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Los Angeles

Metastable States, Magic Polarization, and Hyperfine Spectroscopy in $^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$

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of the requirements for the degree
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by

Thomas Dellaert

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ABSTRACT OF THE DISSERTATION

Metastable States, Magic Polarization, and Hyperfine Spectroscopy in $^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$

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Ytterbium is a popular species both for quantum information and precision measurement. $^{171}\text{Yb}^+$ specifically has a long history of use in quantum computing due to the simplicity of state preparation and long coherence times present in spin-1/2 isotopes. In addition, the species' use in atomic clocks due to their narrow 435 nm and 411 nm resonances have gained prominence. In this thesis, we explore several techniques to improve quantum computation primitives in $^{171}\text{Yb}^+$, and we demonstrate the first trapping of $^{173}\text{Yb}^+$.

We first demonstrate a shelving technique that utilizes the metastable $^2F_{7/2}^o$ manifold as a place to "shelve" quantum information, enabling us to surpass the conventional limits on state preparation and measurement (SPAM) fidelity and greatly increase our state detection duration. Using the narrow 411 nm transition to the $^2D_{5/2}$ state, we optically pump into the metastable manifold without the need to directly drive the E3 transition. We demonstrate a SPAM inaccuracy of $1.5_{-0.7}^{+1.2} \times 10^{-4}$. The final inaccuracy is limited by a previously unobserved M1 decay, which we were able to quench to demonstrate a further reduction in the SPAM error. Finally, we demonstrate the potential of the $^2F_{7/2}^o$ state as a qubit, demonstrating high-fidelity heralded SPAM and single-qubit gates.

Next, we explore an effect in which the magnetic field-induced state mixing in the ground state hyperfine clock qubit in $^{171}\text{Yb}^+$ introduces polarization dependence. With a given choice of magnetic field, this state mixing allows one to tune the polarization of perturbing light to a "magic polarization" such that differential light shift of the qubit is eliminated. We demonstrate this effect in ytterbium, and show an improvement in the coherence time of the qubit by a factor of 2000.

Finally, we discuss the trapping, cooling, and spectroscopy of a previously untrapped isotope: $^{173}\text{Yb}^+$, with the ultimate aim of examining the hyperfine structure of the $^2F_{7/2}^o$ state as a sensitive probe of the isotope's nuclear structure. We measure the hyperfine structure coefficients of the following previously unmeasured $^2D_{3/2}$, $^2D_{5/2}$, $^3[3/2]_{1/2}^o$, and $^1[3/2]_{3/2}^o$ states, laying the groundwork for hyperfine Ramsey spectroscopy of the $^2F_{7/2}^o$ state. We then discuss potential future applications of $^{173}\text{Yb}^+$ in quantum information and precision measurement.

The dissertation of Thomas Dellaert is approved.

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CHAPTER 1

Introduction

1.1 Trapped Ions

Since the first ion traps were produced in the 1950s, they have proved an invaluable resource, allowing quantum control of single particles and enabling experiments probing fundamental physics such as precision measurements of the electron’s magnetic moment [CGG17], atomic clocks that are accurate to the 10^{-18} level [HSL16], and as highly accurate sensors [GAL21].

1.1.1 in Quantum Computing

Ever since the observation that computers composed of quantum particles are theoretically able to solve problems that would be prohibitively difficult to solve classically, the field of quantum computation has arisen. This observation was first made by Peter Shor, who devised an algorithm capable of factoring a prime N in $O(\log(N))$ time [Sho97].

One of the key advantages leveraged in quantum computers is that when a bit is represented as a quantum state, we may refer to superpositions of that state. Therefore, while a classical bit is strictly either 0 or 1, a qubit (quantum bit) can be any superposition

$$c_0 |0\rangle + c_1 |1\rangle$$

If we take a system of two qubits, we must consider that any correlation of the two qubits is possible, and we write the quantum state as

$$c_{00} |00\rangle + c_{01} |01\rangle + c_{10} |10\rangle + c_{11} |11\rangle$$

where we now require four complex amplitudes in order to fully specify the quantum state of the system. In general, to represent a quantum state consisting of n qubits, we require at least 2^n classical bits of information. A 100-qubit system, therefore, contains 2^{100} amplitudes, an amount of information that is impossible to store classically.

1.1.1.1 Physical Quantum Computers

The great challenge in quantum computation, then, is to *utilize* the vast amount of information stored in these amplitudes to perform useful computations. A commonly accepted set of criteria for a quantum computer to be useful was set forth by diVincenzo [DiV00], who stated that a quantum computer must be:

1. Scalable and well-characterized
2. Able to be prepared into a well-defined state
3. Able to maintain quantum coherence over the duration of a computation
4. Have a "universal" set of quantum gates - that is, a set of unitary operations that can create an arbitrary entangled state in the system
5. The ability to read out the state of the system in some basis

This set of criteria is quite general, and a great variety of approaches spanning a number of technologies have appeared, all seeking to fulfill them. These include but are not limited to trapped ions[MBA23], trapped atoms [WBB23], superconducting Josephson junctions [AAB19], quantum dots [Har22], nitrogen vacancy centers [GWI21], nuclei [GC97], and photons [AAA25].

The diVincenzo criteria may set a broad guideline for the minimum requirements needed to make a quantum computer, but the particulars highlight the enormity of the challenge

involved. As any useful general quantum computation will likely require thousands of quantum gates, the fidelity of the states prepared for the computation must be extremely high, as does the accuracy of the individual interactions being performed. For instance, for 1000 two-qubit gates to be performed without any errors 50% of the time, a gate fidelity of roughly 0.9994 is required.

To reduce these requirements and overcome the physical limitations (such as lifetime, temperature, and coupling to noise) of qubits, the field of quantum error correction has emerged to provide ways to encode several imperfect physical qubits together into a single "logical" qubit with a lower error rate than any of its individual constituents. The lower error bound for such encodings to be usable is roughly 1-0.1%, although the number of qubits required for a given logical qubits is large when near this threshold [CTV17]. In order for encodings to be practical on a small number of qubits, error rates must be lower, around 10^{-4} . While this is still a difficult target, it is significantly easier to achieve than developing the perfect qubit.

1.1.1.2 Why trapped ions

Trapped ions are a promising avenue for quantum computation for a number of reasons.

- Each ion is physically identical, reducing (though certainly not eliminating) the calibration requirements
- Trapped ions can be laser-cooled, which mitigates the requirements of cryogenic systems
- The hyperfine clock states of trapped ions with half-integer nuclear spin are insensitive to magnetic fields, and feature coherence times of up to 1 hour [WLQ21].
- Trapped ions are charged, giving them trap depths much higher than those of neutral atoms.

1.1.1.3 This work

In the quantum computing-related work performed in the course of this thesis, we seek to improve the fidelity of the state preparation and measurement (SPAM) of a trapped ytterbium-171 ion. Using a technique in which we shelve the population of one of the qubit states to a long-lived metastable state, we were able to greatly increase the fidelity of the measurement, limited only by a previously unobserved M1 decay.

In addition, we demonstrated an effect that had previously been unobserved in ytterbium, in which the mixing of the qubit states in the presence of a magnetic field allows us to fine-tune the qubit’s sensitivity to a perturbing laser field. Using this effect, we were able to extend the coherence time of the qubit in the presence of a high-power perturbing laser by a factor of roughly 2000.

1.1.1.4 in Probing Nuclear Structure

In addition to their usefulness in quantum computing, trapped ions present the researcher with a physical system that can be controlled and measured to extreme precision. The electronic structure of any trapped atomic system is sensitive to a great number of effects, and experiments in trapped ions have searched for beyond-standard model physics such as a fifth force [HAC22], violations of Lorentz invariance [SHL19, KL99], and shifts of fundamental constants [HSL16, BCH19].

Of particular relevance to the work in this thesis is the sensitivity of the electronic structure to perturbations from the nucleus. This allows us to measure the electronic structure in order to infer nuclear properties such as nuclear size [FM23], magnetization distribution [RG21], and multipole moments [Hof14]. In particular, the metastable $^2F_{7/2}^o$ state in the previously untrapped ytterbium-173 isotope is extremely long-lived and sensitive to the very high-order multipole moments of the nucleus. By leveraging this long lifetime in conjunction with the quantum control that has been developed in the trapped ion field, we can make

measurements of the nuclear moments to an unprecedented precision.

In this work, we have developed techniques for trapping and manipulating the state of $^{173}\text{Yb}^+$, based on the preliminary work performed by Conrad Roman [Rom21].

1.2 Story of the Thesis

As is often the case, the order of the work in this thesis is out of order from the order in which it was performed in the lab. For reasons of flow, I have kept the parts of the work pertaining to $^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$ separate, in the hopes that the reader does not need to keep the level structure of both in their head at once.

Although I also contributed to some work related to background-free state detection using a mode-locked laser [RRI20], I deemed it outside the scope of this work. Chronologically, the first work that was performed and appears in this thesis is the SPAM measurement detailed in Chapter 6. During the execution of this work, the COVID-19 pandemic took place, during which I performed most of the design of the four-rod trap detailed in Section 5.2. Upon returning to the lab, we undertook the ^{173}Yb work in chapter 9. After the observation of the magic polarization in $^{133}\text{Ba}^+$ by my colleagues Sam Vizvary and Zach Wall [VWB23], we moved towards performing measurements of the same effect in $^{171}\text{Yb}^+$, detailed in chapter 8. During the execution of these experiments, we experienced an ion trap failure and were forced to construct a new ion trap as detailed in section 5.2.2. Following the construction of the new trap, the magic polarization work continued.

