

VECTOR LIGHT SHIFT IN A MAGNETIC FIELD

In this section, we explain how a vector light shift arises in the presence of a magnetic field.

The energy shift of an atomic electronic state $|i\rangle \equiv |n, F, m_F, (J, I, \ell, S)\rangle$, in the presence of non-resonant classical electromagnetic radiation is typically approximated to second order in the field as [1]:

$$\Delta\omega_i = - \sum_k \left(\frac{\langle i | (\vec{d} \cdot \vec{E}_+)^{\dagger} | k \rangle \langle k | \vec{d} \cdot \vec{E}_+ | i \rangle}{4(\omega_{ki} - \omega_L)} + \frac{\langle i | \vec{d} \cdot \vec{E}_+ | k \rangle \langle k | (\vec{d} \cdot \vec{E}_+)^{\dagger} | i \rangle}{4(\omega_{ki} + \omega_L)} \right), \quad (1)$$

where $\omega_{ki} = (E_k - E_i)/\hbar$, $\vec{d} = e\vec{r}$ is the E1 transition dipole operator, and the electric field of the laser is $\vec{E} = \vec{E}_+ e^{-i\omega_L t} + \vec{E}_+^{\dagger} e^{i\omega_L t}$ with $\vec{E}_+ = E_+ \hat{e}$, where $\hat{e}$ is a unit vector. For $m_F = 0$ states and σ_+ polarization $\langle k | \vec{d} \cdot \vec{E} | i \rangle$ is only non-zero for k states with $m_F = 1$, while the counter-rotating term $\langle k | (\vec{d} \cdot \vec{E})^{\dagger} | i \rangle$ is only non-zero for k states with $m_F = -1$. Though the intermediate states are different for the rotating and counter-rotating terms, for $m_F = 0$ the numerators of the expression are identical allowing the light shift to be written as:

$$\begin{aligned} \Delta\omega_i &= - \sum_k \frac{\langle i | \vec{d} \cdot \vec{E}^* | k \rangle \langle k | \vec{d} \cdot \vec{E} | i \rangle}{4} \left(\frac{1}{(\omega_{ki} - \omega_L)} + \frac{1}{(\omega_{ki} + \omega_L)} \right) \\ &= - \sum_k \frac{|\langle k | \vec{d} \cdot \vec{E}_+ | i \rangle|^2}{2} \frac{\omega_{ki}}{(\omega_{ki}^2 - \omega_L^2)}. \end{aligned} \quad (2)$$

Comparing to the forms of the scalar, vector, and tensor light shifts [2], we see the expected result that for $m_F = 0$ there is no vector light shift.

To avoid Raman scattering errors during a laser gate, it is beneficial for the gate laser to be red detuned of the lowest excited state [3, 4]. Thus, for ground state, non-degenerate qubit states, with $\omega_{ki} > 0$ in zero magnetic field, it is not possible to have a gate laser with a ‘magic’ wavelength or polarization that nulls the light shift on the qubit.

In the presence of a magnetic field, states of differing F are mixed according to:

$$\hat{H}_B = \mu_B (\hat{L}_z + g_e \hat{S}_z) B_z - g_N \mu_N \hat{I}_z B_z. \quad (3)$$

and the the rotating and counter-rotating terms no longer have identical numerators. As a result, Eqn. (1) is instead factored as:

$$\Delta\omega_i = - \sum_k a_{ki}^{(+)} \frac{\omega_{ki}}{(\omega_{ki}^2 - \omega_L^2)} - a_{ki}^{(-)} \frac{\omega_L}{(\omega_{ki}^2 - \omega_L^2)}, \quad (4)$$

where $a_{ki}^{(+)} = |\langle k | \vec{d} \cdot \vec{E}_+ | i \rangle|^2 + |\langle k | (\vec{d} \cdot \vec{E}_+)^{\dagger} | i \rangle|^2$ and $a_{ki}^{(-)} = |\langle k | \vec{d} \cdot \vec{E}_+ | i \rangle|^2 - |\langle k | (\vec{d} \cdot \vec{E}_+)^{\dagger} | i \rangle|^2$. Though we have not performed the usual decomposition into spherical irreducible tensors, we see the light shift takes the form of a scalar light shift (the $\hat{a}_{ki}^{(+)}$ terms) and a vector light shift (the $\hat{a}_{ki}^{(-)}$ terms). The $a_{ki}^{(\pm)}$ coefficients can have opposite signs for the two qubit states $\{|i=0\rangle, |i=1\rangle\}$ allowing for the possibility of finding a zero differential light shift for certain wavelengths and polarizations – i.e. $\delta\omega_q(\lambda, \vec{B}, \hat{e}) \equiv \Delta\omega_0 - \Delta\omega_1 = 0$.

