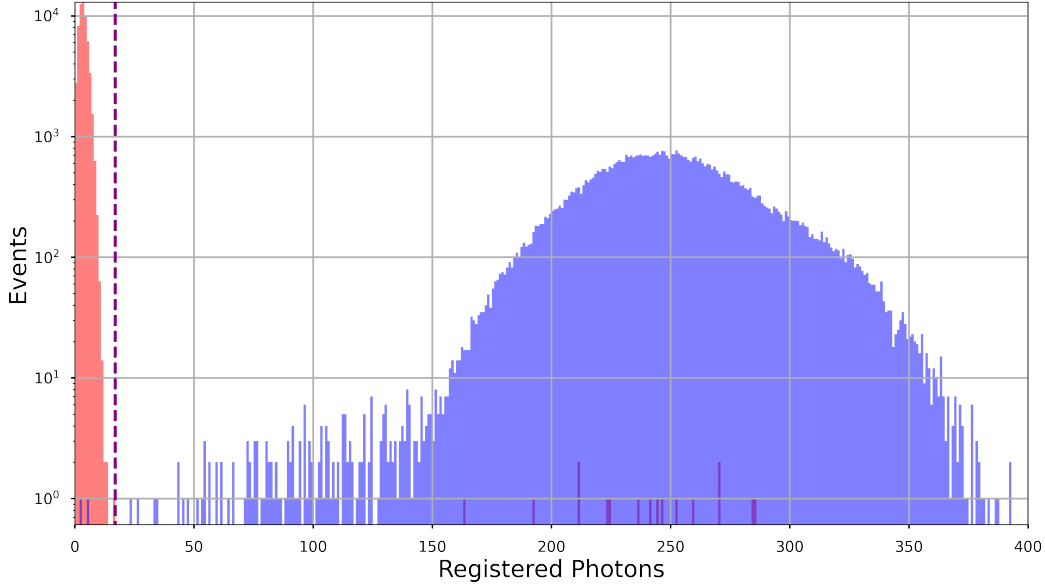


Figure 6.8: Histogram of counts recorded for the $^{171}\text{Yb}^+$ hyperfine qubit prepared in $|0\rangle$ (blue) and $|1\rangle$ (red) when using the shelving detection scheme. The ultimate infidelity achieved using this technique was $2.1^{+0.5}_{-0.4} \times 10^{-4}$.

After the shelving step is complete, we illuminate the ion with normal Doppler cooling light and collect photons for 17ms to determine its state. Population which was shelved successfully will not fluoresce, where population remaining in the $^2S_{1/2}$ state will fluoresce.

We can detect for so long thanks to the incredibly long lifetime of the $^2F_{7/2}^o$ state. This allows us to “separate” the histogram of “bright” and “dark” counts, as seen in figure 6.8.

6.4.1.6 Deshelving

After state detection is complete, we “deshelve” population to reset the ion and begin another experiment. We do this by applying light at 760nm to drive the $^2F_{7/2}^o \rightarrow ^1[3/2]_{3/2}^o$ transition for 35ms. We also apply light at 976nm addressing the $^2D_{5/2} \rightarrow ^1[3/2]_{3/2}^o$ transition, which helps increase the deshelling rate, as the $^1[3/2]_{3/2}^o$ state has a 2.2% branching ratio to the $^2D_{5/2}$ state.

6.4.1.7 Total Statistics and Blinding

We randomly selected 10,000 experiments per state after the SPAM measurement and set our thresholds for the state discriminator and the Doppler cooling threshold based on this data. The state discriminator threshold is set such that if $|1\rangle$ was prepared correctly, it would result in an error at the $\leq 10^{-7}$ level when considering the distribution of counts from the sample dataset. Similarly, the Doppler cooling threshold is set such that a successful cooled ion would show an error at the $\leq 10^{-6}$ level, again based on the sample dataset.

Once the thresholds were set, we unblinded the remaining data. It resulted in a total of 50,000 experiments per state. We identified a total of 13 errors (1 bright, 12 dark). For the dark errors, 8 of them were identified as ion loss events (storage errors). Removing those led to a SPAM infidelity of $2.1_{-0.4}^{+0.5} \times 10^{-4}$, where the errorbars are a 1σ Wilson confidence interval. The published results can be found in [RRD21].

6.5 Potential Improvements

The big drawback of this type of shelving detection in $^{171}\text{Yb}^+$ is the incredibly long time required to achieve high fidelity population transfer. Shelving for 200ms is an incredible overhead when readout is required, especially in mid-circuit readout is necessary. Faster population of the $^2\text{F}_{7/2}^o$ state has been shown however when using 411nm light combined with $3.4\mu\text{m}$ light [YMW21] in a two-step shelving process. Light at $3.4\mu\text{m}$ has been used for depopulation of the $^2\text{F}_{7/2}^o$ state as well [Sug99], and when paired with 976nm light, this repump pathway can be much faster than the 760nm scheme, as it does not rely on a narrow E2 transition. Of course, direct excitation of the E3 transition can be used, but this would be slow and technologically challenging.

Lower SPAM infidelities have been achieved in other systems recently. For instance using the $I = 3/2$ nuclear spin $^{137}\text{Ba}^+$ [ARS22]. The technique employed relies on multiple “cabinet” shelving pulses to different m_F levels in the $^2\text{D}_{5/2}$ state in $^{137}\text{Ba}^+$. It is also possible to use “Heralding” to help eliminate errors when performing SPAM experiments, leading to even lower infidelities [SLV24]. In $^{173}\text{Yb}^+$ however, it is possible that the lifetime of manifolds in the $^2\text{F}_{7/2}^o$ state are “quenched” due to hyperfine mixing [XLC20]. This makes the possibility of direct cabinet shelving with the 467nm E3 transition more feasible.

CHAPTER 7

The Metastable Qubit in Yb^+

In chapter 6, we found that the long-lived $^2\text{F}_{7/2}^o$ state is an excellent resource for hiding quantum information. This long lifetime however means that the $^2\text{F}_{7/2}^o$ state can be more than just a hiding place, it's effectively the *second ground state* in Yb^+ . We can define a zero field hyperfine clock qubit in the $^2\text{F}_{7/2}^o$, and using a similar manifold-selective readout process as the shelving technique for the ground state hyperfine qubit, we can achieve high fidelity readout of this “metastable” qubit.

7.1 *omg* Blueprint for Yb^+

The *omg* blueprint for quantum information processing [ACC21] aims to leverage multiple types of qubits defined in a single atomic ion species in order to circumvent some of the issues that come with operating a mixed-species atomic ion register. The types of qubit defined are the **o**ptical qubit, defined between two long-lived electronic states connected by an optical transition ($\approx 100\text{s}$ of THz), a **m**etastable qubit ($\approx 1\text{MHz}$ - 10GHz), defined on either Zeeman or Hyperfine sublevels in a single long-lived electronic state, and a **g**round state qubit ($\approx 1\text{MHz}$ - 10GHz), defined again on Zeeman or Hyperfine sublevels in the ground state of the atom. The idea of hosting multiple qubit “types” in an atom is not new. For instance, shelving readout with an optical qubit was used to readout the state of a hyperfine qubit in $^{43}\text{Ca}^+$ [KBZ09]. However, the use of multiple qubit types in a single atomic species has increase drastically in the past years.

Many commonly used trapped ions are suitable for operating with the *omg* architecture, so long as a metastable state is accessible, and Yb^+ is no exception. In chapter 6, we showed that the *g*-type qubit has excellent SPAM fidelity, and it's clear that the metastable $^2\text{F}_{7/2}^o$ state's long lifetime is well suited as a host for an *m*-type qubit, which will be discussed in this chapter. The capabilities to convert between *g*- and *m*-type qubits have been shown with a 2-step process with 411nm and $3.4\mu\text{m}$ light [YMW21]. Of course, the E3 transition at 467nm may also be used for this *o*-type qubit, but the incredibly narrow linewidth of such a transition makes creating a suitable laser difficult. Here we will discuss our work with the *m*-type qubit in $^{171}\text{Yb}^+$.

7.2 Heralded State Preparation and Single Qubit Rotations

The method we use to populate the $^2\text{F}_{7/2}^o$ state is narrowband optical pumping on the 411nm transition ($^2\text{S}_{1/2} \rightarrow ^2\text{D}_{5/2}$), as described in chapter 6. However, our goal is to populate the $^2\text{F}_{7/2}^o, F = 3, m_F = 0$ or $^2\text{F}_{7/2}^o, F = 4, m_F = 0$ states to use the zero-field hyperfine clock states to host a qubit.

There are two issues with the high fidelity shelving readout scheme to achieve this goal. Firstly, we are using the near-degenerate Lande-*g* factors of the $^2\text{S}_{1/2}, F = 1$ and $^2\text{D}_{5/2}, F = 3$ manifolds to reduce the required laser tones. Although this is a great feature, the issue is that we are populating the $^2\text{D}_{5/2}, F = 3$ manifold in the first place. Hyperfine selection rules tell us that this manifold will decay into both the $^2\text{F}_{7/2}^o, F = 3$ and $^2\text{F}_{7/2}^o, F = 4$ manifolds, and this will cause state preparation in the metastable state to be rather difficult. We can instead choose to drive population to the $^2\text{F}_{7/2}^o$ state with the $^2\text{S}_{1/2}, F = 0 \rightarrow ^2\text{D}_{5/2}, F = 2$ transition. This will ensure population only accumulates in the $^2\text{F}_{7/2}^o, F = 3$ manifold by selection rules.

The second issue is the polarization used to drive the 411nm transition. For high fidelity shelving, we set the laser polarization $\hat{\epsilon}$, the magnetic field $\hat{B}$, and the laser's propagation

direction $\hat{k}$ to be mutually perpendicular ($\hat{\epsilon} \perp \hat{B} \perp \hat{k}$). This will optimize the $\Delta m_F = \pm 2$ transitions while suppressing all other transitions. However, this means when we are driving population from the $^2S_{1/2}, F = 0, m_F = 0$ to the $^2D_{5/2}$ state, the $m_F = \pm 2$ states will be populated. Than again, E1 selection rules from the $^2D_{5/2}, F = 2$ manifold to the $^2F_{7/2}^o, F = 3$ manifold tell us that population will then only accumulate in the $m_F = -3, -2, -1, 1, 2, 3$ sublevels, critically missing the $m_F = 0$ state we wish to populate. Therefore, we need to choose which Δm_F transitions we wish to drive. There are then two options for us. We can choose to simply rotate the polarization by $\pi/2$ to drive the $\Delta m_F = \pm 1$ transitions, or we can move our magnetic field to be at a 45 degree angle with the laser's propagation direction. Experimentally, it's much easier to just rotate the polarization, so we opted for this solution.

By driving the $^2S_{1/2}, F = 0, m_F = 0 \rightarrow ^2D_{5/2}, F = 2$ transition with $\hat{k} \perp \hat{B} \parallel \hat{\epsilon}$, we end up populating the $^2F_{7/2}^o, F = 3, m_F = -2, -1, 0, 1, 2$ sublevels. This maximizes population in the $m_F = 0$ sublevel without requiring us to move our magnetic field. Of course, this comes with one more complication, which is the decay from the $^2D_{5/2}, F = 2$ manifold to the $^2S_{1/2}, F = 1$ manifold. This manifold will not be shelved, so we can simply apply normal optical pumping light to ring population back into the $^2S_{1/2}, F = 0, m_F = 0$ state for another shelving attempt. A diagram of this process is given in figure 7.1.

This process results in population spread into 5 total Zeeman sublevels in the $^2F_{7/2}^o, F = 3$ manifold. From here, we need a way of removing population from the unwanted Zeeman sublevels ($m_F \neq 0$). Firstly, we'd like to isolate the population that is in the $^2F_{7/2}^o, F = 3, m_F = 0$ state. We can do this by moving that population to the other hyperfine manifold with a microwave π -pulse to the $^2F_{7/2}^o, F = 4, m_F = 0$ state. From there, we may use a similar technique to electron shelving to remove any population left in the $F = 3$ manifold. The method we use to clear out the $^2F_{7/2}^o$ state is an E2 transition at 760nm. We can use the fact that this light is highly manifold-selective, and will only out-couple population in one of the manifolds, leaving any population in the other manifold safe.