1.3 Structure of the Thesis

This thesis is divided into ten chapters, the following nine of which are detailed below:

- Chapter 2 lays a groundwork for the atomic structure theory needed in order to understand the aspects of the ytterbium ion described in this work. It details the gross level

structure, hyperfine structure, Zeeman structure, and contains a very brief primer on nuclear structure.

- Chapter 3 lays out the basic theory of ion traps, outlines the effect of the ion trap on the atom, and addresses ways of compensating the trapping potential.
- Chapter 4 This chapter outlines the experimental techniques that are common to most ytterbium trapped ion experiments. This includes procedures such as photoionization, laser cooling, state preparation and measurement (SPAM).
- Chapter 5 details the experimental apparatus used in this work, and acts as a reference for experimental parameters.
- Chapter 6 details the first major experiment of the thesis [RRD21], in which narrowband optical pumping was used to greatly increase SPAM fidelity by shelving one quantum state in the $^2F_{7/2}^o$ state during state detection, protecting it from mixing during readout.
- Chapter 7 gives a brief treatment of the metastable qubit in the F state of $^{171}\text{Yb}^+$, and the work that has taken place in the Campbell group to operate it. Much more detail on the subject can be found in [Rom21].
- Chapter 8 details the experiment we performed to measure a magic polarization in the ground state of $^{171}\text{Yb}^+$ – that is, a magnetic-field-dependent polarization at which the differential light shift of the ground state qubit is canceled out.
- Chapter 9 details the work that has been performed in trapping $^{173}\text{Yb}^+$ in preparation for performing spectroscopy of the metastable $^2F_{7/2}^o$ state. It covers Doppler cooling, spectroscopy of a number of transitions needed to populate and depopulate the state, and lays out a detailed scheme for the spectroscopy.

- Chapter 10 lists some future directions of research into the $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$, and highlights other work that is currently taking place with the ion.

CHAPTER 2

Atomic Structure

2.1 The Level Structure of Ytterbium

2.1.1 Ytterbium's place among the ions

Ytterbium has a rich level structure compared to most other commonly used trapped ion species. The ionized rare earths only have a single valence electron surrounding a very stable core. As such, the gross level structure very closely resembles that of hydrogen, and can be said to obey quantum defect theory [Foo05]. In ytterbium, on the other hand, a single electron exists, but the core is much more weakly bound — rather than having a fully filled shell, only the f -subshell is filled, giving a $4f^{14}6s\ ^2S_{1/2}$ ground state.

For many purposes, the Ytterbium ion looks a lot like any of the other ion species when you ignore the $4f^{14}$ core. It has a $4f^{14}6s\ ^2S_{1/2}$ ground state, it is cooled and read out using a $4f^{14}6p\ ^2P_{1/2}^o$ excited state, and, like barium and calcium, it possesses low-lying and long-lived $4f^{14}5d\ ^2D_{3/2,5/2}$ states. However, when stepping even slightly outside of these states, the importance of the core becomes apparent.

The first excited state of ytterbium is a $4f^{13}6s^2\ ^2F_{7/2}^o$ state. That is, it does not consist of a single electron orbiting a filled core, but rather consists of a *hole* in the core. When driving an electric octopole transition from the $^2S_{1/2}$, we are actually exciting an electron from the filled core to fill an s -orbital, leaving the core unfilled.

2.1.2 J_1K and J_1J_2 -coupled states

Many other states in ytterbium are similarly multi-electron excited states. Many of the lower-lying energy levels have electron configurations in which electrons have been excited from the core, such as $4f^{13}5d6s$ or $4f^{13}5d6p$ [AYJ23]. For these states, it is often insufficient to assign a Russell-Saunders (LS-coupled) term, and we must turn to one of two intermediate coupling schemes. Rather than considering the angular momentum of a single electron, these coupling schemes split the electrons into a "core" group and an "outer" group.

2.1.2.1 J_1K -coupled states

Several of the states that we use when manipulating the ytterbium ion do not use standard LS coupling. These states like $^3[3/2]_{1/2}$ and $^1[3/2]_{3/2}$ instead use the J_1K coupling scheme. In this coupling scheme, the angular momentum vectors are added as follows:

$$\begin{aligned}\ell_c + s_c &= J_c \\ J_c + \ell_o &= K \\ K + s_o &= J\end{aligned}\tag{2.1}$$

where the good quantum numbers are J_c , K , and J . J_1K -coupled terms are written as $^{2s_o+1}[K]_J$, and often, the angular momentum of the core and outer electrons is explicitly written in the electron configuration as well.

For a worked example, we may write the upper state of our 935 nm transition as $4f^{13}(^2F_{7/2}^o)5d6s(^3D)^2[3/2]_{1/2}^o$. In this example, $\ell_c = 3$, $s_c = \frac{1}{2}$, $\ell_o = 2$, $s_o = 1$, $J_c = \frac{7}{2}$, $K = \frac{3}{2}$, and $J = \frac{1}{2}$. Since these states are simply related by the order of the coupling, we can freely convert between the J_1K and LS representations for states, and the process is given in Appendix A.0.1.

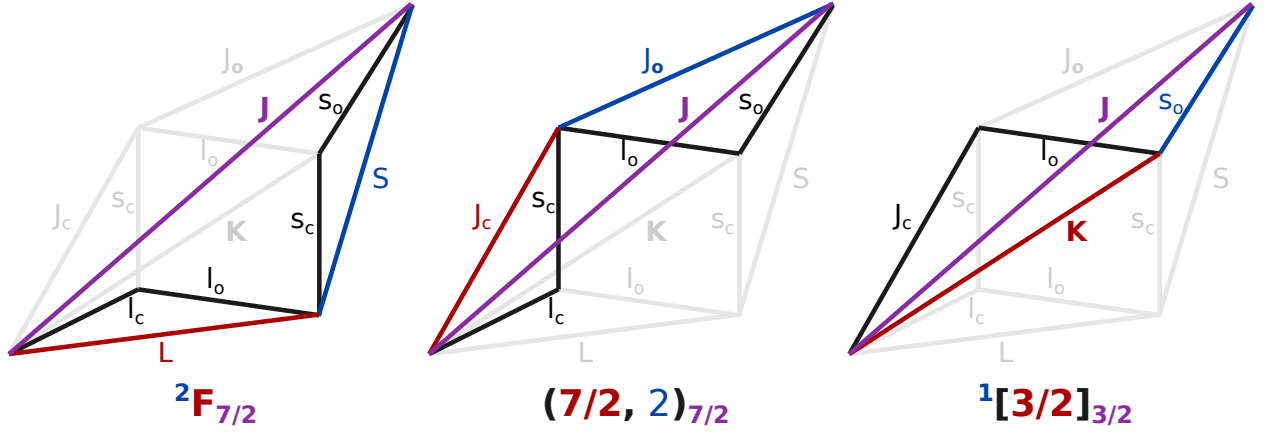


Figure 2.1: A diagram of the three angular momentum coupling schemes that appear in the ytterbium ion, and their vector components. The left diagram represents LS coupling, where the ℓ_c and ℓ_o couple to form L , while s_c and s_o couple to form S . The middle diagram represents J_1J_2 coupling, where each ℓ and s first combine to form J_c and J_o . The right diagram represents J_1K coupling, where J_c and ℓ_o couple to form K , which then couples to s_o .

2.1.2.2 J_1J_2 -coupled states

Many excited states in ytterbium (though none that will be used explicitly in this work) are in the J_1J_2 coupling scheme. In this scheme, the angular momentum vectors add as

$$\begin{aligned}
 \ell_c + s_c &= J_c \\
 \ell_o + s_o &= J_o \\
 J_c + J_o &= J
 \end{aligned}
 \tag{2.2}$$

where the good quantum numbers are J_c , J_o , and J . J_1J_2 -coupled terms are written as $(J_c, J_o)_J$, and often the full terms of the core and outer electrons are also explicitly written in the electron configuration.

2.1.3 A final note on coupling schemes

As is always the case in a many-electron system, the assignment of a single term symbol to an atomic state rarely fully characterizes the state. The assigned coupling simply provides the *best* description of the state's behavior. When assigning term symbols, the assignment process is one in which the state is computed, and then its projection onto various coupling schemes is evaluated. Then, the term with the largest projection onto the state is assigned to the state. Often, the second-leading term symbol is provided.

Coupling schemes are a way of simplifying and labeling terms, and are only an approximate characterization of the full atomic Hamiltonian. It is not always the case that the three electrons couple cleanly in a given order. For a good further discussion on this subject, see any of the fine works by Robert Cowan [CA65, Cow70].

2.2 Hyperfine Structure

In isotopes with nuclear spin, J , the total angular momentum of the electrons, is no longer fully a good quantum number. This is because the nucleus now has its own nuclear spin I that contributes to the total angular momentum of the system. In this section, we will go over various terms in the hyperfine structure of the odd isotopes of Ytterbium.