AN ORDER OF MAGNITUDE ESTIMATE OF THE CRITICAL MAGNETIC FIELD

While a full calculation of the critical field requires a calculation of AC polarizabilities $\alpha_F^{(k), \text{HFI}}(\omega_L)$ treatment [5] to third order in the light-matter and hyperfine interactions, a rough estimation of the critical field is possible by considering only the magnetic field induced mixing of the hyperfine clock states.

To begin we assume that a Ba^{133} ion is exposed to light at frequency ω with σ_+ polarization and $B = 0$ magnetic field. The qubit states, $|6s^2 \ ^2\text{S}_{1/2}, F=0\rangle \equiv |S, 0\rangle$ and $|6s^2 \ ^2\text{S}_{1/2}, F=1, m_F=0\rangle \equiv |S, 1, 0\rangle$, are primarily mixed by the laser light with the $|6s6p \ ^2\text{P}_{1/2}, F=1, m_F=1\rangle \equiv |P_{1/2}, 1, 1\rangle$, $|6s6p \ ^2\text{P}_{3/2}, F=$

$1, 1\rangle \equiv |P_{3/2}, 1, 1\rangle$ and $|6s6p\ ^2P_{3/2}, F = 2, 1\rangle \equiv |P_{3/2}, 2, 1\rangle$ states, leading to a differential light shift for the qubit of:

$$\delta\omega_q(B=0) \approx -\frac{|E|^2}{4} \sum_k \left(\frac{|\langle k|\hat{d}|S, 0\rangle|^2}{4(\omega_1 - \omega)} - \frac{|\langle k|\hat{d}|S, 1, 1\rangle|^2}{4(\omega_k - \omega + \omega_q)} \right), \quad (5)$$

where k indexes the excited $|P_J, F, m_F = 1\rangle$ states, ω_k is the transition frequency between the $|S, 0\rangle$ and state k , ω_q is the hyperfine qubit splitting, $\hat{d}$ is the electric transition dipole operator, $\vec{E}$ is the electric field, we have defined the energy of $|S, 0\rangle$ as zero, and ignored the counter-rotating term. Defining the detuning of the laser from the transition as $\Delta_1 = \omega_1 - \omega$ we have:

$$\Delta\omega_q = -\frac{d_1^2|E|^2}{24} \left(\frac{1}{\Delta_1} - \frac{1}{\Delta_1 + \omega_q} \right) \approx -\frac{d_1^2|E|^2}{24\Delta_1} \frac{\omega_q}{\Delta_1} \quad (6)$$

The same laser also couples the qubit states to the $|6s6p\ ^2P_{3/2}, F = 1, 1\rangle \equiv |P_{3/2}, 1, 1\rangle$ and $|6s6p\ ^2P_{3/2}, F = 2, 1\rangle \equiv |P_{3/2}, 2, 1\rangle$ states.

This leads to a differential light shift of the qubit for $B = 0$ as:

$$\delta\omega_q(B=0) \approx -\frac{\omega_q|E|^2}{24} \left(\frac{d_1^2}{\Delta_1^2} + \frac{d_2^2}{\Delta_2^2} \right) \quad (7)$$