We out-couple all remaining population in the $^2F_{7/2}^o, F = 3$ manifold after the microwave

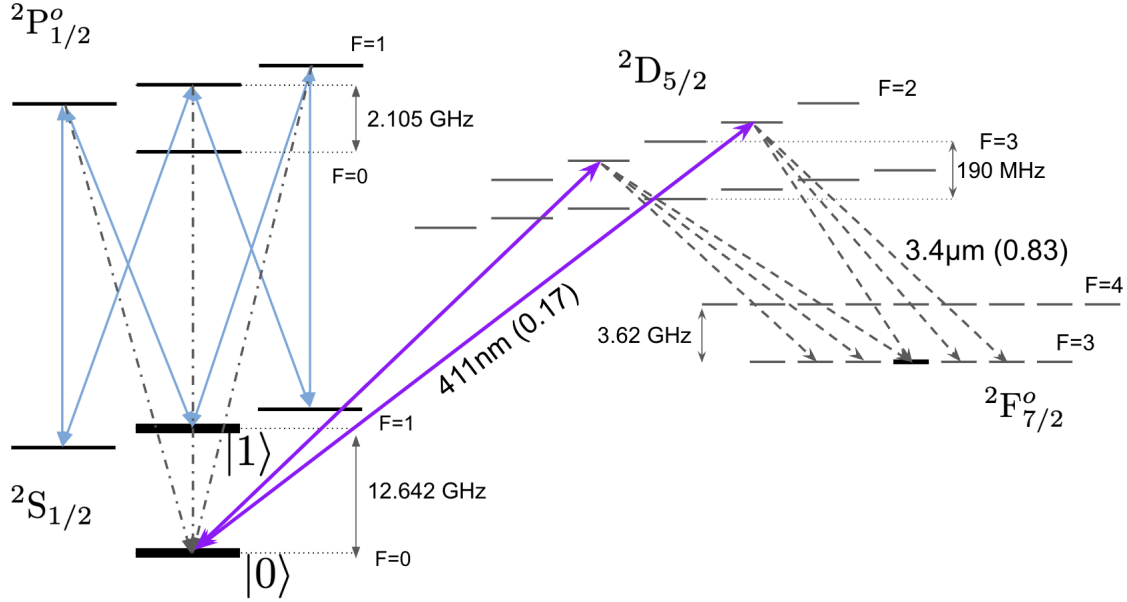


Figure 7.1: Diagram for $F = 3$ manifold preparation in the ${}^2F_{7/2}^o$ state in our experiment. We use the $\Delta m_F = \pm 1$ 411nm transitions from the ground state qubit $|0\rangle$ state to optimize population in the ${}^2F_{7/2}^o, F = 3, m_F = 0$ state. Optical pumping using 369nm light is also used to bring any population pumped into the ${}^2S_{1/2}, F = 1$ manifold back into the $|0\rangle$ state quickly.

π -pulse, bringing it back to the ground state rapidly thanks to the ${}^1[3/2]_{3/2}^o$ state. Population that was successfully transferred to the ${}^2F_{7/2}^o, F = 4, m_F = 0$ state will remain there. At this point we perform Doppler cooling with our normal scheme, and collect photons. If we determine the ion is dark, we have projected the state into ${}^2F_{7/2}^o, F = 4, m_F = 0 \equiv |1\rangle_m$. In the case where we do detect fluorescence in the ${}^2S_{1/2}$ state, we can simply try to perform this process again. This procedure is referred to as “Heralding” the preparation of a state.

To summarize, the sequence for preparing the $|1\rangle_m$ state in the ${}^2F_{7/2}^o$ state in ${}^{171}\text{Yb}^+$ using narrowband optical pumping is:

- Prepare the ground state qubit in $|0\rangle_g$ with optical pumping

- Keep 369nm optical pumping light on, while applying 411nm light in the $\hat{k} \perp \hat{B} \parallel \hat{\epsilon}$ configuration resonant with the $^2S_{1/2} \rightarrow ^2D_{5/2}, F = 2$ transition
- Perform a microwave π -pulse on the zero-field hyperfine clock transition: $^2F_{7/2}^o, F = 3, m_F = 0 \rightarrow ^2F_{7/2}^o, F = 4, m_F = 0$ transition
- Use the 760nm $^2F_{7/2}^o, F = 3 \rightarrow ^1[3/2]_{3/2}^o, F = 1$ transitions to remove population from the entire $^2F_{7/2}^o, F = 3$ manifold
- Perform Doppler cooling on the 369nm transition to determine if population is in the typical Yb^+ cooling cycle states
 - If population is detected, perform the full sequence again
 - If no population is detected, the state is projected into the $|1\rangle_m$ state

It's important to stress the effectiveness of this procedure. Without Heralding, the overall contrast between the $|1\rangle_m$ and $|0\rangle_m$ states is limited by the population in the $^2F_{7/2}^o, F = 3, m_F = 0$ state after the decay from the $^2D_{5/2}, F = 2$ manifold. For a graphical example, see figure 7.2.

7.3 SPAM in the m -type Qubit

Once we can perform good single qubit rotations using our Heralded state preparation, we can perform SPAM experiments to determine the fidelity of our state preparation and measurement process for the m -type qubit. In fact, we can use our process to Herald the preparation of *both* qubit states, since we have independent readout of both $^2F_{7/2}^o$ hyperfine manifolds.

We may Herald the preparation of the qubit $|0\rangle_m$ state by performing the following process *after* the Heralded preparation of the $|1\rangle_m$ state:

- Perform a microwave π -pulse on the $|1\rangle_m \rightarrow |0\rangle_m$ transition

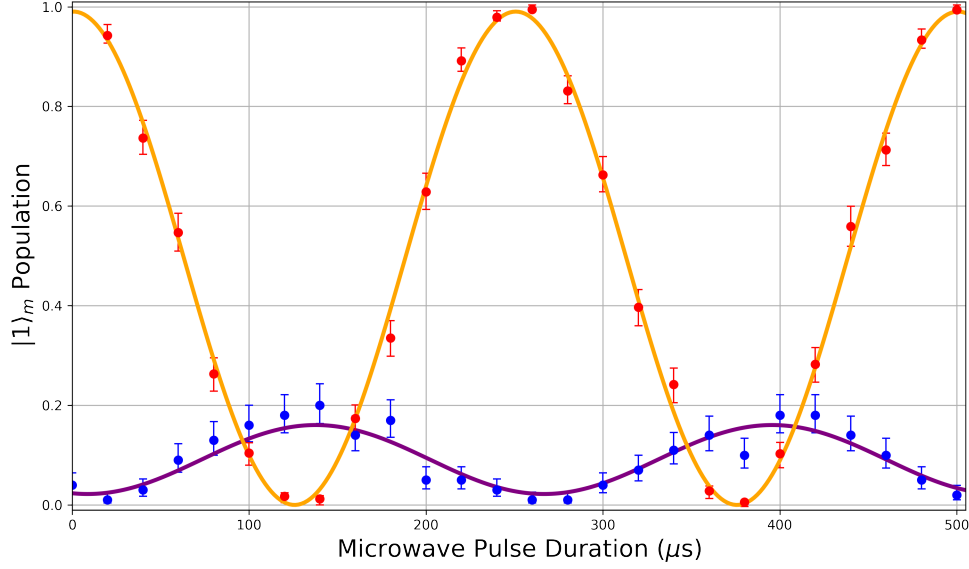


Figure 7.2: Microwave Rabi flopping of the m -type qubit in $^{171}\text{Yb}^+$. Blue datapoints are results from un-Heralded state preparation, while red datapoints are taken with Heralded state preparation. We can see that the contrast of the un-Heralded process is limited by the initial population in the $^2F_{7/2}^o, F = 3, m_F = 0$ state.

- Use 760nm light to drive the $^2F_{7/2}^o, F = 4 \rightarrow ^1[3/2]_{3/2}^o, F = 2$ transition to depopulate any population in $|1\rangle_m$
- Perform Doppler cooling on the 369nm transition to determine if population is in the typical Yb^+ cooling cycle states
 - If population is detected, perform the full sequence again
 - If no population is detected, the state is projected into the $|0\rangle_m$ state

We performed a SPAM experiment by Heralding preparation of both the $|1\rangle_m$ and $|0\rangle_m$ states, and using a 1σ Wilson confidence interval, we achieved a total SPAM infidelity of $1.5_{-0.8}^{+2.4} \times 10^{-4}$. The results can be seen in figure 7.3.

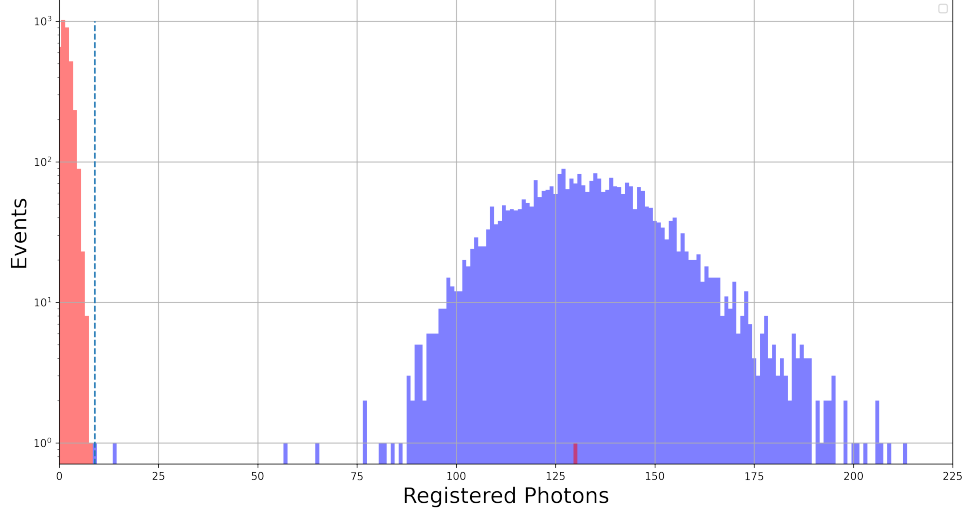


Figure 7.3: Results of the m -type SPAM experiment when Heralded state preparation of both the $|1\rangle_m$ and $|0\rangle_m$ states is performed. The infidelity achieved is $1.5^{+2.4}_{-0.8} \times 10^{-4}$. A total of 6,868 experiments were performed: 3,392 $|1\rangle_m$ experiments, and 3,456 $|0\rangle_m$ experiments

7.3.1 m -type Qubit State Detection Limits

Throughout this section, we'll assume that we have successfully heralded preparation of the $|0\rangle_m$ ($|^2F_{7/2}^o; F = 3, m_F = 0\rangle$) or the $|1\rangle_m$ ($|^2F_{7/2}^o; F = 4, m_F = 0\rangle$) state. In other words, we will consider only state *detection* errors, not state *preparation* errors. To detect the state of the m -type qubit, we must state-selectively ‘deshelve’ population from the $^2F_{7/2}^o$ state into the typical Yb^+ cooling cycle to collect laser induced florescence. To do this, we apply 760 nm laser light resonant with the $^2F_{7/2}^o(F = 3) \rightarrow ^1[3/2]_{3/2}^o(F = 1)$ transition for 35 ms. We also apply light at 976 nm and 935 nm, resonant with the $^2D_{5/2}(F = 2, 3) \rightarrow ^1[3/2]_{3/2}^o(F = 2)$, and $^2D_{3/2}(F = 1, 2) \rightarrow ^3[3/2]_{1/2}^o(F = 0, 1)$ transitions, respectively. We then collect photons for 17 ms while our standard Doppler cooling lasers are on. In the m -type qubit, the $|0\rangle_m$ state is identified as the ‘bright’ state, while $|1\rangle_m$ is the ‘dark’ state.