2.2.1 Dipole hyperfine structure

Since the magnitude of the nuclear dipole moment is much smaller than that of the electron's magnetic dipole moment $\mu_N \sim \frac{m_e}{m_N} \mu_B \approx \frac{1}{2000} \mu_B$, we can treat the hyperfine interaction as a perturbation on the existing fine structure states that we find in spin zero isotopes.

In the case where we only consider a spin-spin interaction, the hyperfine interaction Hamiltonian is

$$H_{M1} = \mu_I \cdot \mathbf{B}_{elec} = A \mathbf{I} \cdot \mathbf{J} \quad (2.3)$$

due to the fact that the electronic magnetic field $\mathbf{B}_{elec}$ is created due to the electron's angular motion around the nucleus and therefore scales with $\mathbf{J}$, and $\mu_I = \mu_N g_I \mathbf{I}$. Since the magnitude of the electronic magnetic field at the nucleus is dependent on the shape of the electronic wave function, we absorb the constant of proportionality between J and into a hyperfine A coefficient and leave its exact calculation to atomic structure theorists.

To extract the energy shift due to the hyperfine interaction, we take the matrix element $A \langle I, J | \mathbf{I} \cdot \mathbf{J} | I, J \rangle$. Defining a new total angular momentum operator $\mathbf{F} = \mathbf{I} + \mathbf{J}$, and using the relation $\mathbf{I} \cdot \mathbf{J} = \frac{1}{2}(\mathbf{F}^2 - \mathbf{I}^2 - \mathbf{J}^2)$, we see that the values of the shift are

$$\Delta E_{M1} = \frac{A}{2} [F(F+1) - J(J+1) - I(I+1)] \quad (2.4)$$

2.2.2 Multipole hyperfine structure

In general, more than just the dipole moments of the nucleus and the electron can interact. We can additionally have multipole interactions such as quadrupole-quadrupole, octopole-octopole, and even higher-order interactions, given that both the nucleus and electron cloud have the relevant multipole moments. Since the spherical harmonics are orthogonal, multipoles of different orders never interact. The most elegant way to derive the forms of these higher-order shifts is given in [BDJ08], and I will provide that derivation here. For a complete relativistic derivation of the multipole hyperfine shift, see [Arm71] (for a copy, check the UC library).

In general, we can write the scalar product of a nuclear and an electronic spherical tensor operator as

$$H_{HF} = T_{\mu}^{(k),e} \cdot T_{\mu}^{(k),n} = \sum_{k,\mu} (-1)^{\mu} T_{\mu}^{(k),e} T_{\mu}^{(k),n} \quad (2.5)$$

$T^{(k),e}$ in the above expression is the rank- k multipole moment operator for the electronic wave function. For the even-order moments ($k = 2, 4, \dots$), these terms simply correspond to the spatial charge distribution of the electrons. For the odd-order moments, the relationship is more complex, as we have to consider the magnetic field generated by the electron cloud

at the nucleus. As such, we can write the following forms for the odd and even terms for a single electron:

$$t_{k,\mu}^e(\mathbf{r}) = \begin{cases} -\frac{1}{r^{k+1}} C_{k,\mu}(\hat{\mathbf{r}}) & \text{electric (even } k \text{)} \\ -\frac{i}{r^{k+1}} \sqrt{\frac{k+1}{k}} \alpha \cdot \mathbf{C}_{k,\mu}^{(0)}(\hat{\mathbf{r}}) & \text{magnetic (odd } k \text{)} \end{cases} \quad (2.6)$$

where α is the relativistic electron momentum operator (defined in page 21 of [Arm71]), $C_{k,q} = \sqrt{\frac{4\pi}{2k+1}} Y_{k,q}(\theta, \phi)$ is the q -th normalized spherical harmonic of rank k , and $\mathbf{C}_{k,q}^{(0)} = \sqrt{\frac{4\pi}{k(k+1)(2k+1)}} \mathbf{L} Y_{(k,q)}(\theta, \phi)$ is the the q -th normalized *vector* spherical harmonic of rank k [Joh06].

$T^{(k),n}$ corresponds to the rank- k nuclear multipole moment operator, up to an arbitrary prefactor defined by convention. Since these prefactors get absorbed into the hyperfine structure coefficients, I will give them later.

Applying perturbation theory, the matrix element that we wish to derive is the diagonal element of the hyperfine interaction $\langle I, J, F, m_F | H_{HF} | I, J, F, m_F \rangle$. We can apply the reduction theorem for scalar products of spherical tensor operators to reduce the problem [Zar91]:

$$\begin{aligned} \langle I, J, F, m_F | T_{\mu}^{(k),e} \cdot T_{\mu}^{(k),n} | I, J, F, m_F \rangle &= (-1)^{I+J+F} \sum_k \begin{Bmatrix} F & J & I \\ k & I & J \end{Bmatrix} \\ &\times \langle J || T^{(k),e} || J \rangle \langle I || T^{(k),n} || I \rangle \end{aligned} \quad (2.7)$$

By rewriting the reduced matrix elements in the above expression in terms of the stretched matrix elements of the interaction by the definition

$$\langle O_k \rangle_j \equiv \langle j, m_j = j | O_k | j, m_j = j \rangle = \begin{pmatrix} j & k & j \\ -j & 0 & j \end{pmatrix} \langle j || O_k || j \rangle \quad (2.8)$$

we get an elegant way to express the k th-order multipole term as

$$\Delta E^{(k)} = X_k \langle T^{(k),e} \rangle_J \langle T^{(k),n} \rangle_I \quad (2.9)$$

where

$$X_k(I, J, F) \equiv (-1)^{I+J+F} \frac{\begin{Bmatrix} F & J & I \\ k & I & J \end{Bmatrix}}{\begin{pmatrix} J & k & J \\ -J & 0 & J \end{pmatrix} \begin{pmatrix} I & k & I \\ -I & 0 & I \end{pmatrix}} \quad (2.10)$$

This lets us define the hyperfine structure coefficients in a compact way:

$$\begin{aligned} A &= \frac{1}{IJ} \langle T^{(1),n} \rangle_J \langle T^{(1),e} \rangle_J = \frac{1}{IJ} \mu \langle T^{(1),e} \rangle_J \\ B &= 4 \langle T^{(2),n} \rangle_J \langle T^{(2),e} \rangle_J = 2Q \langle T^{(2),e} \rangle_J \\ C &= \langle T^{(3),n} \rangle_I \langle T^{(3),e} \rangle_J = -\Omega \langle T^{(3),e} \rangle_J \\ D &= \langle T^{(4),n} \rangle_I \langle T^{(4),e} \rangle_J = \Pi \langle T^{(2),e} \rangle_J \end{aligned} \quad (2.11)$$

where μ, Q, Ω, Π are the nuclear dipole, quadrupole, octopole, and 16-pole moments. The prefactors in the above expression are set by convention. We can now explicitly evaluate the first several terms of the multipole hyperfine shift. We write

$$\Delta E_{HF} = AIJX_1(I, J, F) + \frac{1}{4}BX_2(I, J, F) + CX_3(I, J, F) + DX_4(I, J, F) \quad (2.12)$$

We can explicitly rewrite the terms multiplying the hyperfine coefficients (see page 70 of [Arm71] for the working out):

$$IJX_1(I, J, F) = \mathbf{I} \cdot \mathbf{J} \quad (2.13)$$

$$\frac{1}{4}X_2(I, J, F) = \frac{3(\mathbf{I} \cdot \mathbf{J})^2 + \frac{3}{2}(\mathbf{I} \cdot \mathbf{J}) - I(I+1)J(J+1)}{2IJ(2I-1)(2J-1)} \quad (2.14)$$

$$\begin{aligned} X_3(I, J, F) &= +2(\mathbf{I} \cdot \mathbf{J}) [-3I(I+1)J(J+1) + I(I+1) + J(J+1) + 3] \\ &\quad - 5I(I+1)J(J+1) / [I(I-1)(2I-1)J(J-1)(2J-1)] \end{aligned} \quad (2.15)$$

The key aspect to note about the above terms is that each subsequent term in the hyperfine structure adds an energy shift that is a k th-order polynomial in $(\mathbf{I} \cdot \mathbf{J})$. This means that if we plot the hyperfine structure of a particular state as a function of $F(F+1)$, we can extract the hyperfine coefficients by fitting to a polynomial.