where $\Delta_2 = \omega_2 - \omega$ and $\Delta_2 = \omega_2 - \omega$ are the laser detuning relative to the $|S, 0\rangle \leftrightarrow |P_{1/2}, 1, 1\rangle$ and $|S, 0\rangle \leftrightarrow |P_{3/2}, 1, 1\rangle$, respectively, and we have ignored the hyperfine splitting $\omega_{3/2}$ in the $6s6p\ ^2P_{3/2}$ state. In zero magnetic field the $|S, 0\rangle$ state is closer to the excited states k than the $|S, 1, 0\rangle$ state. This form results from the fact that the contribution to the total light shift of a qubit state from each excited state is proportional to d_i^2/Δ_i , while the difference in qubit light shifts goes as ω_q/Δ_i . Therefore, the qubit has a negative differential light shift that depends linearly on the qubit splitting ω_q . If a magnetic field of strength B is applied along the quantization axis, the two qubit states are mixed as:

$$\begin{aligned} |S, \psi_+\rangle &\approx |S, 0\rangle + \frac{\mu_B B}{\omega_q} |S, 1, 0\rangle \\ |S, \psi_-\rangle &\approx |S, 1, 0\rangle - \frac{\mu_B B}{\omega_q} |S, 0\rangle, \end{aligned} \quad (8)$$

where μ_B is the Bohr magneton. As shown below, this modifies the light shift in two important ways. Before proceeding, we note that the magnetic field also mixes the excited states k , however, since the detuning is large relative to the hyperfine structure the mixing does not affect the differential light shift.

First, due to the mixing, the matrix elements between the qubit states and the $|P_{1/2}, 1, 1\rangle$ state become:

$$\begin{aligned} \langle P_{1/2}, 1, 1 | \hat{d} | S, \psi_+\rangle &\approx \frac{d_1}{\sqrt{6}} \left(1 - \frac{\mu_B B}{\omega_q} \right) \\ \langle P_{1/2}, 1, 1 | \hat{d} | S, \psi_-\rangle &\approx -\frac{d_1}{\sqrt{6}} \left(1 + \frac{\mu_B B}{\omega_q} \right) \end{aligned} \quad (9)$$

Thus, the differential light shift is modified to:

$$\delta\omega_q(B) \approx -\frac{\omega_q|E|^2}{24} \left(\frac{d_1^2}{\Delta_1^2} + \frac{d_2^2}{\Delta_2^2} \right) + \frac{d_1^2|E|^2}{24\Delta_1} \frac{4\mu_B B}{\omega_q}. \quad (10)$$

This change results from the difference in sign of the transition moment of the two qubit states with the $|P_{1/2}, 1, 1\rangle$ state.

Upon application of a magnetic field, the $|S, 0\rangle$ state acquires some $|S, 1, 0\rangle$ character and, due to the opposite sign of that state's transition dipole moment, the transition moment of the mixed state is lowered and its light shift is reduced. The opposite effect occurs for the $|S, 1, 1\rangle$ state and the result is a contribution to the light shift that is opposite the zero-field light shift.

The second important effect of the magnetic field induced mixing stems from the inequality of the qubit state transition moments to the $|P_{3/2}, F, m_F = 1\rangle$ states. In a magnetic field, the matrix elements become:

$$\begin{aligned}
\langle P_{3/2}, 1, 1 | \hat{d} | S, \psi_+ \rangle &\approx \frac{d_2}{\sqrt{6}} \left(1 + \frac{\mu_B B}{2\omega_q} \right) \\
\langle P_{3/2}, 1, 1 | \hat{d} | S, \psi_- \rangle &\approx \frac{d_2}{2\sqrt{6}} \left(1 - \frac{2\mu_B B}{\omega_q} \right) \\
\langle P_{3/2}, 2, 1 | \hat{d} | S, \psi_+ \rangle &\approx 0 + \frac{d_2}{2\sqrt{2}} \frac{\mu_B B}{\omega_q} \\
\langle P_{3/2}, 2, 1 | \hat{d} | S, \psi_- \rangle &\approx \frac{d_2}{2\sqrt{2}}.
\end{aligned} \tag{11}$$

