

Supplementary Material for Suppression of differential light shifts in ground and metastable trapped-ion qubits

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Isotope	I	g_I
⁹ Be	3/2	-0.78495
²⁵ Mg	5/2	-0.34218
⁴³ Ca	7/2	-0.37637
⁸⁷ Sr	9/2	-0.24284
¹³³ Ba	1/2	-1.5433
¹³⁵ Ba	3/2	0.55863
¹³⁷ Ba	3/2	0.62491
¹⁷¹ Yb	1/2	0.98734
¹⁷³ Yb	5/2	-0.272008(12)

TABLE I. Nuclear spins and nuclear g factors used in the numerical calculation of critical magnetic fields. Nuclear magnetic moments are taken from Ref. [1], and the g_I values are obtained from the tabulated moments μ using $g_I = \mu/(I\mu_N)$.

State	Hyperfine A constant (MHz)	Reference
¹⁷¹ Yb ⁺		
4f ¹⁴ 6s ² S _{1/2}	12642.812118466(2)	[2]
4f ¹⁴ 6p ² P _{1/2} ^o	2104.9(1.3)	[3]
4f ¹⁴ 6p ² P _{3/2} ^o	875.4(1.0)	[4]
4f ¹⁴ 5d ² D _{3/2}	430.0	[5]
4f ¹⁴ 5d ² D _{5/2}	-63.6(7)	[6]
¹⁷³ Yb ⁺		
4f ¹⁴ 6s ² S _{1/2}	-3497.24007985(3)	[7]
4f ¹⁴ 6p ² P _{1/2} ^o	-581.2(4)	[3]
4f ¹⁴ 6p ² P _{3/2} ^o	-245(10)	[8]
4f ¹⁴ 5d ² D _{3/2}	-118.258070(120)	[9]
4f ¹⁴ 5d ² D _{5/2}	20(20)	[10]

TABLE II. Hyperfine A constants used to calculate the critical magnetic fields for ¹⁷¹Yb⁺ and ¹⁷³Yb⁺. The reduced electric-dipole matrix elements are given in the main text.

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Ba ⁺		
Transition	Reduced $E1$ matrix element (a.u.)	Reference
$6s\ ^2S_{1/2} - 6p\ ^2P_{1/2}^o$	3.3251(21)	[11]
$6s\ ^2S_{1/2} - 6p\ ^2P_{3/2}^o$	4.7017(27)	[11]
$5d\ ^2D_{3/2} - 6p\ ^2P_{1/2}^o$	3.0413(21)	[12]
$5d\ ^2D_{3/2} - 6p\ ^2P_{3/2}^o$	1.33199(96)	[13]
$5d\ ^2D_{5/2} - 6p\ ^2P_{3/2}^o$	4.1028(25)	[13]
State	Hyperfine A constant (MHz)	Reference
¹³³ Ba ⁺		
$6s\ ^2S_{1/2}$	-9925.45355459(10)	[14]
$6p\ ^2P_{1/2}^o$	-1840(11)	[15]
$6p\ ^2P_{3/2}^o$	311.5(10.0)	[16]
$5d\ ^2D_{3/2}$	-468.5(10.0)	[17]
$5d\ ^2D_{5/2}$	27.7(6.7)	[16]
¹³⁵ Ba ⁺		
$6s\ ^2S_{1/2}$	3591.67011745(29)	[18]
$6p\ ^2P_{1/2}^o$	664.6(3)	[19]
$6p\ ^2P_{3/2}^o$	113.0(1)	[19]
$5d\ ^2D_{3/2}$	169.5892(9)	[20]
$5d\ ^2D_{5/2}$	-10.735(2)	[20]
¹³⁷ Ba ⁺		
$6s\ ^2S_{1/2}$	4018.87083385(18)	[21]
$6p\ ^2P_{1/2}^o$	743.7(3)	[19]
$6p\ ^2P_{3/2}^o$	127.2(2)	[19]
$5d\ ^2D_{3/2}$	189.7296(7)	[20]
$5d\ ^2D_{5/2}$	-12.028(11)	[20]

TABLE III. Reduced electric-dipole matrix elements and hyperfine A constants used to calculate the critical magnetic fields for ¹³³Ba⁺, ¹³⁵Ba⁺, and ¹³⁷Ba⁺.

⁸⁷ Sr ⁺		
Transition	Reduced $E1$ matrix element (a.u.)	Reference
$4p^6 5s\ ^2S_{1/2} - 4p^6 5p\ ^2P_{1/2}^o$	3.078(17)	[22] [*]
$4p^6 5s\ ^2S_{1/2} - 4p^6 5p\ ^2P_{3/2}^o$	4.351(24)	[22] [*]
$4p^6 4d\ ^2D_{3/2} - 4p^6 5p\ ^2P_{1/2}^o$	3.111(31)	[22] [*]
$4p^6 4d\ ^2D_{3/2} - 4p^6 5p\ ^2P_{3/2}^o$	1.382(14)	[22] [*]
$4p^6 4d\ ^2D_{5/2} - 4p^6 5p\ ^2P_{3/2}^o$	4.186(42)	[22] [*]
State	Hyperfine A constant (MHz)	Reference
$4p^6 5s\ ^2S_{1/2}$	-1000.473673(11)	[23]
$4p^6 5p\ ^2P_{1/2}^o$	-177.33	[22] [*]
$4p^6 5p\ ^2P_{3/2}^o$	-36.0(4)	[24]
$4p^6 4d\ ^2D_{3/2}$	-46.7	[22] [*]
$4p^6 4d\ ^2D_{5/2}$	2.1743(14)	[25]

TABLE IV. Reduced electric-dipole matrix elements and hyperfine A constants used to calculate the critical magnetic field for ⁸⁷Sr⁺. An asterisk denotes theoretically calculated values.