7.3.1.1 $|0\rangle_m$ State Detection Limits

The symptom of a $|0\rangle_m$ state detection error is receiving photon counts below the distribution of successful $|0\rangle_m$ state detection events. If we assume that we can reliably pump out of the $^2F_{7/2}^o(F=3)$ manifold with 760nm, 935nm, and 976nm light, the main contribution to these errors will be due to the following chain of events:

- Decay from $^1[3/2]_{3/2}^o(F=1) \rightarrow ^2D_{5/2}(F=2)$ (2.2%)
- Excitation from 976 nm light to $^1[3/2]_{3/2}^o(F=2)$
- Decay from $^1[3/2]_{3/2}^o(F=2) \rightarrow ^2F_{7/2}^o(F=4)$ ($\sim 0.1\%$)

This error channel has a rate of $\sim 2 \times 10^{-5}$ (about 1 out of every 50,000 attempts), and can be prevented if 976nm light is used in a particular way. Instead of using a single tone of light which transfers population from $^2D_{5/2}(F=2,3) \rightarrow ^1[3/2]_{3/2}^o(F=2)$, one could tune the 976nm laser to the $^2D_{5/2}(F=2) \rightarrow ^1[3/2]_{3/2}^o(F=1)$ transition only, preventing the possibility of decay to the $^2D_{5/2}(F=3)$ manifold altogether. However, the large hyperfine splitting in the $^1[3/2]_{3/2}^o$ state ($2\pi \times 8.86$ GHz) makes it difficult to operate the 976nm laser in a mode to assist deshelling of both $^2F_{7/2}^o$ manifolds independently (a function which is needed to facilitate heralded state preparation). A fiber-coupled EOM could be very useful in providing this function, as one could turn on or off certain tones on the laser depending on which state was being depopulated. It's important to note that all other decay pathways from the $^1[3/2]_{3/2}^o(F=1)$ manifold will put population into the state detection cycle, or back into the $^2F_{7/2}^o(F=3)$ manifold for another attempt at deshelling.

7.3.1.2 $|1\rangle_m$ State Detection Limits

The symptom of a $|1\rangle_m$ state detection error is receiving photon counts above the distribution of successful $|1\rangle_m$ state detection events. There are two possible ways to have population return into the cooling cycle from the $|1\rangle_m$ state during m -type state detection: an electric

octupole decay to the $^2S_{1/2}$ state, and off-resonantly deshelve population in $|1\rangle_m$ while addressing the $^2F_{7/2}^o(F=3) \rightarrow ^1[3/2]_{3/2}^o(F=1)$ transition.

The former possibility has an error rate which goes as $\epsilon = 1 - e^{-t/\tau}$, where τ is the lifetime of the $^2F_{7/2}^o$ state (~ 1.6 years), and t is the total state detection time (~ 40 ms when deshelveing of the $|0\rangle_m$ state is considered). This amounts to an error rate of $\sim 8 \times 10^{-10}$, which is far too small to observe in a small scale realistic m -type SPAM experiment.

When performing linescan experiments of the $^2F_{7/2}^o \rightarrow ^1[3/2]_{3/2}^o$ transition, we observe a FWHM linewidth of $\sim 2\pi \times 20$ MHz. This is highly broadened when compared to the estimated natural linewidth, due in part to power broadening and the coarse nature of the wavemeter-based lock used in the scans. Nevertheless, we will take this linewidth to be the worst-case scenario for off-resonant deshelveing. The only allowed transition at 760 nm that addresses the $|1\rangle_m$ is $^2F_{7/2}^o(F=4) \rightarrow ^1[3/2]_{3/2}^o(F=2)$. This transition is $\sim 2\pi \times 5.2$ GHz away from the carrier transition used for m -type state detection. The probability of off-resonant excitation of $|1\rangle_m$ population is 3.7×10^{-6} , and can certainly be improved if a better laser lock is used for the 760nm light.

7.4 m -type Laser Driven Gate Possibilities

A big next milestone for the m -type qubit will be the ability to impart a spin-dependent force on the ion while population is in the metastable $^2F_{7/2}^o$ state. We would like to impart this force with laser light, so we will need to look for electronic states which we can couple to the $^2F_{7/2}^o$ state.

7.4.1 Coupling to Even-Parity States

A well-developed technique for performing single qubit gates with lasers in hyperfine qubits is the stimulated Raman transition. Because the primary driver of transitions is the detuning between the two beams required for the gate, the detuning can be scanned over the ion's

motional sidebands on the qubit’s frequency, allowing for coupling of the ion’s motional and electronic states. These transitions typically use E1 transitions, meaning if we are to implement a stimulated Raman scheme for population in the ${}^2F_{7/2}^o$ state, we require an excited, even-parity state. A great analysis of the coupling to the excited even parity states from the ${}^2F_{7/2}^o$ state is given in [Rom21].

Several potential downsides come with the use of the even-parity states above ${}^2F_{7/2}^o$ though. Firstly, if a spontaneous emission error occurs, it is unclear where population will decay to from the excited state. There are a large number of odd-parity states with energies between the ${}^2F_{7/2}^o$ state and the excited even-parity states, meaning there will likely be many landing spots for population outside of the qubit manifold. Additionally, the transitions are in the ultra-violet, meaning it may not be easy to generate sufficient laser power to obtain high Rabi frequencies for the gates. However, because stimulated Raman transitions rely on the beat note between two beams to drive the transition, is it relatively easy to use high-quality optical modulators and stable microwave sources to tune the frequency offset, thereby converting an overall laser frequency stability problem into a microwave electronics problem.

7.4.2 Coupling to Odd-Parity States

There are some ≈ 42 odd-parity J_1K coupled states with energies between the ${}^2F_{7/2}^o$ state and the first J_1J_2 even-parity state in Yb^+ that have $11/2 \leq J \leq 3/2$. If we are primarily concerned with J and parity being good quantum numbers, it appears these states can be coupled to the ${}^2F_{7/2}^o$ state via an E2 transition. Using narrow E2 transitions also allows one to resolve motional sidebands on the narrow transition. In addition, E2 transitions have been shown to be able to generate a $\sigma_Z\sigma_Z$ “light-shift” gate on clock qubits [BBF21].

CHAPTER 8

Narrow Transitions from ${}^2F_{7/2}^o$

The ${}^2F_{7/2}^o$ state can be a great resource for quantum information processing. In atomic systems, long-lived states can be used as a tool for readout, or as hosts for qubits on their own. One major capability that has yet to be realized for the metastable qubit in Yb^+ is a spin-dependent force which acts on qubits in the metastable manifold. Our goal is to find a transition which can be used to provide this spin dependent force.

Additionally, when evaluating nuclear properties through King plot analysis, narrow transitions are used as their frequencies can be determined to higher precision. While investigating narrow transitions for m -type operations, we are also adding more transitions to multi-dimensional King plot analysis for better identification of higher-order nuclear effects.

8.1 Selection of States

In order to resolve motional sidebands on a transition from the ${}^2F_{7/2}^o$ state, we must find a transition which is much narrower than the secular frequency. Therefore it's natural to begin looking for transitions that are electric dipole forbidden, so we will consider electric quadrupole (E2) transitions. To satisfy this, we can look for excited states that have the same parity as the ${}^2F_{7/2}^o$ state, and that have $\Delta J \leq 2$. The only choice then are the JK coupled states (the so-called “bracket” states). We then have some criteria that must be met in order to consider the transition. We require that $11/2 \geq J > 7/2$, as any lower J value will likely decay quickly to shorter-lived states in Yb^+ , such as the ${}^2D_{5/2}$ or ${}^2D_{3/2}$

Configuration	Term	J	k (1/cm)	g	λ from $^2F_{7/2}^o$ (nm)
$4f^{13}(^2F_{7/2}^o)5d6s(^3D)$	$^3[11/2]^o$	9/2	30,224.33	0.935	1135.6435
$4f^{13}(^2F_{7/2}^o)5d6s(^3D)$	$^3[11/2]^o$	11/2	30,562.79	1.112	1093.6085
$4f^{13}(^2F_{7/2}^o)5d6s(^3D)$	$^3[7/2]^o$	9/2	33,052.29	1.264	859.5836
$4f^{13}(^2F_{7/2}^o)5d6s(^1D)$	$^1[11/2]^o$	11/2	34,784.95	1.119	748.1558
$4f^{13}(^2F_{7/2}^o)5d6s(^3D)$	$^3[9/2]^o$	9/2	35,019.13	1.158	735.2736
$4f^{13}(^2F_{7/2}^o)5d6s(^3D)$	$^3[9/2]^o$	11/2	35,831.68	1.214	693.8215
$4f^{13}(^2F_{7/2}^o)5d6s(^1D)$	$^1[9/2]^o$	9/2	38,342.02	1.093	590.9023

Table 8.1: Table showing the states considered for narrow transitions from the $^2F_{7/2}^o$ state in Yb^+ . The table presents the electron configuration, term symbol, total angular momentum quantum number J , energy level, and wavelength of the transition from the $^2F_{7/2}^o$ state based on values from the NIST ASD [KYR24].

states. For example, take the $^2F_{7/2}^o \rightarrow ^3[9/2]_{7/2}^o$ transition at 828nm, driven in [Sug99]. This transition takes us out of the $^2F_{7/2}^o$ state and brings population back to the laser cooling cycle. Additionally, we will look only at transitions that involve 1 electron, and also we will only consider states which have $> 50\%$ state character. We will also limit our search to have a transition wavelength greater than 500nm, for technological ease. There are 7 states that then satisfy our conditions, presented in table 8.1.

We use a CW Ti:Sapph laser in our lab, so the most natural choices for us to pursue initially are the transitions at 860nm, 748nm, and 735nm, as they fall within the bandwidth of the laser, and the available power is high for the search.

It's also important to point out that the $^3[9/2]_{9/2}^o$ state (transition at 735nm) has a nearly identical Lande g factor as the $^2F_{7/2}^o$ state, making it an attractive candidate for reducing the effects of magnetic field fluctuations.

8.2 Magnetic Dipole Decay Pathways

Just as in chapter 6, we ought to be mindful of the states lower in energy than the excited state, and consider the potential decay pathways, despite the fact that no E1 transitions are possible from these excited states. Luckily, we can solve the matrix elements exactly for the magnetic dipole operator.

The magnetic dipole (M1) operator is defined as $\frac{\vec{\mu}}{\mu_B} = \vec{L} + g_s \vec{S}$, where μ_B is the Bohr Magneton, g_s is the g-factor of the electron, and $\vec{L}$ and $\vec{S}$ are the usual angular momentum operators for orbital and spin angular momentum, respectively. We can use the M1 operator as usual to find the decay rates of $J_1 K$ coupled states though, with some effort to ensure the bases are correct. The spontaneous decay rate for an M1 transition is given by:

$$A_{M1} = \frac{w^3}{3\pi\epsilon_0\hbar c^5} \frac{|\langle n' J' K' m'_J | \vec{\mu} | n J K m_J \rangle|^2}{2J' + 1} \quad (8.1)$$

Here, w is the cyclic frequency of the transition in question, ϵ_0 is the vacuum permittivity, $\hbar$ is the reduced Planck's Constant, c is the speed of light in vacuum, and J' is the total angular momentum of the excited state.

8.2.1 Finding the Reduced Matrix Element

The goal is to repeatedly reduce the matrix element to L and S of the core and orbital electrons, so that we can simply evaluate the matrix element. The M1 operator can be split up into two parts:

$$\vec{\mu} = \vec{\mu}_c + \vec{\mu}_o \quad (8.2)$$

and further into the L and S components as:

$$\vec{\mu}_c = \mu_B(\vec{L}_c + g_s \vec{S}_c) \quad (8.3)$$

$$\vec{\mu}_o = \mu_B(\vec{L}_o + g_s \vec{S}_o) \quad (8.4)$$

And the matrix element we begin with is:

$$\langle J'M'|T_q^{(1)}[\vec{\mu}]|JM\rangle \quad (8.5)$$

These aren't the basis states we need to be working with, so it's a job for the Wigner-Eckhart Theorem.

$$\langle J'M'|T_q^{(1)}[\vec{\mu}]|JM\rangle = (-1)^{(J'-M')} \begin{pmatrix} J' & 1 & J \\ -M' & q & M \end{pmatrix} \langle J'K'J'_cS'_o|T^{(1)}[\vec{\mu}]|JKJ_cS_o\rangle \quad (8.6)$$

Now we must begin using the so-called ‘‘Spectator Theorem’’ (also called the repeated reduction relation) to begin reducing these terms to something that looks like: $\langle j'_i|\mathbf{j}_i|j_i\rangle$. That will allow us to simply identify the reduced matrix element as: $\delta_{j'_i,j_i} \sqrt{j_i(j_i+1)(2j_i+1)}$.