To first order in $\mu_B B / \omega_q$ the modification of the differential light shift due to the change in transition moments to the $|P_{3/2}, 2, 1\rangle$ is zero. Therefore, the light shift is modified only from the change in the transition moment to the $|P_{3/2}, 1, 1\rangle$ state and becomes:

$$\delta\omega_q(B) \approx -\frac{\omega_q |E|^2}{24} \left(\frac{d_1^2}{\Delta_1^2} + \frac{d_2^2}{\Delta_2^2} \right) + \frac{d_1^2 |E|^2}{24\Delta_1} \frac{4\mu_B B}{\omega_q} - \frac{d_2^2 |E|^2}{24\Delta_2} \left(\frac{2\mu_B B}{\omega_q} \right). \tag{12}$$

Thus, the mixing leads to a contribution of the same sign as the zero-field light shift. This results from the increase in the transition moment of the $|S, 0\rangle$ state to the $|P_{3/2}, 1, 1\rangle$ state and the decrease in the transition moment of the $|S, 1, 1\rangle$ state to the $|P_{3/2}, 1, 1\rangle$ in a magnetic field.

We can then estimate the critical magnetic field by finding B such that $\omega_q(B) = 0$:

$$B_c \approx \frac{\omega_q^2 \left(\frac{d_1^2}{\Delta_1^2} + \frac{d_2^2}{\Delta_2^2} \right)}{2\mu_B \left(\frac{2d_1^2}{\Delta_1} - \frac{d_2^2}{\Delta_2} \right)} \tag{13}$$

Given the reduced matrix elements $d_1 = 3.3251(21)$ a.u. and $d_2 = 4.7017(27)$ a.u. [6], we estimate $B_c \approx 1.07$ G at 532 nm for $^{133}\text{Ba}^+$. This approximation differs by about 10 % from the full theoretical treatment due to higher order n state couplings.

M QUBIT OPERATIONS

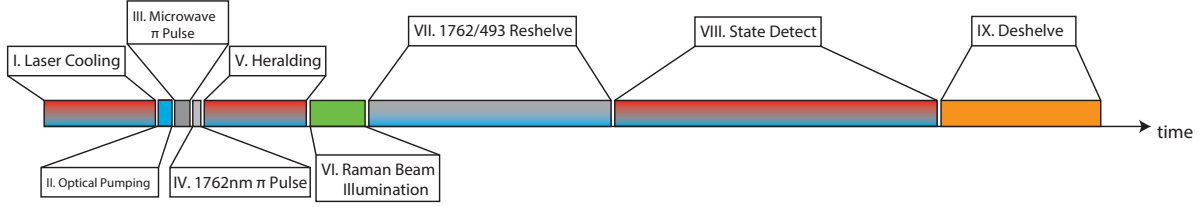


FIG. 1. Experimental sequence diagram of metastable operations.

Experimental m-qubit Raman Spectroscopy Sequence

The narrow electric quadrupole 1762 nm optical transition is used to transfer population between the g and m -qubit manifolds. The experimental sequence can be seen in Fig. 1.

I. Laser cooling for 8 ms with 493 and 650 nm light, both using an electro-optical modulator (EOM) to produce multi-tone light resonant with the hyperfine splitting of the $S_{1/2}$ and $D_{3/2}$.

II. 493 nm-light hyperfine tones are turned off and a 1840 MHz tone resonant with the $|S_{1/2}, F = 1, m_F = 0\rangle$ to $|P_{1/2}, F = 1\rangle$ populates the $|S_{1/2}, F = 0, m_F = 0\rangle$ state in 150 μ s.

III. 130 μ s microwave π pulse from $|S_{1/2}, F = 0, m_F = 0\rangle$ to $|S_{1/2}, F = 1, m_F = 0\rangle$.

IV. A 45 μ s 1762 nm pulse transfers around 90-95% population to the $|D_{5/2}, F = 3, m_F = 0\rangle$ state.

V. To herald transferred population into the $|D_{5/2}, F = 3, m_F = 0\rangle$ state, the cooling lasers are turned on for 8 ms and photon counts are collected.

VI. Two, co-propagating, detuned, 532 nm beams illuminate the ion, leading to coherent transfer of population between $|D_{5/2}, F = 3\rangle$ and $|D_{5/2}, F = 2\rangle$ hyperfine clock states. The frequency of a single beam is varied with an acousto-optical modulator (AOM) to produce data seen in Fig 2.