2.2.3 Second-order corrections

In general, it is not sufficient to compute the hyperfine shifts using first order perturbation theory. When the magnitudes of the magnetic octupole shift or electric hexadecapole shifts are kilohertz or less, second-order corrections to the magnetic dipole and electric quadrupole shift become important [XLC20]. In general, the second-order perturbation to the energy is

$$\Delta E_{HF}^{(2)} = \sum_{\gamma', J'} \frac{\langle \gamma, I, J, F, m_F | H_{HF} | \gamma', I, J', F, m_F \rangle \langle \gamma', I', J', F, m_F | H_{HF} | \gamma, I, J, F, m_F \rangle}{E - E'} \quad (2.16)$$

If we rewrite the matrix elements in the same way as as we did in equation 2.7, we can express this as

$$\begin{aligned} \Delta E_{HF}^{(2)} &= \sum_{\gamma', J'} \frac{1}{E - E'} \sum_{k_1, k_2} \begin{Bmatrix} I & J & F \\ J' & I & k_1 \end{Bmatrix} \begin{Bmatrix} I & J & F \\ J' & I & k_2 \end{Bmatrix} \\ &\quad \times \langle I || T^{(k_1), n} || I \rangle \langle I || T^{(k_2), n} || I \rangle \langle \gamma, J || T^{(k_1), e} || \gamma, J' \rangle \langle \gamma, J || T^{(k_2), e} || \gamma, J' \rangle \\ &= \sum_{\gamma', J'} \frac{1}{E - E'} \sum_{k_1, k_2} \frac{\begin{Bmatrix} I & J & F \\ J' & I & k_1 \end{Bmatrix} \begin{Bmatrix} I & J & F \\ J' & I & k_2 \end{Bmatrix}}{\begin{pmatrix} I & k_1 & I \\ -I & 0 & I \end{pmatrix} \begin{pmatrix} I & k_2 & I \\ -I & 0 & I \end{pmatrix}} \\ &\quad \times \langle T^{(k_1), n} \rangle_I \langle T^{(k_2), n} \rangle_I \langle \gamma, J || T^{(k_1), e} || \gamma, J' \rangle \langle \gamma, J || T^{(k_2), e} || \gamma, J' \rangle \end{aligned} \quad (2.17)$$

In practice, if we only consider second order dipole and quadrupole effects, three terms arise: a dipole-dipole term ($k_1 = k_2 = 1$), a dipole-quadrupole term ($k_1 = 1, k_2 = 2$), and a quadrupole-quadrupole term ($k_1 = k_2 = 2$). Calculating these directly is outside the scope of this thesis, but a discussion of the impact of these terms on the $^2F_{7/2}$ state of $^{173}\text{Yb}^+$ can be found in Section 9.1.

2.2.4 Hyperfine Rescaling

One thing to note about all of the above work is that the nuclear and electronic parts of the hyperfine coefficients are always separate and can be factored out. This has a convenient consequence that allows us to estimate the hyperfine structure of a state in one isotope if we know its hyperfine structure in another isotope, along with some properties of the nucleus.

Take, for example, the $^2S_{1/2}$ ground state in $^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$. Since $A \propto \mu I J \langle T^{(1),e} \rangle_J$, and the electronic states are identical, we can rescale the hyperfine structure of one isotope to predict that of the other. Knowing that the nuclear magnetic dipole moments of the two species are $\mu_{171} = +0.49367(1)$, $\mu_{173} = -0.67989(3)$ [Sto05], and that $I_{171} = \frac{1}{2}$, $I_{173} = \frac{5}{2}$ we can deduce that the ratio of the hyperfine A coefficients is -3.6305.

Although this method only allows one to estimate the hyperfine structure when another isotope has a state with similar hyperfine structure, being able to deduce the hyperfine coefficients of $^{173}\text{Yb}^+$ from that of $^{171}\text{Yb}^+$ is a valuable tool. In states where $J < 1$, only the magnetic dipole contribution applies to the hyperfine structure, and we can accurately predict the hyperfine splittings in $^{173}\text{Yb}^+$. Even in states with $J > 1$, being able to determine the value of the A coefficient can help when trying to assign lines or can allow you to infer the higher-order hyperfine coefficients from fairly limited spectroscopic data.

2.2.4.1 Anomalous Hyperfine Structure

Although it will not be mentioned elsewhere in this work, it is worth briefly touching on a subtlety of the previous calculation. Upon estimation of the hyperfine coefficient A of $^{173}\text{Yb}^+ ^2S_{1/2}$ we may observe that the predicted ratio of -3.6305 is slightly different from the actual ratio of -3.61508 measured in spectroscopic data [HXZ24, MBG87]. This difference is physical and is called the *magnetic hyperfine anomaly*

This anomaly appears as a correction to the hyperfine structure due to the finite size of the nucleus [Per13, BS70]. It comes from two predominant sources:

- The Bohr-Weisskopf effect, in which the finite magnetization distribution of the nucleus is taken into account
- The “Breit-Rosenthal-Crawford-Schwalow” correction, in which we take into account the change of the electronic wavefunction due to nuclear size effects

Generally, these effects are only observable in the S and P states where the electron wavefunction significantly overlaps the nucleus, and generally the Bohr-Weisskopf effect is dominant. Given pairs of atoms, this effect can be used to glean information about nuclear magnetization distribution, which is difficult to measure by any other means, and can be used to evaluate different models of nuclear structure [RG21]. In particular, the hyperfine anomaly can also be used to investigate the precise degree of state mixing that occurs in complicated atomic terms, as was pointed out for ytterbium in ref. [BS70]. For an introductory discussion of hyperfine anomaly and a thorough tabulation of experimentally measured hyperfine anomalies, see ref. [Per13].

2.3 Zeeman Structure

2.3.1 Fine structure Zeeman

On top of the hyperfine structure, we must consider one last level of atomic structure – namely that which arises from the interaction of the atom with an external magnetic field. In the weak-field limit, we can treat the effect of the magnetic field as a perturbation, and we can assume that F and m_F are still good quantum numbers. The Hamiltonian of the interaction is

$$H_Z = \boldsymbol{\mu} \cdot \mathbf{B} \tag{2.18}$$

where the total magnetic moment of the atom in the absence of the hyperfine interaction is $\boldsymbol{\mu} = g_L \mu_B \mathbf{L} + g_S \mu_B \mathbf{S}$. The interaction we would like to evaluate the expectation of is

therefore

$$H_Z = \mu_B g_L \mathbf{B} \cdot \mathbf{L} + \mu_B g_S \mathbf{B} \cdot \mathbf{S} \quad (2.19)$$

and we would like to evaluate this expectation value in the J, m_J basis. Without loss of generality we may assume that the magnetic field is pointing in the z-direction. We will evaluate the $\mathbf{S}$ term first. As we have assumed that J is a good quantum number, we can assume that the expectation value of $\mathbf{S}$ is closely approximated by its projection onto $\mathbf{J}$. In this limit, we can state that [Sob96]

$$\langle \mathbf{S} \rangle \approx \frac{\langle \mathbf{S} \cdot \mathbf{J} \rangle}{J(J+1)} \mathbf{J} \quad (2.20)$$

Using the fact that $\mathbf{S} \cdot \mathbf{J} = \frac{1}{2} \mathbf{J}^2 - \mathbf{L}^2 + \mathbf{S}^2$, we can write the expectation value of the $\mathbf{S}$ term as:

$$\langle \mu_B g_S \mathbf{B} \cdot \mathbf{S} \rangle = \mu_B B g_S m_J \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)} \quad (2.21)$$

Following the same procedure for the $\mathbf{L}$ term, we can write the total perturbation as

$$\langle H_Z \rangle = \mu_B B m_J \left[g_L \frac{(J(J+1) - S(S+1) + L(L+1))}{2J(J+1)} + g_S \frac{(J(J+1) + S(S+1) - L(L+1))}{2J(J+1)} \right] \quad (2.22)$$

$$\equiv \mu_B g_J m_J B \quad (2.23)$$

For a finite nuclear mass $g_L = 1 - m_e/m_{\text{atom}}$.

2.3.2 Hyperfine structure Zeeman

Should we want to add in the hyperfine interaction structure, our Hamiltonian becomes

$$H_Z = -g_J \mu_B \mathbf{B} \cdot \mathbf{J} + g_I \mu_N \mathbf{B} \cdot \mathbf{I} \quad (2.24)$$

We can make the same approximation that we did for $\mathbf{L}$ and $\mathbf{S}$, claiming that the expectation values of $\mathbf{I}$ and $\mathbf{J}$ both project mostly onto $\mathbf{F}$. In that case, we can reuse equation 2.23 to state that

$$\langle H_Z \rangle = m_F B \left[\mu_B g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} + \mu_N g_I \frac{F(F+1) + I(I+1) - J(J+1)}{2F(F+1)} \right] \quad (2.25)$$

$$\approx \mu_B m_F B g_J \frac{F(F+1) - I(I+1) + J(J+1)}{2F(F+1)} \quad (2.26)$$

$$\equiv \mu_B m_F g_F B \quad (2.27)$$

where we have made the approximation that the g_I term is small, as the nuclear magneton $\mu_N \ll \mu_B$.

2.4 Intermediate-field Zeeman structure

Performing hyperfine-sensitive measurements of the structure of the F-state to a given precision requires the states to be resolved to that precision. One factor that limits the precision we can achieve is the sensitivity of the states to magnetic field noise. As such we should use the MCDHF computation of the hyperfine structure (further discussed in section 9.1) [XLC20] of $^{173}\text{Yb}^+$ as a basis to predict the magnetic field sensitivity of the clock states in the $^2F_{7/2}^o$ manifold.

To compute the magnetic sensitivity of the clock qubits to all orders, we employ a generalized Breit-Rabi Hamiltonian. That is, we want to diagonalize the atomic Hamiltonian in the regime where the hyperfine splitting and the Zeeman splitting are comparable. Although this can be done by hand in the $^2S_{1/2}$ ground state of an alkali element, for instance, a general approach involves a large number of Zeeman states; the hyperfine manifold of our F state contains 48 Zeeman sublevels.