If operator $T^{(k)}$ acts only on space j_1 but the basis is given by j , j_1 , and j_2 (where $j = j_1 + j_2$), then one can remove the j and j_2 dependence by applying:

$$\langle \gamma'j'_1j'_2j'|T^k|\gamma j_1j_2j\rangle = \delta_{j'_2,j_2}(-1)^{j'_1+j'_2+j+k} \sqrt{(2j'+1)(2j+1)} \begin{Bmatrix} j'_1 & j' & j'_2 \\ j & j_1 & k \end{Bmatrix} \langle \gamma'j'_1|T^k|\gamma j_1\rangle \quad (8.7)$$

Similarly, if the operator $T^{(k)}$ acts only on space j_2 but the basis is given by j, j_1 , and j_2 (where $j = j_1 + j_2$), then one can remove the j and j_1 dependence by applying:

$$\langle \gamma'j'_1j'_2j'|T^k|\gamma j_1j_2j\rangle = \delta_{j'_1,j_1}(-1)^{j'_1+j_2+j'+k} \sqrt{(2j'+1)(2j+1)} \begin{Bmatrix} j'_2 & j' & j'_1 \\ j & j_2 & k \end{Bmatrix} \langle \gamma'j'_2|T^k|\gamma j_2\rangle \quad (8.8)$$

To decouple the angular momenta, we need to consider the order in which they were coupled to begin with, and since these are J_1K coupled states, we know the order, as given in table

4.3. If we apply these formulae all the way to the L and S basis, we arrive at:

$$\begin{aligned}
\langle J'M'|T_q^{(1)}[\vec{\mu}]|JM\rangle = & \sum_{M,M',q} (-1)^{J'-M'} \begin{pmatrix} J' & 1 & J \\ -M' & q & M \end{pmatrix} \delta_{S'_o, S_o} \sqrt{(2J'+1)(2J+1)} (-1)^{K'+1} \left(\times \right. \\
& \times g_s (-1)^{S_o+J'} \delta_{K', K} \sqrt{S_o(S_o+1)(2S_o+1)} \begin{Bmatrix} S'_o & J' & K' \\ J & S_o & 1 \end{Bmatrix} + \\
& (-1)^{S'_o+J+J'_c+1} \delta_{L'_o, L_o} \sqrt{(2K'+1)(2K+1)} \begin{Bmatrix} K' & J' & S'_o \\ J & K & 1 \end{Bmatrix} \left(\times \right. \\
& \times \delta_{J'_c, J_c} (-1)^{L_o+K'} \sqrt{L_o(L_o+1)(2L_o)+1} \begin{Bmatrix} L'_o & K' & J'_c \\ K & L_o & 1 \end{Bmatrix} + \\
& \delta_{L'_c, L_c} \delta_{S'_c, S_c} (-1)^{L'_o+K+L'_c+1} \sqrt{(2J'_c+1)(2J_c+1)} \begin{Bmatrix} J'_c & K & L'_o \\ K & J_1 & 1 \end{Bmatrix} \left(\times \right. \\
& \times (-1)^{S'_c+J_c} \sqrt{L_c(L_c+1)(2L_c+1)} \begin{Bmatrix} L'_c & J'_c & S'_c \\ J_c & L_c & 1 \end{Bmatrix} + \\
& \left. \left. \left. g_s (-1)^{S_c+J'_c} \sqrt{S_c(S_c+1)(2S_c+1)} \begin{Bmatrix} S'_c & J'_c & L_c \\ J_c & S_c & 1 \end{Bmatrix} \right) \right) \right)
\end{aligned}$$

To verify this equation's validity, we can estimate the g-factor of high-purity J_1K coupled states in Yb^+ and compare the results to that of the NIST ASD. We can do this by calculating the expectation value of a diagonal matrix element for the Zeeman Hamiltonian. We know that this should simply be:

$$\langle JM|\vec{\mu} \cdot \vec{B}|JM\rangle = g_s BM \quad (8.9)$$

If we assume $\vec{\mu} \cdot \vec{B} = \mu B$, then we can cancel the magnetic field and divide our matrix element by the M quantum number to get the g-factor. Note that this needs the full matrix element though, not just the reduced matrix element. Therefore we will need the 3j symbol in order to calculate it correctly.

Additionally, since these JK coupled states do not purely overlap with a single term symbol, it's important to consider the second leading order state percentage, as given in the NIST ASD. Now, we don't know the phase between the primary and secondary state contributions, so assigning a full M1 lifetime based on these values is not a perfect estimate, since the phase plays an important part when evaluating the matrix element. However, we can still obtain some rough estimates by assuming the phase is simply equal to 1. We consider all possible decay paths from the excited state and it's second leading order term contribution, and look for decay paths to all lower terms and second percentages of the lower terms.

Going through this process, we arrive at the M1 lifetimes shown in table 8.2. We can see that the states which we have chosen have M1 lifetimes on the order of 1 second, the longest being the 4 seconds of the 860nm transition. However, the two transitions which are in the near infrared have much longer M1 lifetimes, or no available M1 decay pathways. These states may be more attractive for narrow spectroscopy, as their M1 lifetimes are longer, thus spontaneous emission events will be much less frequent. Work is ongoing to estimate the true second highest state character percentages to obtain better lifetime estimates.

8.3 Initial Detection of Transitions

8.3.1 Aligning the Beam

Before beginning our search, it's logical to consider how we will align our light to the ion, to reduce the number of variable in our system. Luckily, Yb^+ has a wealth of transitions available to it. Specifically, we can use an alternative Doppler cooling repump scheme which utilizes light at 861nm, as described in section 4.4. If we can use this light as our primary repump light rather than the 935nm transition, we can use the ion's fluorescence as a clear indicator for the beam alignment.

We set up this alternative cooling scheme, and aligned the Ti:Sapph beam to the ion,

Term	k (1/cm)	λ from $^2F_{7/2}^o$ (nm)	M1 lifetime (s)
$^3[11/2]_{9/2}^o$	30,224.33	1135.6435	N/A
$^3[11/2]_{11/2}^o$	30,562.79	1093.6085	1356.4
$^3[7/2]_{9/2}^o$	33,052.29	859.5836	4.0
$^1[11/2]_{11/2}^o$	34,784.95	748.1558	1.8
$^3[9/2]_{9/2}^o$	35,019.13	735.2736	0.7
$^3[9/2]_{11/2}^o$	35,831.68	693.8215	0.5
$^1[9/2]_{9/2}^o$	38,342.02	590.9023	1.1

Table 8.2: Table showing the calculated M1 lifetimes of the considered excited states for narrow transitions from the $^2F_{7/2}^o$ state in Yb^+ . The table presents the term symbol, energy level, wavelength of the transition from the $^2F_{7/2}^o$ state based on values from the NIST ASD [KYR24].

iteratively lowering the power to ensure we are as aligned as possible. After this process was completed, we moved the Ti:Sapph to 860nm, our first target transition.

8.3.2 Search Space

The best estimate on the frequencies of the transitions we have is the value of the energy levels given in the NIST ASD [KYR24]. We chose to begin our searches with $^{174}\text{Yb}^+$, so we began to look at literature values for frequencies of transitions in $^{174}\text{Yb}^+$ as compared to the given value in the NIST ASD. By comparing several transitions, we found that most measured values lie within $\pm 5\text{GHz}$ of the NIST ASD value for the energy of the levels. This gives us an approximate range of searchable space, about a total of 10GHz centered around the value provided in the NIST ASD. If we are to compare the potential strengths of these transitions to known E2 transitions in Yb^+ , as a first pass, we can estimate the linewidth to be roughly similar ($\approx 10\text{Hz}$). This means we are searching for a feature that is about 1 part

in 10^9 , showing the difficulty of our search.

8.3.3 Method of Search

We started the search for these transitions in a “CW” (continuous-wave) way, where we cycled the ion between the $^2S_{1/2}$ and $^2F_{7/2}^o$ states via the 411nm transition, and repumped from the $^2F_{7/2}^o$ state with the 760nm transition all while cooling the ion with 369nm and 935nm light. We tuned the power of the 411nm and 760nm lasers such that the overall 369nm fluorescence of the ion was about half of that when the 411nm laser was turned off. The cycling is rapid enough such that in our 100ms PMT collection bins, the ion never fully went “dark”. While cycling, we applied light near the estimated transition frequency and scanned the frequency at a rate of about 0.5 MHz per second. The evidence of the transition is multi-bin “dark” periods in the ion fluorescence while the ion undergoes quantum jumps to and from the target state. To confirm this was indeed a transition, we would block the spectroscopy light while the ion was “dark”, and observe it remain such until we unblocked the light. Occasionally, we would fully lose the ion’s fluorescence, indicating there exist decay paths which take population to undesired states (see section 8.2). A PMT trace is shown in figure 8.1 of our first evidence of driving the 860nm transition from the $^2F_{7/2}^o$ to $^3[7/2]_{9/2}^o$ state.

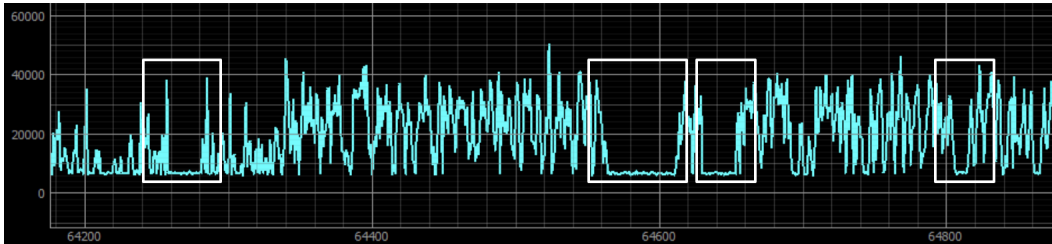


Figure 8.1

8.4 Frequency Determinations

Frequency determinations of the transitions are made by quantum jump spectroscopy primarily, as discussed above. At the time of writing this, the primary focus of our frequency determination work has been for the 860nm transition in $^{174}\text{Yb}^+$ specifically, but by using quantum jump spectroscopy method, we were able to determine the frequency of the transitions to within a few 10's of megahertz (due to Zeeman broadening) for the 735nm, 748nm, and 860nm transitions to the $^3[9/2]_{9/2}^o$, $^1[11/2]_{11/2}^o$, $^3[7/2]_{9/2}^o$ states respectively for 170, 172, 174, 176. Work is ongoing to determine the frequencies of these transitions in the methods described below for better measurements.

Additional work has been done on trying to refine our measurement for the frequency of the 860nm transition, as it appears to be the easiest of the options to drive given our experience with all three of the transitions. For the 860nm transition, The beam is the easiest to align with our alternative Doppler cooling scheme, whereas we do not have nearby transitions for the other two wavelengths. The closest alignment wavelength we could use is the 760nm transition out of the $^2F_{7/2}^o$ state, but it is still about 16THz away from our target frequencies.

8.4.1 860nm Transition

To better determine the frequency of the 860nm transition, we began an experiment to measure the number of potential transitions in a fixed time bin as a function of laser frequency. The results may be seen in figure 8.2. We began each datapoint by collecting photon counts for 1 minute with the spectroscopy laser off. This gave us a calibrated background signal, where we can see the ion's signal telegraph due to cycling to and from the $^2F_{7/2}^o$ state. We need this calibration signal to reject quantum jump events that are due to this process, and better identify the longer dark events that are indicative of the transition we wish to find. Then, we perform photon collection for 1 minute with the spectroscopy laser on, and then

we begin to analyze the data.

We use our calibration dataset to find the maximum “dark” time due to cycling population to and from the $^2F_{7/2}^o$ state. In our spectroscopy dataset, dark times which exceed this maximum value are counted as potential transitions. Figure 8.2 shows the number of suspected transitions which pass this threshold as a function of the 860nm laser’s frequency. We can see in this figure that multiple Δm_J transitions are driven (blue bands correspond to $\Delta m_J = 1$, yellow to $\Delta m_J = 2$). Additionally, there is some excess micromotion present in these scans, represented by the red bands on the plot.

Work is ongoing to perform this procedure for all three transitions (860nm, 748nm, and 735nm) for each isotope. Our focus after performing this spectrum was to obtain the estimated transition strength for the 860nm transition, and use the developed techniques when considering that transition to apply to the other transitions.