VII. To read out the m -qubit state we selectively shelve the $|D_{5/2}, F = 3\rangle$ hyperfine clock state by turning on the 1726 nm laser along with the 493 nm beam for 7 ms. The combination of these lasers shelves this state to the $D_{3/2}$ manifold with a fidelity of 0.9989(2).

VIII. The cooling lasers are turned on for 10 ms to measure photon counts. All population that was originally in the $|D_{5/2}, F = 3\rangle$ will have been shelved to the $D_{3/2}$ manifold via step VII, therefore projecting these measured states as bright (high photon counts). Any population not affected by step VII would have been in the $|D_{5/2}, F = 2\rangle$ hyperfine clock state and will be dark (low photon counts).

IX. A 614 nm laser resonant with the $D_{5/2} \rightarrow P_{3/2}$ manifold returns any population in the m -qubit subspace back to the g -qubit subspace to be Doppler cooled at the start of the next experimental cycle.

Measurement of $D_{5/2}$ Hyperfine clock state Splitting

Fig. 2 shows a Raman spectroscopy scan of the hyperfine m -qubit splitting in $^{133}\text{Ba}^+$. These Raman beams are created from a single laser using two acousto-optical modulators (AOM). Running the sequence described above, while keeping the illumination pulse time constant, creates the data shown in Fig. 2. The illumination time is 268 μ s, and the power in each beam was roughly 35 mW. We use a large beam ($\sim 100 \mu\text{m}$) to reduce the effects of beam pointing fluctuations. The splitting measured here is 89.6697(4) MHz at 5.036(1) Gauss. Accounting for the Zeeman affect of the $D_{5/2}$ levels, the zero field hyperfine splitting is approximately 89.2695(4) MHz. The m -qubit spacing was measured multiple times, each with a corresponding magnetic field measurement to account for magnetic field fluctuations on the measured splitting. The magnetic field amplitude was measured via a detuned microwave Ramsey sequence on the g qubit in $^{133}\text{Ba}^+$. The standard error of the magnetic field measurements was $\approx 4 \times 10^{-4}$ Gauss.

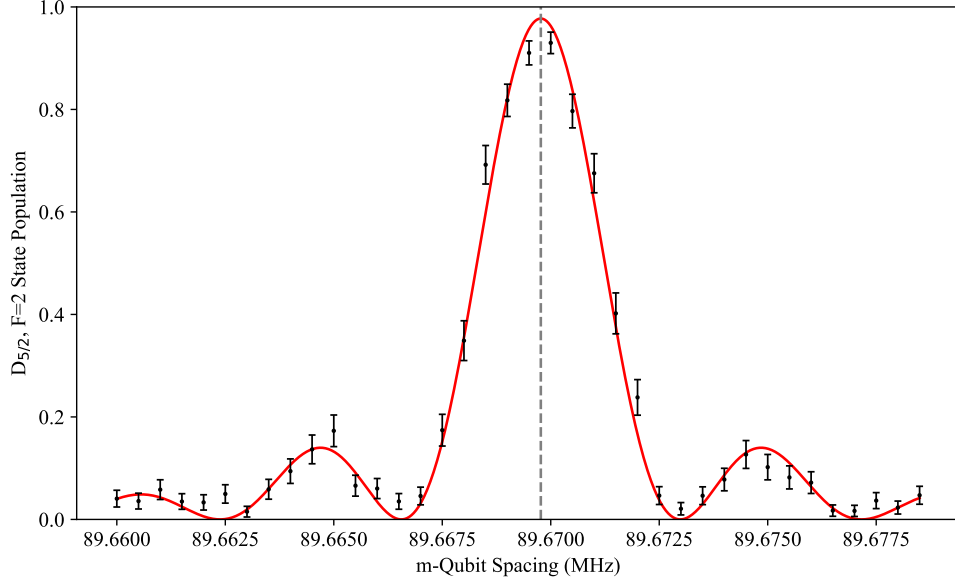


FIG. 2. Experimental measurement of the $D_{5/2}$ hyperfine splitting using high power, 532 nm laser to drive a stimulated Raman transition