2.4.1 Matrix elements of the Breit-Rabi Hamiltonian

The Hamiltonian in which we are interested is that of a magnetic field acting on an eigenstate of the atom without a magnetic field. Such an atom has quantum numbers $|J, I, F, m_F\rangle$. In this basis, the Hamiltonian is diagonal, and we can simply populate our unperturbed Hamiltonian with the measured or computed energies for each F sublevel. The magnetic field Hamiltonian takes the form

$$H_B = \mu_B g_J \mathbf{B} \cdot \mathbf{J} - \mu_N g_I \mathbf{B} \cdot \mathbf{I} \quad (2.28)$$

where g_J is defined in equation 2.23. For a finite nuclear mass $g_L = 1 - m_e/m_{Yb}$. This Hamiltonian is diagonal in the $|J, I, m_J, m_I\rangle$ basis. To express the matrix elements of this Hamiltonian in the $|J, I, F, m_F\rangle$ basis, we have to convert using angular momentum recoupling. Specifically:

$$\begin{aligned} \langle F', m_F' | H_B | F, m_F \rangle &= \sum_{m_J m_I} \langle F', m_F' | m_{J'}, m_I' \rangle \langle m_{J'}, m_I' | H_B | m_J, m_I \rangle \langle m_J, m_I | F, m_F \rangle \\ &= B \sum_{m_J m_I} (\mu_B g_J m_J - \mu_N g_I m_I) (-1)^{m_F' + m_F + 2J - 2I} \sqrt{(2F' + 1)(2F + 1)} \\ &\quad \times \begin{pmatrix} J & I & F' \\ m_J & m_I & -m_F' \end{pmatrix} \begin{pmatrix} J & I & F \\ m_J & m_I & -m_F \end{pmatrix} \delta_{J, J'} \delta_{I, I'} \delta_{m_F, m_F'} \end{aligned} \quad (2.29)$$

Alternatively, we can use the Wigner-Eckart theorem to find an alternate expression for the matrix element that can be a bit faster to compute, as it does not require explicit

summation:

$$\begin{aligned}
& \langle F', m'_F | H_B | F, m_F \rangle \\
&= B \left(\mu_B g_J \langle F', m'_F | T_0^{(1)J} | F, m_F \rangle - \mu_N g_I \langle F', m'_F | T_0^{(1)I} | F, m_F \rangle \right) \\
&= B \left[\mu_B g_J \langle F || T_0^{(1)J} || F \rangle (-1)^{F-m_F} \begin{pmatrix} F & 1 & F' \\ -m_F & 0 & m'_F \end{pmatrix} \right. \\
&\quad \left. - \mu_N g_I \langle F || T_0^{(1)I} || F \rangle (-1)^{F-m_F} \begin{pmatrix} F & 1 & F' \\ -m_F & 0 & m'_F \end{pmatrix} \right] \\
&= B (-1)^{F+F'-m'_F+J+I+1} \\
&\quad \times \left[\mu_B g_J \langle J || T_0^{(1)J} || J \rangle \sqrt{(2F'+1)(2F+1)} \begin{pmatrix} F & 1 & F' \\ -m_F & 0 & m'_F \end{pmatrix} \begin{Bmatrix} J & F' & I \\ F & J & 1 \end{Bmatrix} \right. \\
&\quad \left. + \mu_N g_I \langle I || T_0^{(1)I} || I \rangle \sqrt{(2F'+1)(2F+1)} \begin{pmatrix} F & 1 & F' \\ -m_F & 0 & m'_F \end{pmatrix} \begin{Bmatrix} I & F' & J \\ F & I & 1 \end{Bmatrix} \right] \\
&= B (-1)^{F+F'-m'_F+J+I+1} \sqrt{(2F'+1)(2F+1)} \begin{pmatrix} F & 1 & F' \\ -m_F & 0 & m'_F \end{pmatrix} \\
&\quad \times \left[\mu_B g_J \sqrt{J(J+1)(2J+1)} \begin{Bmatrix} J & F' & I \\ F & J & 1 \end{Bmatrix} + \mu_N g_I \sqrt{I(I+1)(2I+1)} \begin{Bmatrix} I & F' & J \\ F & I & 1 \end{Bmatrix} \right] \tag{2.30}
\end{aligned}$$

For a graphical proof of the equivalence of these two expressions, see Appendix B.

2.4.2 Calculation of the full Zeeman Structure

Looking at equation 2.29, we can exploit the fact that there is a $\delta_{m_F, m_{F'}}$ term (due to the nature of the Zeeman Hamiltonian as $T_0^{(1)}$) to split an otherwise large Hamiltonian into multiple smaller matrices, each containing only one value of m_F . For instance, this turns the 48×48 matrix of the $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$ into 13 matrices: 1×1 , 2×2 , 3×3 , 4×4 , 5×5 , 6×6 , 6×6 , 6×6 , 5×5 , 4×4 , 3×3 , 2×2 , 1×1 . This is much more tractable.

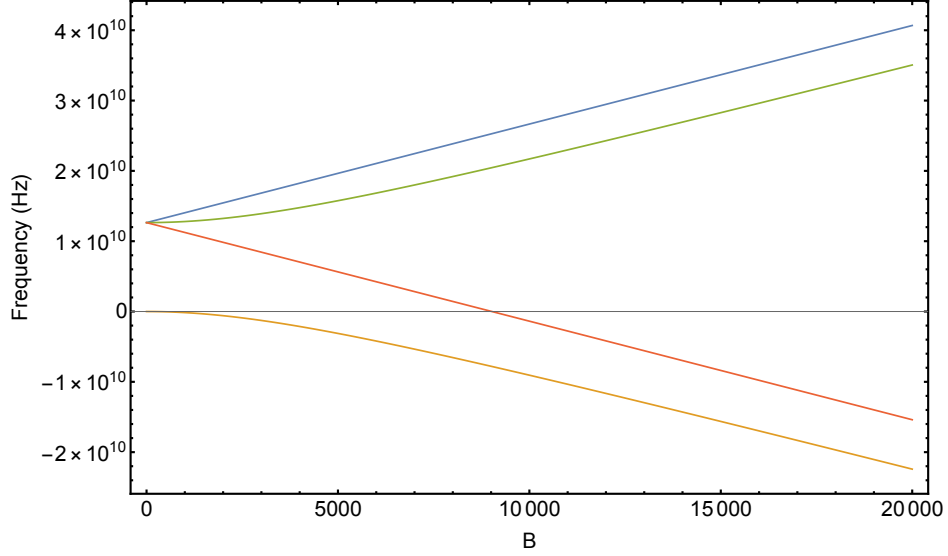


Figure 2.2: The intermediate field Zeeman structure of the $^2S_{1/2}$ ground state in $^{171}\text{Yb}^+$. Note the quadratic field dependence of the $m_F = 0$ states in the low-field limit.

Figure 2.2 shows the result of diagonalizing the above Hamiltonian for the $^2S_{1/2}$ ground state of $^{171}\text{Yb}^+$. It accurately reproduces the observed magnetic field dependence of the clock state, with the qubit frequency changing by 310.778Hz/G^2 .

Having validated our results, we can plot the magnetic field dependence in the $^2F_{7/2}^o$ state of $^{173}\text{Yb}^+$. It is plotted in Figure 2.3, and Appendix F gives the field dependence of each Zeeman state to fourth order.

The second-order magnetic field dependence of the clock qubits is of the most importance, as we plan to operate our qubit or qudit at less than 5 Gauss. The second-order sensitivity of each microwave transition between clock qubits is given in Table 9.2. To compute the degree of broadening of the transition due to field noise, we can look at the shift induced by a signal of a given strength.

For more discussion on the systematics of the F state hyperfine interval measurement, see Section 9.3.

There are some interesting features to note about the Zeeman spectrum that can be