8.5 King Plots

8.5.1 Isotope Shift and Linearity

The isotope shift ($\delta\nu$) of transition A is given by $\delta\nu_A = k_A\mu + F_A\delta\langle r^2 \rangle$, where μ is the change in mass between isotopes, and $\delta\langle r^2 \rangle$ is the change in charge distribution due to the different number of neutrons in the nucleus, and K_A and F_A are constants related to the “mass” and “volume” shifts for transition A . For a different transition B , the coefficients K and F will be different, but μ and $\delta\langle r^2 \rangle$ will be the same for the same two isotopes, allowing one to write an equation for the isotope shift $\delta\nu_A$ as a function of the isotope shift $\delta\nu_B$:

$$\delta\nu_A(\delta\nu_B) = \frac{F_A}{F_B}\delta\nu_B + \mu(K_A - \frac{F_A}{F_B}K_B) \quad (8.10)$$

This equation shows us we can plot the isotope shift $\delta\nu_A$ against the isotope shift $\delta\nu_B$, and the result should be a line with slope F_A/F_B , and y intercept of $\mu(K_A - \frac{F_A}{F_B}K_B)$. This is referred to as a King Plot, and it’s expected to be linear because the isotope shift depends

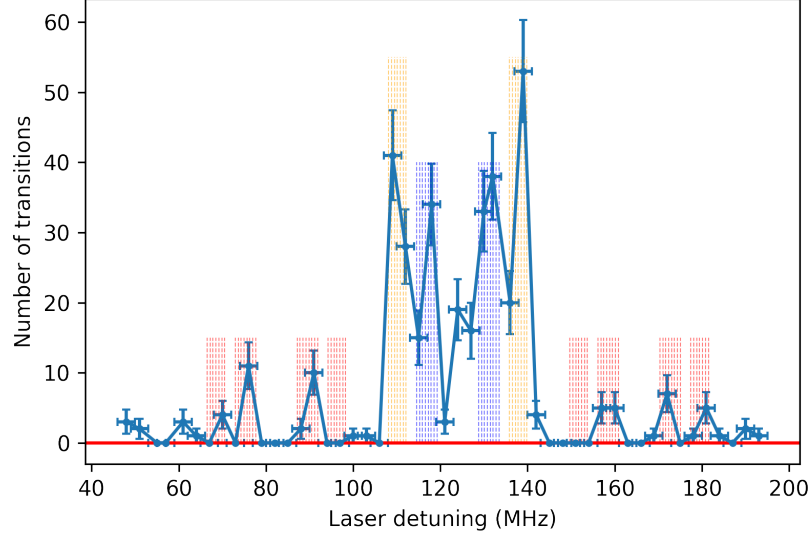


Figure 8.2: Number of suspected 860nm transitions in $^{174}\text{Yb}^+$ which pass our thresholding of requisite dark time form a calibration dataset as a function of laser frequency. Blue bands correspond to theoretically estimated $\Delta m_J = 1$ transitions, and yellow bands correspond to $\Delta m_J = 2$ transitions. Position of the bands are determined relative to the center of the observed spectrum, and calculated based off of the magnetic field in the trap.

on the volume and mass shift linearly.

8.5.2 Nonlinearity of King Plots in Yb

An active area of study is the search for King Plot nonlinearities. These nonlinearities can point toward the existence of new Bosons, however they also help identify and measure higher-order Standard Model nuclear effects. Narrow transitions are best suited for this search, as one can more accurately determine the transition frequency, giving better bounds on the linear fit. Yb^+ is a fantastic choice for this search, as it hosts many narrow transitions, and 5 stable even isotopes. The transitions used for these studies include the E3 transition at 467nm ($^2\text{S}_{1/2} \rightarrow ^2\text{F}_{7/2}^o$), the E2 transition at 435nm ($^2\text{S}_{1/2} \rightarrow ^2\text{D}_{3/2}$), and the E2 transition

Isotope	860nm (THz)	748nm (THz)	735nm (THz)
$^{170}\text{Yb}^+$	348.768479(32)	400.712316(27)	407.732323(27)
$^{172}\text{Yb}^+$	348.765819(32)	400.709365(27)	407.729740(27)
$^{174}\text{Yb}^+$	348.763760(32)	400.707080(27)	407.727740(27)
$^{176}\text{Yb}^+$	348.761796(32)	400.704901(27)	407.725833(27)

Table 8.3: Absolute frequency measurements of the newly observed transitions.

at 411nm $^2\text{S}_{1/2} \rightarrow ^2\text{D}_{5/2}$ [HAC22]. Neutral Yb also has several narrow transitions which have been measured and can be added to the analysis. These include the $^1\text{S}_0 \rightarrow ^3\text{P}_0^o$ transition at 578nm [OSI22], the $^1\text{S}_0 \rightarrow (7/2, 3/2)_2^o$ transition at 431nm [IOT23], and the two photon $^1\text{S}_0 \rightarrow ^1\text{D}_2$ transition at $2 \times 723\text{nm}$ [FBD22].

Our search for new states which we can use to impart a spin-dependent force on the metastable qubit open up new possibilities to add to the multi-dimensional King Plot analysis. We have driven 3 transitions at 860nm ($^2\text{F}_{7/2}^o \rightarrow ^3[7/2]_{9/2}^o$), 748nm ($^2\text{F}_{7/2}^o \rightarrow ^1[11/2]_{11/2}^o$), and 735nm ($^2\text{F}_{7/2}^o \rightarrow ^3[9/2]_{9/2}^o$), and performed measurements of their frequencies in 4 of the 5 even isotopes of Yb^+ : $^{170}\text{Yb}^+$, $^{172}\text{Yb}^+$, $^{174}\text{Yb}^+$, $^{176}\text{Yb}^+$.

8.6 Hyperfine Splittings in $^{171}\text{Yb}^+$

To determine the hyperfine A coefficients of the bracket states in $^{171}\text{Yb}^+$, we must determine the splitting between the hyperfine levels and the order of the levels. To simplify the bracket state spectroscopy process in $^{171}\text{Yb}^+$, we leverage selection rules to populate only 1 hyperfine manifold in the $^2\text{F}_{7/2}^o$ state for the initial search. By driving only to the F=2 manifold in the $^2\text{D}_{5/2}$ state with 411nm light, we can ensure only the F=3 manifold in the $^2\text{F}_{7/2}^o$ state is populated due to selection rules.

We perform initial spectroscopy of these transitions in the same manner as described for

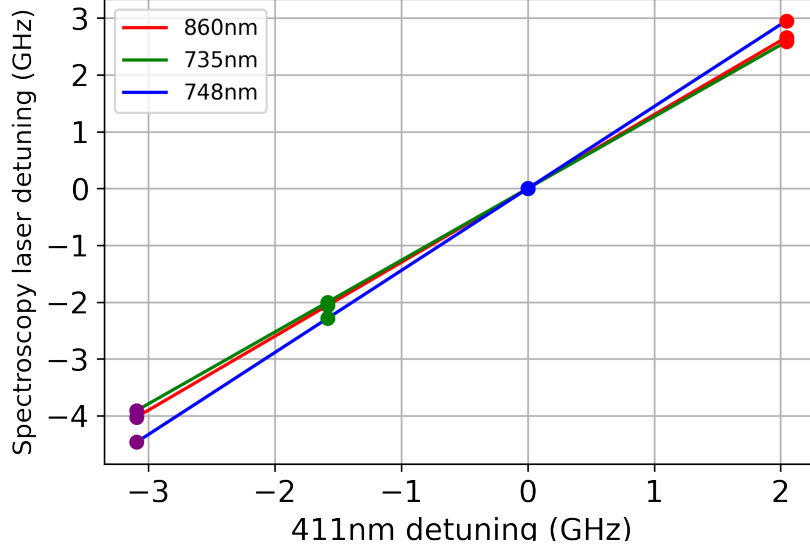


Figure 8.3: King plot of measured transitions. Plotted is the spectroscopy laser frequency vs the 411nm transition frequency for $^{176}\text{Yb}^+$ (purple), $^{174}\text{Yb}^+$ (green), $^{172}\text{Yb}^+$ (blue), $^{170}\text{Yb}^+$ (red).

the spinless isotopes, where we cycle population rapidly between the ground state and the $^2\text{F}_{7/2}^o$ state with the 411nm laser (tuned to the $^2\text{S}_{1/2}, F=1 \rightarrow ^2\text{D}_{5/2}, F=2$ transition), and the 760nm laser to bring population back to the ground state for fluorescence collection. When we scan over a transition with our spectroscopy laser, the ion will exit this cycle and appear dark. This gives us rough frequency information about the transition.

When performing spectroscopy of the target bracket states in $^{171}\text{Yb}^+$, we were able to estimate the hyperfine centroid with our King plot. Once we find one transition from the $^2\text{F}_{7/2}^o$ state, we can use the centroid, the $^2\text{F}_{7/2}^o$ state hyperfine A coefficient, and the frequency of the identified transition to determine the only 2 possible A coefficients of the upper state (one positive, and one negative). We will go through an example using the $^3[7/2]_{9/2}^o$ state.

From figure 8.4, we can write the transition frequency $\nu_{3 \rightarrow f}$ as:

$$\nu_{3 \rightarrow f} = \nu_c - \Delta\text{HF}_{^2\text{F}_{7/2}^o, F=3} + \Delta\text{HF}_{^3[7/2]_{9/2}^o, F=f} \quad (8.11)$$

where $\Delta\text{HF}_{2F_{7/2}^o, F=3}$ is the hyperfine shift of the $F = 3$ manifold in the $2F_{7/2}^o$ state, and $\Delta\text{HF}_{3[7/2]_{9/2}^o, F=f}$ is the hyperfine shift of the $F = f$ manifold in the $3[7/2]_{9/2}^o$ state. The hyperfine shifts can be expanded by using:

$$\Delta\text{HF} = \frac{A}{2} [F(F+1) - I(I+1) - J(J+1)] \quad (8.12)$$

where F , I , and J are the usual quantum numbers to describe atomic states with hyperfine structure. We can solve for $A_{3[7/2]_{9/2}^o}$, and plug in the known quantum numbers to rewrite equation 8.11, as the following.

$$A_{3[7/2]_{9/2}^o} = \frac{2(\nu_{3 \rightarrow f} - \nu_c - \frac{9}{4}A_{2F_{7/2}^o})}{f(f+1) - \frac{51}{2}} \quad (8.13)$$

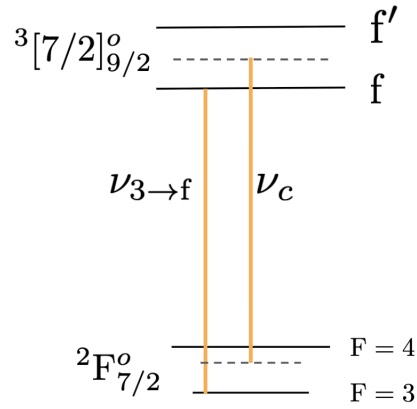


Figure 8.4: Simple diagram of the hyperfine spectroscopy scheme for the 860nm transition. The centroid of the transition is shown as ν_c , and the transition from the lower ($F = 3$) manifold in the $2F_{7/2}^o$ state to one of the excited manifolds ($F = f$) in the $3[7/2]_{9/2}^o$ state.

From equation 8.13, we can see that we can have two different values of $F = f$, and A will either be positive or negative depending on the value of f . For finding the other hyperfine manifold though, this is very helpful, since we now have only 2 possible values of the A coefficient for the upper state, and we can use those two values to find the transition laser frequencies. Using this procedure, we were able to detect the 860nm, 748nm, and 735nm

Upper State	Hyperfine A (MHz)
$^3[7/2]_{9/2}^o$	2131(5)
$^1[11/2]_{11/2}^o$	1616(4)
$^3[9/2]_{9/2}^o$	288(3)

Table 8.4: Measured hyperfine A coefficients of the excited states in $^{171}\text{Yb}^+$.

transitions in $^{171}\text{Yb}^+$, and determine each bracket state’s hyperfine A coefficient. We verified the splitting of each bracket state by populating the $F = 4$ manifold in the $^2\text{F}_{7/2}^o$ state and driving the available transitions from the $F = 4$ manifold as well. Ongoing work is occurring to perform more precise measurements.

8.7 Transition Moments

To evaluate the effectiveness of these transitions for coupling the m -type qubit to motion, we need to have an understanding of the transition moments. This will allow us to calculate the estimated Rabi frequency for coupling to a motional sideband.