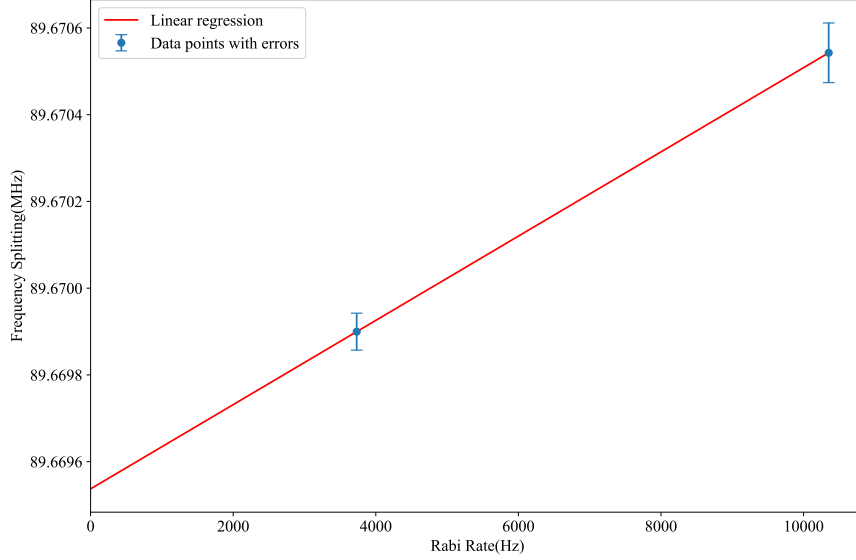


FIG. 3. Intensity on ion given in Rabi rate vs measured m-qubit splitting. The measured frequency splitting shifts due to the differential light shift of the m qubit $D_{5/2}$ clock states

Fig. 3 shows the relationship between the measured frequency splitting of the m qubit and its Rabi rate. An increase in the intensity of the Raman spectroscopy lasers applied to the ion leads to a larger measured splitting due to the differential light shift between the two m -qubit levels. The y-intercept of the linear fit corresponds to the frequency splitting at zero differential light shift. The data point at a 4 kHz Rabi rate corresponds to the intensity that was used for the scans described above, including the one seen in Fig. 2. Notably, the y-intercept is about 400 Hz lower than the measured m -qubit splitting. This difference was subtracted from in the measured splitting values to accurately calculate the zero-field hyperfine coefficient.

G-QUBIT COHERENCE MEASUREMENT

Detuned Ramsey Sequence

To measure the cross-talk of global m -qubit Raman gate beams on the g qubit we measure the coherence of the g qubit under illumination by the m -qubit Raman beams. We use a detuned Ramsey sequence as follows: prepare the $F=0$ state of the g qubit, perform a detuned $\pi/2$ microwave pulse in the g qubit, wait some period of time, and then perform another detuned $\pi/2$ microwave pulse. As the wait time is varied, a Ramsey fringe forms, fluctuating at a frequency of f_{Ramsey} . This Ramsey oscillation frequency corresponds to the detuning of the microwave pulse from the true g -qubit hyperfine splitting. The sequence is repeated 300 times for each unique wait time. The decay of this fringe amplitude is fit to a decaying exponential to obtain a coherence time of the g qubit.

As shown in Fig. 2 of the main text, we perform three different Ramsey fringe measurements at a magnetic field magnitude of 5.00(1) Gauss. Fig. 2(a) is taken with no laser light on during the wait time, leading to no differential light shift of the g -qubit spacing. The frequency fit corresponds to a microwave shift of 440.4 ± 0.13 Hz. The fit gives a decoherence rate consistent with zero within the 30 ms of measurement time.

Fig. 2(b) shows a Ramsey fringe when illuminating the ion with two co-propagating high power 532 nm Raman beams with a combined intensity of 150 MW/m^2 . These beams have frequencies that are split by the m -qubit spacing and have their polarization set to $A = -1$ (σ^-). The coherence time of the g qubit illuminated by this light is measured to be 46.80 ± 3.8 ms. During the illumination period, the ion experiences both short and long timescale intensity fluctuations. Fluctuations that occur on timescales equal to or shorter than the Ramsey interrogation time, greatly contribute to decoherence, leading to a notably short coherence time for this clock-state qubit. Low frequency fluctuations, likely due to thermal effects can be seen as slow fringe frequency drifts of roughly $20 \text{ Hz}/60 \text{ min} = 0.33 \text{ mHz/min}$. This represents a change in laser intensity of roughly 10 MW/m^2 over a typical 60 minute Ramsey experimental cycle.