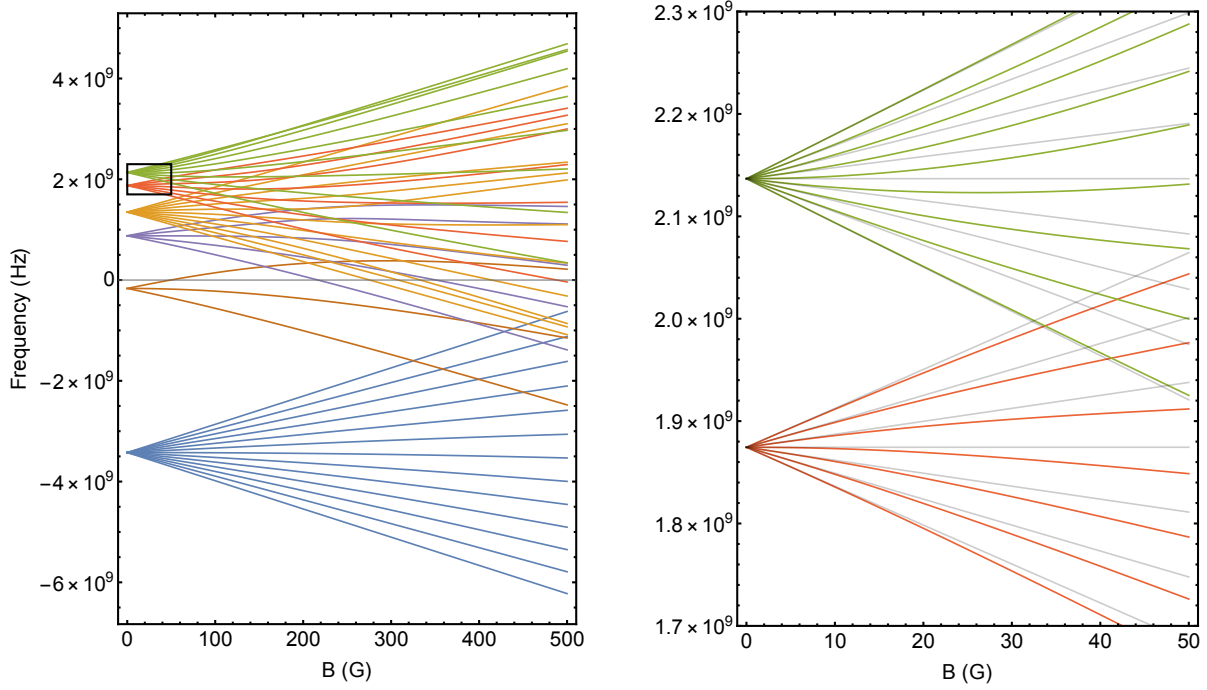


Figure 2.3: The predicted intermediate-field Zeeman structure of the $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$, showing the complicated interactions between the multiple hyperfine manifolds. The figure on the right shows in more detail the shifts of the $F = 3$ and $F = 4$ manifolds, where the strongest nonlinearity is expected in the low-field limit. Light gray lines show the linear Zeeman effect.

Transition	Second-order sensitivity (kHz/G ²)	Linewidth at 10mG with 5mG noise
1 - 2	12.2	1.22
2 - 3	56.0	5.6
3 - 4	242.9	24.3
4 - 5	-4.8	0.48
5 - 6	-6.4	0.64

Table 2.1: The second-order magnetic field sensitivity of the five zero-field clock qubits in the $^{173}\text{Yb}^+ \ ^2F_{7/2}^o$ state, and the uncertainty in the qubit frequency that is induced by residual field noise at the planned magnetic field.

educational. For example, there are several examples of avoided crossings in the Zeeman structure. Since the Zeeman Hamiltonian only links states of the same m_F , levels are free to cross unless they have the same m_F , in which case we observe an avoided crossing. Specifically, the zero-field states $F = 1, m_F = -1$ and $F = 6, m_F = -1$ have an avoided crossing around 900 G, and the zero-field states $F = 1, m_F = 0$ and $F = 6, m_F = 0$ have an avoided crossing around 1400 G. These avoided crossings are highlighted in Figure 2.4.

2.5 Interaction with Radiation

Now that we have established the eigenstates of the atomic Hamiltonian to enough precision to describe $^{173}\text{Yb}^+$, we should discuss the dynamics of the atom in the presence of light.

2.5.1 E1 Transitions

The Hamiltonian for an electric field interacting with the atom in the electric dipole approximation (where $\lambda \gg r$) is

$$H_{E1} = \hat{\epsilon} \cdot \mathbf{d} = eE_0\hat{\epsilon} \cdot \mathbf{r} \quad (2.31)$$

Where $\hat{\epsilon}$ is the light's polarization unit vector, E_0 is the perturbing electric field, and

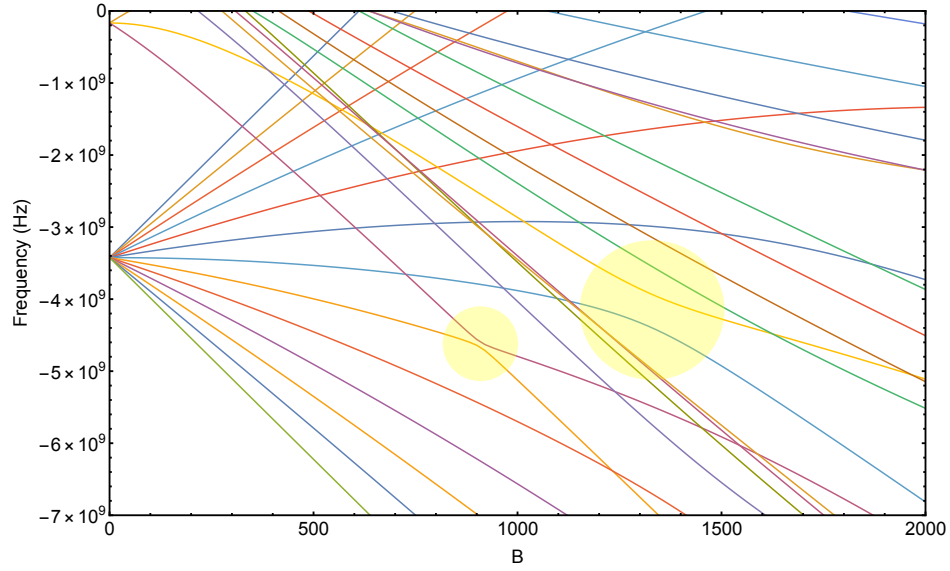


Figure 2.4: Avoided crossings in the Zeeman structure of the $^2F_{7/2}^o$ state of $^{173}\text{Yb}^+$. Highlighted are the avoided crossings of the (zero-field) $|F = 0, m_F = -1\rangle$ and $|F = 6, m_F = -1\rangle$ states, and of the (zero-field) $|F = 0, m_F = 0\rangle$ and $|F = 6, m_F = 0\rangle$ states.

$\mathbf{d} = e\mathbf{r}$ is the dipole moment operator. From this, we can write the Rabi frequency of the transition as

$$\Omega_{E1} = \frac{eE_0}{2\hbar} |\langle g | \hat{\varepsilon} \cdot \mathbf{r} | e \rangle| = \frac{eE_0}{2\hbar} |\hat{\varepsilon} \cdot \langle g | \mathbf{r} | e \rangle| \quad (2.32)$$

We can express $\mathbf{r}$ as a scaled version of the rank-1 spherical tensor $rT_q^{(1)}$. If we also express $\hat{\varepsilon}$ as a rank-1 spherical tensor, then in the spherical tensor basis, we can write $\hat{\varepsilon} \cdot \mathbf{r} = r \sum_{q \in \{-1, 0, 1\}} \hat{\varepsilon}_q^{(1)} \hat{r}_{-q}^{(1)}$. The conversion between the Cartesian basis and the spherical basis for a vector operator $\hat{\varepsilon}$ is

$$\begin{aligned} \hat{\varepsilon}_{\pm} &= \frac{1}{\sqrt{2}} (\varepsilon_x \pm i\varepsilon_y) \\ \hat{\varepsilon}_0 &= \varepsilon_z \end{aligned} \quad (2.33)$$

Explicitly, our Rabi frequency is now

$$\Omega_{E1} = \frac{eE_0}{2\hbar} \left| \sum_{q \in \{-1,0,1\}} \hat{\varepsilon}_{-q} \langle g | \mathbf{r}_q | e \rangle \right| \quad (2.34)$$

Now that we know the composition of our matrix element, we can proceed to use the rules of angular momentum operators to begin simplifying our matrix element. Let us assign a value of J , F , and m_F to each of our states and proceed with the reduction. We will first use the Wigner-Eckart theorem to factor m_F out of our matrix element:

$$\Omega_{E1} = \frac{eE_0}{2\hbar} \left| \sum_{q \in \{-1,0,1\}} \hat{\varepsilon}_{-q} \langle \gamma', J', F', m'_F | \mathbf{r}_q | \gamma, J, F, m_F \rangle \right| \quad (2.35)$$

$$= \frac{eE_0}{2\hbar} \left| \sum_{q \in \{-1,0,1\}} \begin{pmatrix} F' & 1 & F \\ -m'_F & q & m_F \end{pmatrix} \hat{\varepsilon}_{-q} \langle \gamma', J', F' || \mathbf{r} || \gamma, J, F \rangle \right| \quad (2.36)$$

We can furthermore apply the repeated reduction formula to extract the F -dependence of the Rabi frequency:

$$\Omega_{E1} = \frac{eE_0 \sqrt{(2F+1)(2F'+1)}}{2\hbar} \left| \sum_q \begin{Bmatrix} J & J' & 2 \\ F' & F & I \end{Bmatrix} \begin{pmatrix} F' & 1 & F \\ -m'_F & q & m_F \end{pmatrix} \hat{\varepsilon}_{-q} \langle \gamma', J' || \mathbf{r} || \gamma, J \rangle \right| \quad (2.37)$$

From the Rabi rate, we can compute the Einstein A coefficient, which represents the state lifetime associated with the transition:

$$A_{eg} = \frac{4\alpha\nu^3}{3c^2(2J'+1)} |\langle e | \mathbf{r} | g \rangle|^2 \quad (2.38)$$

where α is the fine structure constant, and J' refers to the J of the *upper* state.