8.7.1 Atomic Scale Units

To compare the transition moments to other transitions, we need a common way to compare their magnitudes. The atomic scale electric quadrupole moment is given by ea_0^2 , where e is the electron’s charge, and a_0 is the Bohr radius. In order to compare the magnitude of the transition moments of the newly observed transitions to other E2 transitions, we can scale them to atomic units.

Following [Jam98], Fermi’s golden rule for electric quadrupole transitions is given by

$$A_{E2} = \frac{c\alpha k^5}{15} \sum_{q=-2}^2 |\langle f | r^2 \mathbf{C}_q^{(2)} | i \rangle|^2 \quad (8.14)$$

Where c is the speed of light, α is the fine structure constant, and k is the wavenumber. We then can use the Wigner-Eckart theorem to reduce the matrix element and remove the q dependence in the tensor operator:

$$A_{E2} = \frac{c\alpha k^5}{15} |\langle f | r^2 \mathbf{C}^{(2)} | i \rangle|^2 \sum_{q=-2}^2 \begin{pmatrix} j & 2 & j' \\ -m_j & q & m'_j \end{pmatrix}^2 \quad (8.15)$$

Where the primed quantities refer to the quantum numbers of the excited states. We then sum over all possible lower m_j values, to obtain a more simple expression for A_{E2} :

$$A_{E2} = \frac{c\alpha k^5}{15(2j' + 1)} |\langle f | r^2 \mathbf{C}^{(2)} | i \rangle|^2 \quad (8.16)$$

And we can simply re-arrange and solve for the reduced matrix element:

$$|\langle f | r^2 \mathbf{C}^{(2)} | i \rangle| = \sqrt{\frac{15(2j' + 1)A_{E2}}{c\alpha k^5}} \quad (8.17)$$

To obtain the value of the matrix element in atomic scale units, we simply need to divide this result by a_0^2 , as the resultant reduced matrix element in equation 8.17 is already in units of em^2 .

To validate our equation, we can estimate the reduced matrix element of the two common E2 transitions in Yb^+ ($^2\text{S}_{1/2} \rightarrow ^2\text{D}_{5/2}$ at 411nm, and $^2\text{S}_{1/2} \rightarrow ^2\text{D}_{3/2}$ at 435nm). The lifetime of the $^2\text{D}_{5/2}$ ($F = 3$) state in $^{171}\text{Yb}^+$ is given in [TEM21] as 7.1(4)ms, with a branching ratio back to the $^2\text{S}_{1/2}$ state of 17.6(4)%. The lifetime of the $^2\text{D}_{3/2}$ state in $^{174}\text{Yb}^+$ is given in [SYM23] as 54.83(18)ms. Using these values, we obtain $\approx 12.5ea_0^2$ for the 411 transition in $^{171}\text{Yb}^+$, and $\approx 10.1ea_0^2$ for the 435nm transition in $^{174}\text{Yb}^+$.

For the 411nm transition in Yb^+ , we can look to the literature value given in [TEM21] of $12.5(4)ea_0^2$. In addition, the matrix elements of both were calculated theoretically in [SD11], and the results line up with our estimated values.

This gives us confidence that our calculations are correct, and therefore we can begin to

use our equation to estimate the reduced matrix element (transition moment) of the new transitions to the bracket states based off of lifetime estimates that we extract from data.

8.7.2 Measurement of the Transfer Rate

We can estimate the lifetime of the excited state by performing variable length pulses, and determining the fraction of population transferred to the excited state as a function of time. This is in effect very similar to the 411nm “shelving rate” experiments. In fact, it’s useful to use the 411nm transition to test our procedure for measuring the transition moments. A simplified system of equations for the shelving process can be written upon inspection in the form of Einstein A and B coefficients as:

$$S'(t) = A\zeta D(t) + B\rho(w)D(t) - B\rho(w)S(t) \quad (8.18)$$

$$D'(t) = -AD(t) - B\rho(w)D(t) + B\rho(w)S(t) \quad (8.19)$$

$$F'(t) = A(1 - \zeta)D(t) \quad (8.20)$$

Where ζ is the branching ratio from the $^2D_{5/2} \rightarrow ^2S_{1/2}$ transition, and $\rho(w)$ is the *energy* spectral density of the laser at the ion. We can write ρ as

$$\rho(w) = \frac{P}{cA\Delta w} \quad (8.21)$$

Where P is the power of the laser at the ion, c is the speed of light, A is the cross sectional area of the laser beam, and Δw is the linewidth of the laser.

Measuring the linewidth of a laser is an incredibly challenging problem, and our estimated of the linewidth of our light sources is ongoing. For our purposes here, we will assume both the 411nm laser and our Ti:Sapph laser have a similar linewidth of 1MHz, with an error of 50%. The cross sectional area of the beam will be calculated by $A = \pi w_0^2$, where w_0 is the $1/e^2$ beam waist radius.

Using data collected in the experiment for figure 6.2, we can extract the Einstein A coefficient for the curve when fitting to the system of differential equations 8.18, along with our values for the power ($597\mu\text{W}$), beam waist ($44.5\mu\text{m}$), and linewidth (1MHz). The results give an Einstein A coefficient of $A = 2\pi \times 16\text{Hz}$, which is reasonably close to the measured value of $A = 2\pi \times 22\text{Hz}$ [TEM21].

We can now try to apply this process to our spectroscopy targets. The sequence of events that we use for this measurement is:

- Collect Doppler cooling counts, and require presence of a bright ion before beginning the sequence
- Perform electron shelving on the $^2\text{S}_{1/2}, m_J = 1/2 \rightarrow ^2\text{D}_{5/2}, m_J = 5/2$ transition
- Collect photon counts to Herald the preparation of population in the $^2\text{F}_{7/2}^o$ state, only experiments that return a dark ion are considered
- Perform a variable length 860nm pulse
- Deshelve population
- Collect photons to determine if population was transferred to the excited state

The results of such an experiment is given in figure 8.5. From our fitted parameters, we find the fitted Einstein A coefficient of the 860nm transition to be $A = (2\pi) \times 0.6 \pm 0.1\text{mHz}$, where the error here is the error in the fit. The fractional error on $\rho(\omega)$ is about 0.54, since we do not know our laser linewidth well. Using this as our fractional error, we end up with a final value for the A coefficient of $A = 2\pi \times 6 \pm 3\text{mHz}$. The transition moment is then given by equation 8.17 divided by the Bohr radius squared: $(1.26 \pm 0.3)ea_0^2$. Work is ongoing to reduce our error in the linewidth estimate.

This means our initial estimate of the linewidth of this transition is rather poor, as the measured value is three orders of magnitude smaller than the estimate based on the linewidth

of the other E2 transitions from the ground state of Yb^+ . Instead of a 10Hz feature in a 10GHz range, we are looking for a 1mHz feature in a 10GHz range, 1 part in 10^{13} . This experiment also tells us that the lifetime of the excited state is M1 *limited*, meaning the primary contribution to the lifetime is the M1 decay processes.

Work is currently ongoing to obtain estimates of the lifetime and transition moments of the other two transitions (735nm and 748nm).

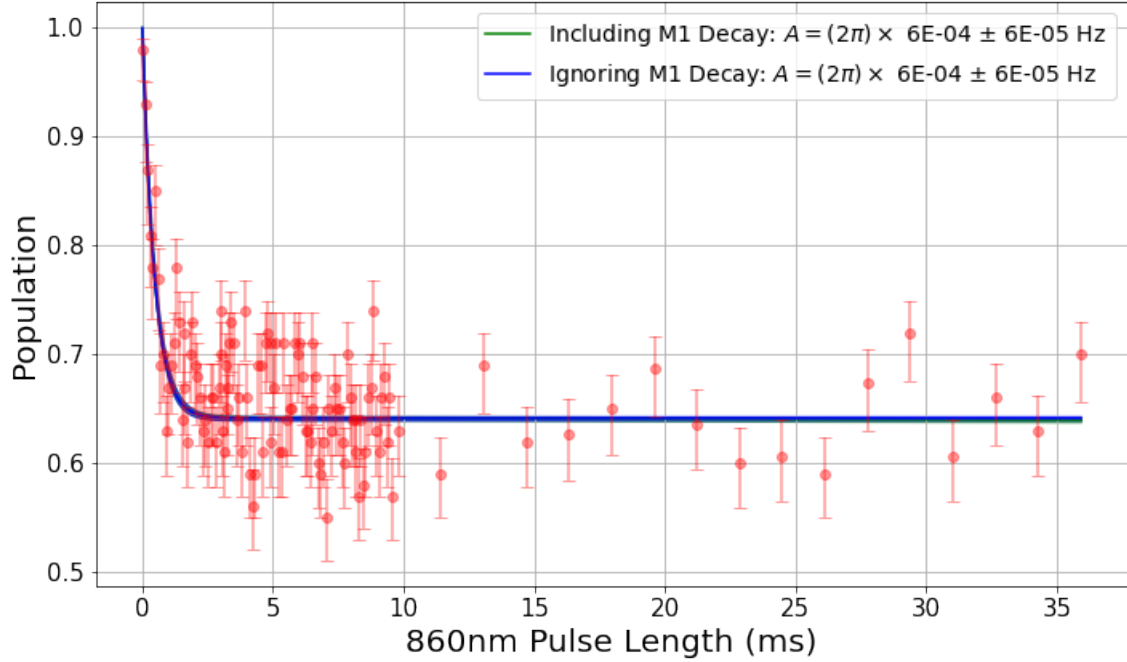


Figure 8.5: Transfer rate experiment for the 860nm transition in $^{174}\text{Yb}^+$. Population in the $^2\text{F}_{7/2}^o$ state is measured after a variable length spectroscopy pulse. Data are fit to a 2- and 3-level rate equation model. The 3-level model accounts for the M1 lifetime of the state (calculated to be 4s), whereas the 2-level model ignores this decay. The results of both models are consistent for this data. The presented errorbar is the error in the fit.

8.7.3 Estimating Rabi Frequencies

We can use our measurement of the A_{E2} coefficient on the 860nm transition to estimate the Rabi frequency we can achieve with reasonable laser parameters, and compare it to a commonly trapped ion: $^{40}\text{Ca}^+$. For this comparison, we will use the calculations made in [Roo00]. For a fair comparison, we will assume the secular frequency of both ions is $\omega_s = 2\pi \times 1\text{MHz}$. This leads to Lamb-Dicke parameters of $\eta_{\text{Ca}} = 0.09$, and $\eta_{\text{Yb}} = 0.04$. We will also use the same laser parameters as in [Roo00], with 100mW of light focused onto a $30\mu\text{m}$ $1/e^2$ waist radius. Finally, we will assume a starting motional occupation of $n = 10$.

The quadrupole transition used in $^{40}\text{Ca}^+$ is the $^2\text{S}_{1/2} \rightarrow ^2\text{D}_{5/2}$ transition at 729nm. For sideband cooling, the $^2\text{S}_{1/2}, m_J = -1/2 \rightarrow ^2\text{D}_{5/2}, m_J = -5/2$ transition is used. For the 860nm transition in $^{174}\text{Yb}^+$, we will use the $^2\text{F}_{7/2}^o, m_J = 7/2 \rightarrow ^3[7/2]_{9/2}^o, m_J = 9/2$ transition. We can use equation 3.53 to obtain the Rabi frequency of both of the E2 transitions given the specific quantum numbers for each ion, along with the transitions wavelength. In the Lamb-Dicke regime $\eta^2(2n+1) \ll 1$, the carrier ($n' = n$) Rabi frequency is given by $\Omega_{n,n} = \Omega(1 - \eta^2 n)$, while the “red” sideband ($n' = n - 1$) Rabi frequency is given by $\Omega_{n,n-1} = \Omega\eta\sqrt{n}$.

With these parameters, we find the carrier Rabi frequency for the 729nm transition in $^{40}\text{Ca}^+$ is $\Omega_{n,n} = 2\pi \times 1.1\text{MHz}$, while the red sideband is $\Omega_{n,n-1} = 2\pi \times 342\text{kHz}$. The carrier Rabi frequency for the 860nm transition is $\Omega_{n,n} = 2\pi \times 69\text{kHz}$, and the red sideband Rabi frequency is $\Omega_{n,n-1} = 2\pi \times 8.9\text{kHz}$. If we are able to stabilize our laser to $\approx 1\text{kHz}$, we may be able to drive a motional sideband on this transition. Of course, since the Rabi frequency scales inversely to the waist, we can achieve higher rates if we reduce the beam size, which is certainly achievable. Stabilization to 1kHz will be difficult, but with a high finesse cavity it may be possible.