Fig. 2(c) is a Ramsey fringe while the ion is illuminated with the similar Raman beams as Fig. 2(b) but now with a ‘magic polarization’. This polarization is defined as $A = 0.212$ for 5 Gauss, corresponding to elliptical polarization. Due to the tuning of this polarization to a ‘magic condition’ we minimize the differential light shift in the g qubit and shield this manifold from both high and low frequency intensity fluctuations. The f_{Ramsey} fit value for this fringe is 455.67 ± 0.1 Hz. This represents a roughly 15 Hz shift from the laser-free fringe seen in Fig. 2(a) of the main text. Due to experimental uncertainties regarding the magnetic field we are unable to fully cancel the differential shift. Using the ‘magic condition’ technique, we are able to increase the coherence time of the g qubit from 46 ms as shown Fig. 2(b) to 3.5 ± 2.6 s. The relatively large error of the measured coherence time is due to the minimal decoherence that occurs during the allowed data taking time period for the experiment. To further quantize this effect, we can calculate the differential light shift sensitivity of the g qubit as $19 \text{ Hz}/(\text{kW/cm}^2)$. Tuning the polarization of our m -qubit beams, we are able to mitigate the equivalent of roughly 141 MW/m^2 of laser intensity on the ion.

Fitting Ramsey Curves

The decay of the Ramsey curves taken in Fig. 2 is fit with the function:

$$A \exp\left(\frac{-t}{\tau}\right) \sin^2(\pi f_{\text{Ramsey}} t + \alpha t^2) \quad (14)$$

where A is the starting amplitude of the fringe, τ is the coherence time and f_{Ramsey} is the frequency of the fringe. There are two timescales of amplitude noise that affect the decay, the first is on the order of kHz and causes the decoherence on the timescale of milliseconds, the second occurs on the time scale of 10s of minutes due to thermal drifts in the laser intensity on the ion. This causes the periodic decaying exponential function to also have a time dependent phase and is defined as α in the fit function. The benefit of the magic polarization technique is highlighted here as decoherence due to both of these types of noise is alleviated when using this technique.

Estimation of Beam Divergence

The uncertainty in the angle between the lasers wave vector $\vec{k}$ and the measured magnetic field $\vec{B}$ is given as:

$$\theta = \tan^{-1} \left(\frac{w - w_0}{2F} \right)$$

where w_0 is the beam waist at the focal length, w is the beam waist at the focusing lens, and F is the focal length. The beam is focused through a 400 mm focal length lens and is positioned with a high accuracy 3D micrometer stage. Given the focal length of 400 mm, we find a beam divergence of $\theta=13.5$ mrad.

Estimation of Cross-Talk Fidelity vs Two-qubit Gate Number

The fidelity of quantum information stored in the g -qubit after a given number of π -pulses of the m qubit is given by

$$\mathcal{F} = (1 - e^{\tau_{gate}/\tau_{coherence}})^n,$$

where τ_{gate} is the gate time, $\tau_{coherence}$ is the coherence time and n is the number of gate operations. Using the decaying exponential envelope of the coherence measurement in Fig. 2 of the main text we can estimate a g -qubit infidelity during an m -qubit π -pulse. After calculating τ_{gate} from the powers and beam sizes used in the coherence measurement, the non-magic condition gives an error per π pulse of $2.7(2) \times 10^{-5}$, while the magic condition gives $3(2) \times 10^{-7}$. Furthermore, a similar 100-fold reduction in cross-talk error for two-qubit gates is expected. Calculating this two-qubit cross-talk error gives a value of $2.4(2) \times 10^{-4}$ and $3(2) \times 10^{-6}$ for the anti-magic and magic conditions, respectively. As these infidelities are below state-of-the-art single qubit gate infidelities [2], they should not significantly affect use of the *omg* protocol.

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