This is as far as we will take the computation. Reducing it to this point allows us several conveniences. Generally, the lifetimes of fine structure levels are tabulated. Since we have extracted the dependence on F and m_F into easily-computable 3j and 6j symbols, we can now easily determine the lifetime of a hyperfine or even Zeeman sublevel from the reported

lifetime of the fine structure level. Even if we don't know the lifetime of a state, we can predict the relative transition strengths of two sub-transitions in a given transition between two fine structure states. This is convenient because computing the reduced matrix element of a transition generally involves computing complicated radial integrals.

2.5.2 E2 Transitions

The electric quadrupole (E2) interaction can be seen to be derived from the interaction between the gradient of the driving electric field and the quadrupole moment of the ion. This means that we need to add a term to the electric dipole approximation. Our interaction Hamiltonian is

$$H_{E2} = Q_{ij} \nabla_i E_j = -Q_{ij} \hat{k}_i \hat{\epsilon}_j E_0 \quad (2.39)$$

where $Q_{ij} = r_i r_j - \frac{1}{2} r^2 \delta_{ij}$ is the electric quadrupole operator. Since for a plane wave $\hat{k} \cdot \hat{\epsilon}$ is already always zero, we can simplify our expression for the quadrupole operator to just be $Q_{ij} = r_i r_j$. This means that we can now write the Rabi rate of our transition as

$$\Omega_{E2} = \left| \frac{eE_0}{2\hbar} \hat{\epsilon}_i \hat{k}_j \langle g | r_i r_j | e \rangle \right| \quad (2.40)$$

Converting into a spherical tensor basis, we have

$$\Omega_{E2} = \left| \frac{eE_0}{2\hbar} \sum_q c_{ij}^{(q)} \hat{\epsilon}_i \hat{k}_j \langle g | r^2 T_q^{(2)} | e \rangle \right| \quad (2.41)$$

where $T_q^{(2)}$ is the unit rank-2 spherical tensor operator, and $c_{ij}^{(q)}$ is a tensor that converts $\hat{\epsilon}_i \hat{k}_j$ into a spherical tensor basis. If we take $v^{\{-,0,+\}}$ to be the rank-1 spherical basis vectors defined in equation 2.33, we can define [Jam98]

$$c_{ij}^{(q)} = \sqrt{\frac{10}{3}} (-)^q \sum_{n,m=-1}^1 \begin{pmatrix} 1 & 1 & 2 \\ n & m & -q \end{pmatrix} v_i^{(n)} v_j^{(m)} \quad (2.42)$$

Similarly to in equation 2.36, we can now be a bit more explicit in how we define our excited and ground states, and pull out geometric factors. We first apply the Wigner-Eckart

theorem:

$$\begin{aligned}\Omega_{E2} &= \left| \frac{eE_0}{2\hbar} \sum_q c_{ij}^{(q)} \hat{\varepsilon}_i \hat{k}_j \langle \gamma', J', F', m'_F | r^2 T_q^{(2)} | \gamma, J, F, m_F \rangle \right| \\ &= \left| \frac{eE_0}{2\hbar} \sum_q c_{ij}^{(q)} \hat{\varepsilon}_i \hat{k}_j \begin{pmatrix} F' & 2 & F \\ -m'_F & q & m_F \end{pmatrix} \langle \gamma', J', F' || r^2 T^{(2)} || \gamma, J, F \rangle \right| \end{aligned} \quad (2.43)$$

We can use the repeated reduction formula to pull out the F-dependence as well, leaving us with

$$\begin{aligned}\Omega_{E2} &= \frac{eE_0 \sqrt{(2F+1)(2F'+1)}}{2\hbar} \\ &\times \left| \begin{Bmatrix} J & J' & 2 \\ F' & F & I \end{Bmatrix} \langle \gamma', J' || r^2 T^{(2)} || \gamma, J \rangle \sum_q c_{ij}^{(q)} \hat{\varepsilon}_i \hat{k}_j \begin{pmatrix} F' & 2 & F \\ -m'_F & q & m_F \end{pmatrix} \right| \end{aligned} \quad (2.44)$$

Following [Roo00], we can also express the term $c_{ij}^{(q)} \hat{\varepsilon}_i \hat{k}_j$ in an easier-to-understand way by assuming that 1) the magnetic field is in the z-direction, 2) that $\hat{\varepsilon}$ and $\hat{k}$ are orthogonal, 3) that our light is linearly polarized, and 4) that our light propagates in the x-z plane. Defining φ as the angle between the k-vector and the magnetic field, and θ as the angle between the polarization and the magnetic field, we can write $\hat{k} = (\sin \varphi, 0, \cos \varphi)$ and $\hat{\varepsilon} = (\cos \theta \cos \varphi, \sin \theta, -\cos \theta \sin \varphi)$. We can then express the geometric factor $g^{(q)} = c_{ij}^{(q)} \hat{\varepsilon}_i \hat{k}_j$ as

$$\begin{aligned}g^{(0)} &= \frac{1}{2} |\cos \theta \sin 2\varphi| \\ g^{(\pm 1)} &= \sqrt{\frac{1}{6}} |\cos \theta \sin 2\varphi + i \sin \theta \cos \varphi| \\ g^{(\pm 2)} &= \frac{1}{2\sqrt{6}} |\cos \theta \sin 2\varphi + 2i \sin \theta \sin \varphi| \end{aligned} \quad (2.45)$$

The resulting coupling strengths as a function of θ and φ are shown in Figure 2.5.

We can further convert this Rabi frequency to an Einstein A coefficient using the formula from [Jam98]:

$$A_{eg} = \frac{\alpha \nu^5}{15c^4(2J'+1)} \left| \langle e || r^2 T^{(2)} || g \rangle \right|^2 \quad (2.46)$$

where α is the fine structure constant, and J' refers to the J of the *upper* state.

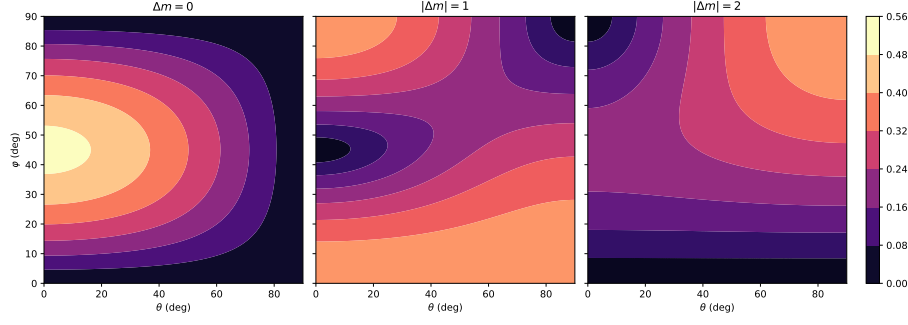


Figure 2.5: The relative coupling strength of an E2 transition as a function of θ , the angle between the magnetic field and the polarization, and φ , the angle between the magnetic field and the k-vector.

2.5.3 M1 Transitions

2.5.3.1 Optical M1 Transitions

Unlike with the E1 and E2 matrix elements, the electric dipole (M1) matrix element can in principle be solved ab-initio for LS coupled states. We begin by writing our Hamiltonian:

$$H_{M1} = \langle e | \mu \cdot \mathbf{B} | g \rangle = \frac{\mu_B E_0}{c} (\mathbf{k} \times \epsilon) \cdot \langle e | \mathbf{L} + g_s \mathbf{S} | g \rangle \quad (2.47)$$

where we have used that the magnetic field of our light in the electric dipole approximation is $B = \frac{E_0}{c} k \times \hat{\epsilon}$. Taking the atomic states to be LS-coupled, we can immediately apply the Wigner-Eckart theorem and the repeated reduction formula to extract the dependence on m_F and F , respectively, giving us

$$H_{M1} = \frac{\mu_B E_0}{c} \sqrt{(2F' + 1)(2F + 1)} \begin{Bmatrix} J' & I & F' \\ F & 1 & J \end{Bmatrix} \langle n', L', S', J' | \mathbf{L} + g_s \mathbf{S} | n, L, S, J \rangle \quad (2.48)$$

$$\times \sum_q \begin{pmatrix} F' & 1 & F \\ m'_F & q & m_F \end{pmatrix} (\mathbf{k} \times \epsilon)_q$$

However, we can go further. By applying one more reduction of the reduced matrix element, once in L and once in S, we end up reducing the problem to two reduced matrix elements of

angular momentum operators:

$$\begin{aligned}
& \langle n', L', S', J' | \mathbf{L} + g_s \mathbf{S} | n, L, S, J \rangle \\
&= \langle n', L', S', J' | \mathbf{L} | n, L, S, J \rangle + g_s \langle n', L', S', J' | \mathbf{S} | n, L, S, J \rangle \\
&= \sqrt{(2J' + 1)(2J + 1)} \left((-1)^{L' + S + J + 1} \begin{Bmatrix} L & J & S \\ J' & L' & 1 \end{Bmatrix} \langle n', L' | \mathbf{L} | n, L \rangle \right. \\
&\quad \left. + g_s (-1)^{L' + S + J + 1} \begin{Bmatrix} S & J & L \\ J' & S' & 1 \end{Bmatrix} \langle n', S' | \mathbf{S} | n, S \rangle \right) \tag{2.49}
\end{aligned}$$