$^{25}\text{Mg}^+$		
Transition	Reduced $E1$ matrix element (a.u.)	Reference
$2p^6 3s^2 S_{1/2} - 2p^6 3p^2 P_{1/2}^o$	2.3691(12)	[22] [*]
$2p^6 3s^2 S_{1/2} - 2p^6 3p^2 P_{3/2}^o$	3.3509(17)	[22] [*]
$2p^6 3p^2 P_{1/2}^o - 2p^6 4s^2 S_{1/2}$	1.69395(85)	[22] [*]
$2p^6 3p^2 P_{1/2}^o - 2p^6 3d^2 D_{3/2}$	4.1587(21)	[22] [*]
$2p^6 3p^2 P_{3/2}^o - 2p^6 4s^2 S_{1/2}$	2.4041(12)	[22] [*]
$2p^6 3p^2 P_{3/2}^o - 2p^6 3d^2 D_{3/2}$	1.86247(93)	[22] [*]
$2p^6 3p^2 P_{3/2}^o - 2p^6 3d^2 D_{5/2}$	5.5875(28)	[22] [*]
State	Hyperfine A constant (MHz)	Reference
$2p^6 3s^2 S_{1/2}$	-596.2542487(42)	[26]
$2p^6 3p^2 P_{1/2}^o$	-103.4	[22] [*]
$2p^6 3p^2 P_{3/2}^o$	-19.29	[22] [*]
$2p^6 4s^2 S_{1/2}$	-163.4	[22] [*]
$2p^6 3d^2 D_{3/2}$	-1.14	[22] [*]
$2p^6 3d^2 D_{5/2}$	0.1196	[22] [*]

TABLE V. Reduced electric-dipole matrix elements and hyperfine A constants used to calculate the critical magnetic field for $^{25}\text{Mg}^+$. An asterisk denotes theoretically calculated values.

$^{43}\text{Ca}^+$		
Transition	Reduced $E1$ matrix element (a.u.)	Reference
$3p^6 4s^2 S_{1/2} - 3p^6 4p^2 P_{1/2}^o$	2.8928(43)	[27]
$3p^6 4s^2 S_{1/2} - 3p^6 4p^2 P_{3/2}^o$	4.092(6)	[27]
$3p^6 3d^2 D_{3/2} - 3p^6 4p^2 P_{1/2}^o$	2.463(13)	[22] [*]
$3p^6 3d^2 D_{3/2} - 3p^6 4p^2 P_{3/2}^o$	1.092(2)	[22] [*]
$3p^6 3d^2 D_{5/2} - 3p^6 4p^2 P_{3/2}^o$	3.283(6)	[22] [*]
State	Hyperfine A constant (MHz)	Reference
$3p^6 4s^2 S_{1/2}$	-806.40207160(8)	[28]
$3p^6 4p^2 P_{1/2}^o$	-145.4(1)	[29]
$3p^6 4p^2 P_{3/2}^o$	-31.0(2)	[29]
$3p^6 3d^2 D_{3/2}$	-47.3(2)	[29]
$3p^6 3d^2 D_{5/2}$	-3.8931(2)	[29]

TABLE VI. Reduced electric-dipole matrix elements and hyperfine A constants used to calculate the critical magnetic field for $^{43}\text{Ca}^+$. An asterisk denotes theoretically calculated values.

$^9\text{Be}^+$		
Transition	Reduced $E1$ matrix element (a.u.)	Reference
$1s^2 2s^2 S_{1/2} - 1s^2 2p^2 P_{1/2}^o$	1.85057(93)	[22] [*]
$1s^2 2s^2 S_{1/2} - 1s^2 2p^2 P_{3/2}^o$	2.6172(13)	[22] [*]
$1s^2 2p^2 P_{1/2}^o - 1s^2 3s^2 S_{1/2}$	0.86751(43)	[22] [*]
$1s^2 2p^2 P_{1/2}^o - 1s^2 3d^2 D_{3/2}$	2.5084(13)	[22] [*]
$1s^2 2p^2 P_{3/2}^o - 1s^2 3s^2 S_{1/2}$	1.22708(61)	[22] [*]
$1s^2 2p^2 P_{3/2}^o - 1s^2 3d^2 D_{3/2}$	1.12191(56)	[22] [*]
$1s^2 2p^2 P_{3/2}^o - 1s^2 3d^2 D_{5/2}$	3.3658(17)	[22] [*]
State	Hyperfine A constant (MHz)	Reference
$1s^2 2s^2 S_{1/2}$	-625.008837044(12)	[30]
$1s^2 2p^2 P_{1/2}^o$	-118.00(4)	[31]
$1s^2 2p^2 P_{3/2}^o$	-1.01(2)	[32]
$1s^2 3s^2 S_{1/2}$	-158.897(7)	[33] [*]

TABLE VII. Reduced electric-dipole matrix elements and hyperfine A constants used to calculate the critical magnetic field for $^9\text{Be}^+$. An asterisk denotes theoretically calculated values.

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