REFERENCES

- [ACC21] D. T. C. Allcock, W. C. Campbell, J. Chiaverini, I. L. Chuang, E. R. Hudson, I. D. Moore, A. Ransford, C. Roman, J. M. Sage, and D. J. Wineland. “*omg* blueprint for trapped ion quantum computing with metastable states.” *Applied Physics Letters*, **119**(21):214002, 11 2021.
- [ARS22] Fangzhao Alex An, Anthony Ransford, Andrew Schaffer, Lucas R. Sletten, John Gaebler, James Hostetter, and Grahame Vittorini. “High Fidelity State Preparation and Measurement of Ion Hyperfine Qubits with $I > \frac{1}{2}$.” *Physical Review Letters*, **129**(13):130501, September 2022. Publisher: American Physical Society.
- [BBF21] C. H. Baldwin, B. J. Bjork, M. Foss-Feig, J. P. Gaebler, D. Hayes, M. G. Kokish, C. Langer, J. A. Sedlacek, D. Stack, and G. Vittorini. “High-fidelity light-shift gate for clock-state qubits.” *Physical Review A*, **103**(1):012603, January 2021. Publisher: American Physical Society.
- [BDL00] P. A. Barton, C. J. S. Donald, D. M. Lucas, D. A. Stevens, A. M. Steane, and D. N. Stacey. “Measurement of the lifetime of the $3d\ ^2D_{5/2}$ state in $^{40}\text{Ca}^+$.” *Physical Review A*, **62**(3):032503, August 2000. Publisher: American Physical Society.
- [Ben08] J. Benhelm. “Precision Spectroscopy and Quantum Information Processing with Trapped Calcium Ions.” 2008.
- [BHR04] R. Blatt, H. Häffner, C. F. Roos, C. Becher, and F. Schmidt-Kaler. “Ion Trap Quantum Computing with Ca^+ Ions.” *Quantum Information Processing*, **3**(1):61–73, October 2004.
- [BMB98] D. J. Berkeland, J. D. Miller, J. C. Bergquist, W. M. Itano, and D. J. Wineland. “Minimization of ion micromotion in a Paul trap.” *Journal of Applied Physics*, **83**(10):5025–5033, May 1998.
- [BSS99] S. Bize, Y. Sortais, M. S. Santos, C. Mandache, A. Clairon, and C. Salomon. “High-accuracy measurement of the ^{87}Rb ground-state hyperfine splitting in an atomic fountain.” *Europhysics Letters*, **45**(5):558, March 1999. Publisher: IOP Publishing.
- [CCV19] Stephen Crain, Clinton Cahall, Geert Vrijsen, Emma E. Wollman, Matthew D. Shaw, Varun B. Verma, Sae Woo Nam, and Jungsang Kim. “High-speed low-crosstalk detection of a $^{171}\text{Yb}^+$ qubit using superconducting nanowire single photon detectors.” *Communications Physics*, **2**(1):97, August 2019. Publisher: Nature Publishing Group.

- [CXN21] Nathanaël Cottet, Haonan Xiong, Long B. Nguyen, Yen-Hsiang Lin, and Vladimir E. Manucharyan. “Electron shelving of a superconducting artificial atom.” *Nature Communications*, **12**(1):6383, November 2021. Publisher: Nature Publishing Group.
- [Del25] Thomas Dellaert. *Metastable States, Magic Polarization, and Hyperfine Spectroscopy in $^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$* . PhD thesis, UCLA, 2025.
- [DF16] V. A. Dzuba and V. V. Flambaum. “Hyperfine-induced electric dipole contributions to the electric octupole and magnetic quadrupole atomic clock transitions.” *Physical Review A*, **93**(5):052517, May 2016. Publisher: American Physical Society.
- [DiV00] David P. DiVincenzo. “The Physical Implementation of Quantum Computation.” *Fortschritte der Physik*, **48**(9-11):771–783, 2000. _eprint: [https://onlinelibrary.wiley.com/doi/pdf/10.1002/1521-3978%28200009%2948%3A9/11%3C771%3A%3AAID-PROP771%3E3.0.CO%3B2-E](https://onlinelibrary.wiley.com/doi/pdf/10.1002/1521-3978%28200009%2948%3A9%3A11%3C771%3A%3AAID-PROP771%3E3.0.CO%3B2-E).
- [DRM24] N. A. Diepeveen, C. Robalo Pereira, M. Mazzanti, Z. E. D. Ackerman, L. P. H. Gallagher, T. Timmerman, R. Gerritsma, and R. X. Schüssler. “Single-ion spectroscopy of four metastable-state clear-out transitions in Yb^+ : Isotope shifts and hyperfine structure.” *Physical Review A*, **110**(4):042809, October 2024. Publisher: American Physical Society.
- [ETH10] S. Ejtemaee, R. Thomas, and P. C. Haljan. “Optimization of Yb^+ fluorescence and hyperfine-qubit detection.” *Physical Review A*, **82**(6):063419, December 2010. Publisher: American Physical Society.
- [FBD22] N.L. Figueroa, J.C. Berengut, V.A. Dzuba, V.V. Flambaum, D. Budker, and D. Antypas. “Precision Determination of Isotope Shifts in Ytterbium and Implications for New Physics.” *Physical Review Letters*, **128**(7):073001, February 2022. Publisher: American Physical Society.
- [FTS24] Ran Finkelstein, Richard Bing-Shiun Tsai, Xiangkai Sun, Pascal Scholl, Su Direkci, Tuvia Gefen, Joonhee Choi, Adam L. Shaw, and Manuel Endres. “Universal quantum operations and ancilla-based read-out for tweezer clocks.” *Nature*, **634**(8033):321–327, October 2024. Publisher: Nature Publishing Group.
- [GTL16] J.P. Gaebler, T.R. Tan, Y. Lin, Y. Wan, R. Bowler, A.C. Keith, S. Glancy, K. Coakley, E. Knill, D. Leibfried, and D.J. Wineland. “High-Fidelity Universal Gate Set for $^9\text{Be}^+$ Ion Qubits.” *Physical Review Letters*, **117**(6):060505, August 2016. Publisher: American Physical Society.

- [HAC22] Joonseok Hur, Diana P.L. Aude Craik, Ian Counts, Eugene Knyazev, Luke Caldwell, Calvin Leung, Swadha Pandey, Julian C. Berengut, Amy Geddes, Witold Nazarewicz, Paul-Gerhard Reinhard, Akio Kawasaki, Honggi Jeon, Wonho Jhe, and Vladan Vuletić. “Evidence of Two-Source King Plot Nonlinearity in Spectroscopic Search for New Boson.” *Physical Review Letters*, **128**(16):163201, April 2022. Publisher: American Physical Society.
- [HHJ11] J P Home, D Hanneke, J D Jost, D Leibfried, and D J Wineland. “Normal modes of trapped ions in the presence of anharmonic trap potentials.” *New Journal of Physics*, **13**(7):073026, July 2011.
- [HOL12] N. Huntemann, M. Okhapkin, B. Lipphardt, S. Weyers, Chr. Tamm, and E. Peik. “High-Accuracy Optical Clock Based on the Octupole Transition in $^{171}\text{Yb}^+$.” *Physical Review Letters*, **108**(9):090801, February 2012. Publisher: American Physical Society.
- [HXZ24] J. Z. Han, N. C. Xin, J. W. Zhang, Y. M. Yu, J. G. Li, L. Qian, and L. J. Wang. “Determination of the ground-state hyperfine splitting of trapped $^{171}\text{Yb}^+$ ions.” *Applied Physics Letters*, **125**(8):084002, August 2024.
- [IOT23] Taiki Ishiyama, Koki Ono, Tetsushi Takano, Ayaki Sunaga, and Yoshiro Takahashi. “Observation of an Inner-Shell Orbital Clock Transition in Neutral Ytterbium Atoms.” *Physical Review Letters*, **130**(15):153402, April 2023. Publisher: American Physical Society.
- [Ita06] Wayne Itano. “Quadrupole moments and hyperfine constants of metastable states of Ca^+ , Sr^+ , Ba^+ , Yb^+ , Hg^+ , and Au .” *Physical Review A*, **73**(2):022510, February 2006.
- [Jam98] D.F.V. James. “Quantum dynamics of cold trapped ions with application to quantum computation.” *Applied Physics B*, **66**(2):181–190, February 1998.
- [JBE11] M. Johanning, A. Braun, D. Eiteneuer, C. Paape, C. Balzer, W. Neuhauser, and C. Wunderlich. “Resonance-enhanced isotope-selective photoionization of YbI for ion trap loading.” *Applied Physics B*, **103**(2):327–338, May 2011.
- [KBZ09] G. Kirchmair, J. Benhelm, F. Zähringer, R. Gerritsma, C. F. Roos, and R. Blatt. “High-fidelity entanglement of $^{43}\text{Ca}^+$ hyperfine clock states.” *Physical Review A*, **79**(2):020304, February 2009. Publisher: American Physical Society.
- [KGB16] Michaela Kleinert, M. E. Gold Dahl, and Scott Bergeson. “Measurement of the $\text{Yb I } ^1\text{S}_0\text{-}^1\text{P}_1$ transition frequency at 399 nm using an optical frequency comb.” *Physical Review A*, **94**(5):052511, November 2016. Publisher: American Physical Society.

- [KGM23] Nikhil Kotibhaskar, Noah Greenberg, Sainath Motlakunta, Chung-You Shih, and Rajibul Islam. “Fast and high-yield fabrication of axially symmetric ion-trap needle electrodes via two step electrochemical etching.” *Review of Scientific Instruments*, **94**(3):033201, March 2023.
- [KLR08] E. Knill, D. Leibfried, R. Reichle, J. Britton, R. B. Blakestad, J. D. Jost, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland. “Randomized benchmarking of quantum gates.” *Physical Review A*, **77**(1):012307, January 2008. Publisher: American Physical Society.
- [Kum15] Muir Kumph. “2D Arrays of Ion Traps for Large Scale Integration of Quantum Information Processors.” 2015.
- [KYR24] A. Kramida, Yu. Ralchenko, J. Reader, and and NIST ASD Team, 2024.
- [LBB03] John R. de Laeter, John Karl Böhlke, P. De Bièvre, H. Hidaka, H. S. Peiser, K. J. R. Rosman, and P. D. P. Taylor. “Atomic weights of the elements. Review 2000 (IUPAC Technical Report).” *Pure and Applied Chemistry*, **75**(6):683–800, January 2003. Publisher: De Gruyter.
- [LPH21] R. Lange, A.A. Peshkov, N. Huntemann, Chr. Tamm, A. Surzhykov, and E. Peik. “Lifetime of the $^2F_{7/2}$ Level in Yb^+ for Spontaneous Emission of Electric Octupole Radiation.” *Physical Review Letters*, **127**(21):213001, November 2021. Publisher: American Physical Society.
- [MBA23] S.A. Moses, C.H. Baldwin, M.S. Allman, R. Ancona, L. Ascarrunz, C. Barnes, J. Bartolotta, B. Bjork, P. Blanchard, M. Bohn, J.G. Bohnet, N.C. Brown, N.Q. Burdick, W.C. Burton, S.L. Campbell, J.P. Campora, C. Carron, J. Chambers, J.W. Chan, Y.H. Chen, A. Chernoguzov, E. Chertkov, J. Colina, J.P. Curtis, R. Daniel, M. DeCross, D. Deen, C. Delaney, J.M. Dreiling, C.T. Ertsgaard, J. Esposito, B. Estey, M. Fabrikant, C. Figgatt, C. Foltz, M. Foss-Feig, D. Francois, J.P. Gaebler, T.M. Gatterman, C.N. Gilbreth, J. Giles, E. Glynn, A. Hall, A.M. Hankin, A. Hansen, D. Hayes, B. Higashi, I.M. Hoffman, B. Horning, J.J. Hout, R. Jacobs, J. Johansen, L. Jones, J. Karcz, T. Klein, P. Lauria, P. Lee, D. Liefer, S.T. Lu, D. Lucchetti, C. Lytle, A. Malm, M. Matheny, B. Mathewson, K. Mayer, D.B. Miller, M. Mills, B. Neyenhuis, L. Nugent, S. Olson, J. Parks, G.N. Price, Z. Price, M. Pugh, A. Ransford, A.P. Reed, C. Roman, M. Rowe, C. Ryan-Anderson, S. Sanders, J. Sedlacek, P. Shevchuk, P. Siegfried, T. Skripka, B. Spaun, R.T. Sprenkle, R.P. Stutz, M. Swallows, R.I. Tobey, A. Tran, T. Tran, E. Vogt, C. Volin, J. Walker, A.M. Zolot, and J.M. Pino. “A Race-Track Trapped-Ion Quantum Processor.” *Physical Review X*, **13**(4):041052, December 2023. Publisher: American Physical Society.

