Supplemental Materials: Phonon lasing from optical frequency comb illumination of trapped ions

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CALCULATION OF THE SECULAR-CYCLE AVERAGE OF THE SCATTERING RATE $\Gamma(\delta, x_0)$

For the large-amplitude oscillations associated with all but the lowest fixed point in this system, fairly good agreement between theory and the data can be achieved without including rf micromotion. However, the inclusion of micromotion to the model does improve the quantitative agreement with the data since it provides a more accurate spectrum than the secular approximation. Since $\Omega_{\rm rf} > \gamma$, we include the effects of rf micromotion as an incoherent sum over the scattering rates from secular-phase-dependent micromotion sidebands [1], with each sideband yielding fluorescence with the functional form of $\Gamma_{\rm comb}$. Defining $\xi \equiv \omega_x t$, the secular-cycle-averaged scattering rate is given by

$$\Gamma(\delta, x_0) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\xi \sum_n \mathrm{J}_n^2\left[k x_0 \frac{q}{2}\cos(\xi)\right] \Gamma_{\rm comb}\left(\delta + n\Omega_{\rm rf} + k\omega_x x_0 \sin(\xi)\right), \tag{S1}$$

where $q \approx 2\sqrt{2}\omega_x/\Omega_{\rm rf}$ is the Mathieu q-parameter for this normal mode, k is the projection of the laser's k-vector on the oscillation axis, and $\mathrm{J}_n[\alpha]$ is the nth Bessel function of the first kind.

Micromotion can be eliminated from this expression by taking $q, n = 0$ (which may be appropriate, for instance, for the axial mode in a linear Paul trap) and yields

$$\Gamma_{\rm sec}(\delta, x_0) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\xi \Gamma_{\rm comb}\left(\delta + k\omega_x x_0 \sin(\xi)\right). \tag{S2}$$

CALCULATION OF THE DAMPING COEFFICIENT $\beta(E)$

To calculate the damping coefficient, we compute the secular-cycle average of the power delivered to the secular motion $\langle \mathbf{F}\cdot \boldsymbol{v}_{\rm sec}\rangle_{T_{\rm sec}}$, where $\mathbf{F} = \hbar \mathbf{k}\Gamma_{\rm comb}$ and $\boldsymbol{v}_{\rm sec}(t) = \hat{\mathbf{x}}\,\omega_x x_0 \sin(\omega_x t)$ is the instantaneous secular velocity. This yields an expression for the amplitude damping coefficient, $\beta = -(m/2E)\langle \mathbf{F}\cdot \boldsymbol{v}_{\rm sec}\rangle$:

$$\beta(E) = \frac{\hbar k}{2\pi\omega_x x_0}\int_0^{2\pi} \mathrm{d}\xi \sin(\xi) \sum_n \mathrm{J}_n^2\left[k x_0 \frac{q}{2}\cos(\xi)\right] \Gamma_{\rm comb}\left(\delta + n\Omega_{\rm rf} + k\omega_x x_0 \sin(\xi)\right). \tag{S3}$$

For applications where micromotion should not be included, the elimination of the micromotion from this expression proceeds the same way as above.

DETUNING DEPENDENCE OF FIXED POINTS

The main detuning dependence of the fixed points in the theoretical model comes as the detuning of the near resonant tooth changes sign, which results in a large change in fixed point. However, there is still some δ dependence in the theory even within the range where δ does not change sign. Figure S1 shows the numerical solutions for the fixed points for the parameters in this work. The fixed points are fairly constant within each range, and we do not study this weak dependence experimentally in this work.

[1] R. Blümel, C. Kappler, W. Quint, and H. Walther, "Chaos and order of laser-cooled ions in a paul trap," Phys. Rev. A **40**, 808 (1989).

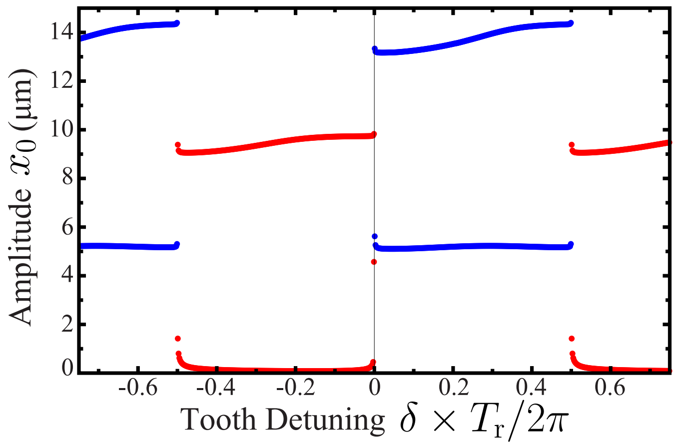

Figure S1. Numerical fixed points (from Eq. 4 in the main text) as a function of detuning. When the near resonant tooth is red(blue) detuned the points are shown in red(blue). The natural linewidth of the transition spans a range of $\gamma \times T_\mathrm{r}/2\pi \approx 0.25$ on the horizontal axis.