- [MBG87] A. Münch, M. Berkler, Ch. Gerz, D. Wilsdorf, and G. Werth. “Precise ground-state hyperfine splitting in ^{173}Yb ii.” *Physical Review A*, **35**(10):4147–4150, May 1987. Publisher: American Physical Society.
- [MBL22] Shuo Ma, Alex P. Burgers, Genyue Liu, Jack Wilson, Bichen Zhang, and Jeff D. Thompson. “Universal Gate Operations on Nuclear Spin Qubits in an Optical Tweezer Array of ^{171}Yb Atoms.” *Physical Review X*, **12**(2):021028, May 2022. Publisher: American Physical Society.
- [MCD25] Mason Marshall, Daniel Rodriguez Castillo, Willa Dworschack, Alexander Aepli, Kyungtae Kim, Dahyeon Lee, William Warfield, Nicholas Nardelli, Tara Fortier, Jun Ye, David Ray Leibbrandt, and David Hume. “High-Stability Single-Ion Clock with 5.5×10^{-19} Systematic Uncertainty.” April 2025. Publisher: arXiv.
- [Meg67] William F. Meggers. “The Second Spectrum of Ytterbium (Yb II).” *Journal of Research of the National Bureau of Standards. Section A, Physics and Chemistry*, **71A**(6):396–546, 1967.
- [MGH94] Ann-Marie Mårtensson-Pendrill, David S. Gough, and Peter Hannaford. “Isotope shifts and hyperfine structure in the 369.4-nm $6s-6p_{1/2}$ resonance line of singly ionized ytterbium.” *Physical Review A*, **49**(5):3351–3365, May 1994. Publisher: American Physical Society.
- [MSW23] Jorge Moreno, Fabian Schmid, Johannes Weitenberg, Savely G. Karshenboim, Theodor W. Hänsch, Thomas Udem, and Akira Ozawa. “Toward XUV frequency comb spectroscopy of the $1S-2S$ transition in He^+ .” *The European Physical Journal D*, **77**(4):67, April 2023.
- [MZL24] Xueying Mai, Liyun Zhang, and Yao Lu. “High-Fidelity Detection on $^{171}\text{Yb}^+$ Qubit via $^2D_{3/2}$ Shelving.”, 2024.
- [NHT78] W. Neuhauser, M. Hohenstatt, P. Toschek, and H. Dehmelt. “Optical-Sideband Cooling of Visible Atom Cloud Confined in Parabolic Well.” *Physical Review Letters*, **41**(4):233–236, July 1978. Publisher: American Physical Society.
- [NVG13] Rachel Noek, Geert Vrijsen, Daniel Gaultney, Emily Mount, Taehyun Kim, Peter Maunz, and Jungsang Kim. “High speed, high fidelity detection of an atomic hyperfine qubit.” *Optics Letters*, **38**(22):4735–4738, November 2013. Publisher: Optica Publishing Group.
- [OHM09] S. Olmschenk, D. Hayes, D. N. Matsukevich, P. Maunz, D. L. Moehring, K. C. Younge, and C. Monroe. “Measurement of the lifetime of the $6p\ ^2P_{1/2}^o$ level of Yb^+ .” *Physical Review A*, **80**(2):022502, August 2009. Publisher: American Physical Society.

- [Olm09] Steven Matthew Olmschenk. *Quantum Teleportation Between Distant Matter Qubits*. Thesis, 2009. Accepted: 2009-09-03T14:41:18Z.
- [OSI22] Koki Ono, Yugo Saito, Taiki Ishiyama, Toshiya Higomoto, Tetsushi Takano, Yosuke Takasu, Yasuhiro Yamamoto, Minoru Tanaka, and Yoshiro Takahashi. “Observation of Nonlinearity of Generalized King Plot in the Search for New Boson.” *Physical Review X*, **12**(2):021033, May 2022. Publisher: American Physical Society.
- [OYM07] S. Olmschenk, K. C. Younge, D. L. Moehring, D. N. Matsukevich, P. Maunz, and C. Monroe. “Manipulation and detection of a trapped Yb^+ hyperfine qubit.” *Physical Review A*, **76**(5):052314, November 2007. Publisher: American Physical Society.
- [PK15] Thaned Pruttivarasin and Hidetoshi Katori. “Compact field programmable gate array-based pulse-sequencer and radio-frequency generator for experiments with trapped atoms.” *Review of Scientific Instruments*, **86**(11):115106, November 2015.
- [QP20] Pascal Quinet and Patrick Palmeri. “Current Status and Developments of the Atomic Database on Rare-Earths at Mons University (DREAM).” *Atoms*, **8**(2):18, June 2020. Number: 2 Publisher: Multidisciplinary Digital Publishing Institute.
- [Ran20] Anthony M. Ransford. *Old Dog, New Trick: High Fidelity, Background-free State Detection of an Ytterbium Ion Qubit*. PhD thesis, UCLA, 2020.
- [Rom21] Conrad Roman. *Expanding the $^{171}\text{Yb}^+$ toolbox: the $^2F_{7/2}$ state as resource for quantum information science*. PhD thesis, UCLA, 2021.
- [Roo00] Roos, Christian. *Controlling the quantum state of trapped ions*. PhD thesis, Leopold-Franzens-Universität Innsbruck, 2000.
- [RRD21] Anthony Ransford, Conrad Roman, Thomas Dellaert, Patrick McMillin, and Wesley C. Campbell. “Weak dissipation for high-fidelity qubit-state preparation and measurement.” *Physical Review A*, **104**(6):L060402, December 2021. Publisher: American Physical Society.
- [RTG99] M. Roberts, P. Taylor, S. V. Gateva-Kostova, R. B. M. Clarke, W. R. C. Rowley, and P. Gill. “Measurement of the $^2\text{S}_{1/2}$ - $^2\text{D}_{5/2}$ clock transition in a single ^{171}Yb ion.” *Physical Review A*, **60**(4):2867–2872, October 1999. Publisher: American Physical Society.
- [SD11] B. K. Sahoo and B. P. Das. “Parity nonconservation in ytterbium ion.” *Physical Review A*, **84**(1):010502, July 2011. Publisher: American Physical Society.

- [SDN09] M. M. Schauer, J. R. Danielson, A.-T. Nguyen, L.-B. Wang, X. Zhao, and J. R. Torgerson. “Collisional population transfer in trapped Yb^+ ions.” *Physical Review A*, **79**(6):062705, June 2009. Publisher: American Physical Society.
- [SLV24] A. S. Sotirova, J. D. Leppard, A. Vazquez-Brennan, S. M. Decoppet, F. Pokorny, M. Malinowski, and C. J. Ballance. “High-fidelity heralded quantum state preparation and measurement.”, 2024.
- [SSW12] J. D. Sivers, L. R. Simkins, S. Weidt, and W. K. Hensinger. “On the application of radio frequency voltages to ion traps via helical resonators.” *Applied Physics B*, **107**(4):921–934, June 2012.
- [Sug99] Kazuhiko Sugiyama. “Laser Cooling of Single $^{174}\text{Yb}^+$ Ions Stored in a RF Trap.” *Japanese Journal of Applied Physics*, **38**(4R):2141, April 1999. Publisher: IOP Publishing.
- [SYM23] H. Shao, H. Yue, Z. Ma, Y. Huang, H. Guan, and K. Gao. “Precision determination of the $5d\ ^2D_{3/2}$ state lifetime of single $^{174}\text{Yb}^+$ ion.” *Physical Review Research*, **5**(2):023193, June 2023. Publisher: American Physical Society.
- [TEM21] T. R. Tan, C. L. Edmunds, A. R. Milne, M. J. Biercuk, and C. Hempel. “Precision characterization of the $^2D_{5/2}$ state and the quadratic Zeeman coefficient in $^{171}\text{Yb}^+$.” *Physical Review A*, **104**(1):L010802, July 2021. Publisher: American Physical Society.
- [TKH04] Y. Takasu, K. Komori, K. Honda, M. Kumakura, T. Yabuzaki, and Y. Takahashi. “Photoassociation Spectroscopy of Laser-Cooled Ytterbium Atoms.” *Physical Review Letters*, **93**(12):123202, September 2004. Publisher: American Physical Society.
- [TPL21] A. Tretiakov, C. A. Potts, Y. Y. Lu, J. P. Davis, and L. J. LeBlanc. “Engineering atomic polarization with microwave-assisted optical pumping.” October 2021. eprint: 2110.10673.
- [WDW78] D. J. Wineland, R. E. Drullinger, and F. L. Walls. “Radiation-Pressure Cooling of Bound Resonant Absorbers.” *Physical Review Letters*, **40**(25):1639–1642, June 1978. Publisher: American Physical Society.
- [WHD24] A. Wilzewski, L. I. Huber, M. Door, J. Richter, A. Mariotti, L. J. Spieß, M. Wehrheim, S. Chen, S. A. King, P. Micke, M. Filzinger, M. R. Steinel, N. Huntemann, E. Benkler, P. O. Schmidt, J. Flannery, R. Matt, M. Stadler, R. Oswald, F. Schmid, D. Kienzler, J. Home, D. P. L. Aude Craik, S. Eliseev, P. Filianin, J. Herkenhoff, K. Kromer, K. Blaum, V. A. Yerokhin, I. A. Valuev, N. S. Oreshkina, C. Lyu, S. Banerjee, C. H. Keitel, Z. Harman, J. C. Berengut, A. Viatkina, J. Gilles, A. Surzhykov, M. K. Rosner, J. R. Crespo López-Urrutia,

- and E. Fuchs. “Nonlinear calcium King plot constrains new bosons and nuclear properties.”, December 2024. arXiv:2412.10277 [physics].
- [XLC20] Di Xiao, Jiguang Li, Wesley C. Campbell, Thomas Dellaert, Patrick McMillin, Anthony Ransford, Conrad Roman, and Andrei Derevianko. “Hyperfine structure of $^{173}\text{Yb}^+$: Toward resolving the Yb 173 nuclear-octupole-moment puzzle.” *Physical Review A*, **102**(2):022810, August 2020.
- [YMW21] H.-X. Yang, J.-Y. Ma, Y.-K. Wu, Y. Wang, M.-M. Cao, W.-X. Guo, Y.-Y. Huang, L. Feng, Z.-C. Zhou, and L.-M. Duan. “Realizing coherently convertible dual-type qubits with the same ion species.” *arXiv:2106.14906 [physics, physics:quant-ph]*, June 2021. arXiv: 2106.14906.
- [YSD15] Bryce Yoshimura, Marybeth Stork, Danilo Dadić, Wesley C. Campbell, and James K. Freericks. “Creation of two-dimensional Coulomb crystals of ions in oblate Paul traps for quantum simulations.” *EPJ Quantum Technology*, **2**(1):2, December 2015.
- [YSJ96] Jun Ye, Steve Swartz, Peter Jungner, and John L. Hall. “Hyperfine structure and absolute frequency of the ^{87}Rb $5\text{P}_{3/2}$ state.” *Optics Letters*, **21**(16):1280, August 1996. Publisher: Optica Publishing Group.
- [ZSB21] I. V. Zalivako, I. A. Semerikov, A. S. Borisenko, M. D. Aksenov, K. Yu. Khabarova, and N. N. Kolachevsky. “Experimental Study of the Optical Qubit on the 435-nm Quadrupole Transition in the $^{171}\text{Yb}^+$ Ion.” *JETP Letters*, **114**(2):59–64, July 2021.