We can now use the identity $\langle j' | \mathbf{j}^{(1)} | j \rangle = \delta_{j,j'} \sqrt{(2j + 1)(j + 1)j}$ [Zar91]. Thus, the total expression for the magnetic dipole Hamiltonian becomes:

$$\begin{aligned}
H^{M1} &= \frac{\mu_B E}{c} \sqrt{(2F' + 1)(2F + 1)(2J' + 1)(2J + 1)} (-1)^{F + F' + J + J' + I - m_F + L + S} \begin{Bmatrix} J' & I & F' \\ F & 1 & J \end{Bmatrix} \\
&\quad \left(\delta_{L,L'} \begin{Bmatrix} L & J & S \\ J' & L & 1 \end{Bmatrix} \sqrt{(2L + 1)(L + 1)L} + g_s \delta_{S,S'} \begin{Bmatrix} S & J & L \\ J' & S & 1 \end{Bmatrix} \sqrt{(2S + 1)(S + 1)S} \right) \\
&\quad \times \sum_q \begin{pmatrix} F' & 1 & F \\ -m'_F & q & m_F \end{pmatrix} (\mathbf{k} \times \boldsymbol{\epsilon})_q \tag{2.50}
\end{aligned}$$

We can write down the Einstein A coefficient associated with the transition as

$$A = \frac{\nu^3}{24\pi^4 \epsilon_0 \hbar c^3} \frac{|\langle n', L', S', J', F' | \mathbf{L} + g_s \mathbf{S} | n, L, S, J, F \rangle|^2}{2F' + 1} \tag{2.51}$$

Lande g factor Interestingly, this expression provides us with an alternative and less hand-wavy way to derive the Lande g factor g_J , defined previously in equation 2.23. By applying the Wigner-Eckart theorem to the Zeeman Hamiltonian, we may write that the

effect of a static magnetic field is

$$\langle \gamma, J, m_J | \mu \cdot \mathbf{B} | \gamma, J, m_J \rangle = \mu_B B (-)^{J-m_J} \begin{pmatrix} J & 1 & J \\ m_J & 0 & -m_J \end{pmatrix} \langle \gamma, J | |\mathbf{L} + g_s \mathbf{S}| | \gamma, J \rangle \quad (2.52)$$

$$= \mu_B B m_J \frac{\langle \gamma, J | |\mathbf{L} + g_s \mathbf{S}| | \gamma, J \rangle}{\sqrt{J(J+1)(2J+1)}} \quad (2.53)$$

we may therefore define g_J as

$$g_J = \frac{\langle \gamma, J | |\mathbf{L} + g_s \mathbf{S}| | \gamma, J \rangle}{\sqrt{J(J+1)(2J+1)}} \quad (2.54)$$

With a great deal of effort, one can indeed prove that we recover the expression for g_J given in equation 2.23 from the above equation.

2.5.3.2 Coherent Microwaves

Although optical M1 transitions do occur between fine structure levels (notably between the $^2D_{5/2}$ and $^2D_{3/2}$ states in ytterbium, see Section 6.5), the main way M1 transitions occur in our atom is when we deliberately drive them using microwaves. This is our primary method of manipulating our qubit, and it is worth going into detail on the dynamics of a two-level system interacting with a microwave field.

Let us write the two-level atomic Hamiltonian as

$$H_0 = -\frac{\omega_0}{2} |0\rangle\langle 0| + \frac{\omega_0}{2} |1\rangle\langle 1| = \frac{\omega_0}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = -\frac{\omega_0}{2} \sigma_z \quad (2.55)$$

When discussing microwaves it is more intuitive to write our perturbing Hamiltonian as that on an oscillating magnetic field, rather than an electric one:

$$\begin{aligned} H_1 &= \mu \cdot \mathbf{B} \cos(\omega t + \phi) (|1\rangle\langle 0| + |0\rangle\langle 1|) \\ &= B_z \cos(\omega t + \phi) (\mu |1\rangle\langle 0| + \mu^* |0\rangle\langle 1|) \\ &= B_z \cos(\omega t + \phi) \mu \sigma_x \end{aligned} \quad (2.56)$$

Given these two terms of our Hamiltonian, we can write down the Schrödinger equation for an arbitrary state $\psi = c_0 |0\rangle + c_1 |1\rangle$:

$$\begin{aligned} i \frac{dc_0}{dt} &= -\frac{\omega_0}{2} c_0 - B_z \mu \cos(\omega t + \phi) c_1 \\ i \frac{dc_1}{dt} &= \frac{\omega_0}{2} c_1 - B_z \mu^* \cos(\omega t + \phi) c_0 \end{aligned} \quad (2.57)$$

If we transform into the rotating frame with respect to ω_0 by making the substitution $c_i = \tilde{c}_i e^{\pm i\omega_0 t/2}$, we can cancel out the terms related to the qubit's fundamental splitting and focus on the interaction Hamiltonian:

$$\begin{aligned} i \frac{d\tilde{c}_0}{dt} &= -\frac{\Omega}{2} \tilde{c}_1 (e^{i(\omega-\omega_0)t+i\phi} + e^{-i(\omega+\omega_0)t-i\phi}) \\ i \frac{d\tilde{c}_1}{dt} &= -\frac{\Omega^*}{2} \tilde{c}_0 (e^{-i(\omega-\omega_0)t+i\phi} + e^{i(\omega+\omega_0)t+i\phi}) \end{aligned} \quad (2.58)$$

Here we have defined $\Omega = \mu B_z$, and we have expressed $\cos(\omega t + \phi)$ in terms of exponentials. We can now argue that, in the dynamics of the system, the fast-oscillating terms will not affect the overall dynamics, as they will integrate to zero over longer timescales. With this assumption, we can drop the counter-rotating terms, which oscillate at frequency $\omega + \omega_0$. This approximation gives us:

$$\begin{aligned} i \frac{d\tilde{c}_0}{dt} &= -\frac{\Omega}{2} \tilde{c}_1 e^{i\delta t+i\phi} \\ i \frac{d\tilde{c}_1}{dt} &= -\frac{\Omega^*}{2} \tilde{c}_0 e^{-i\delta t-i\phi} \end{aligned} \quad (2.59)$$

where $\delta = \omega - \omega_0$ is the detuning between the driving field and the qubit frequency. Entering the rotating frame again, this time at frequency δ allows us to remove the time dependence from the system. Thus, we make the substitution $\tilde{c}_i = a_i e^{i\delta t/2}$:

$$\begin{aligned} i \frac{da_0}{dt} &= -\frac{\Omega}{2} a_1 e^{i\phi} + \frac{\delta}{2} a_0 \\ i \frac{da_1}{dt} &= -\frac{\Omega^*}{2} a_0 e^{-i\phi} - \frac{\delta}{2} a_1 \end{aligned} \quad (2.60)$$

In the case where we are resonantly driving our system (i.e. $\delta = 0$), we see that the dynamics are of a simple oscillation between zero and one. For off-resonant dynamics, we

can solve the system. A full derivation is given in [Rom21], and we will give the resulting dynamics here as a propagator: the matrix $U(\delta, \phi, t)$ can be applied to a state vector in the rotating frame to evolve it in time:

$$U(\delta, \phi, t) = \begin{pmatrix} \cos\left(\frac{\tilde{\Omega}t}{2}\right) - i\frac{\delta}{\tilde{\Omega}}\sin\left(\frac{\tilde{\Omega}t}{2}\right) & -i\frac{\Omega}{\tilde{\Omega}}e^{-i\phi}\sin\left(\frac{\tilde{\Omega}t}{2}\right) \\ -i\frac{\Omega}{\tilde{\Omega}}e^{i\phi}\sin\left(\frac{\tilde{\Omega}t}{2}\right) & \cos\left(\frac{\tilde{\Omega}t}{2}\right) + i\frac{\delta}{\tilde{\Omega}}\sin\left(\frac{\tilde{\Omega}t}{2}\right) \end{pmatrix} \quad (2.61)$$

where we have defined the *generalized Rabi frequency* $\tilde{\Omega} \equiv \sqrt{\delta^2 + \Omega^2}$. Applying this matrix determines the outcome of applying microwaves at a given phase ϕ for a duration t . Plotting the effect of a single such pulse as a function of t and δ yields 2.6. The effect of more complicated series of pulses – such as Ramsey sequences, spin-echo sequences, or Knill sequences [SAS11] – can also be considered via multiple applications of this time propagation matrix.

In the case of interaction with a single pulse of radiation at a detuning δ and interaction time t , we may write that

$$|c_0(t)|^2 = \frac{\Omega^2 \cos^2\left(\tilde{\Omega}t/2\right)}{\tilde{\Omega}^2} \quad (2.62)$$

2.5.4 Transitions between different couplings

In the evaluation of the above optical matrix elements and transition rates, we have made no assumptions about LS coupling. When such assumptions can be made, it is often useful to talk about selection rules, such as those given in [Foo05], that give a useful shorthand for whether a matrix element will be 0. However, in ytterbium specifically, we often talk about transitions between LS coupled states and J_1K coupled states. How do we make qualitative statements about the strength of such transitions? Here, we will work through a heuristic example. For a more nuanced and quantitative discussion, see ref. [CA65].

Ultimately, the rule of thumb when considering these kinds of transitions is to consider the full configuration and to try and break down the problem into a simpler subproblem. Take,

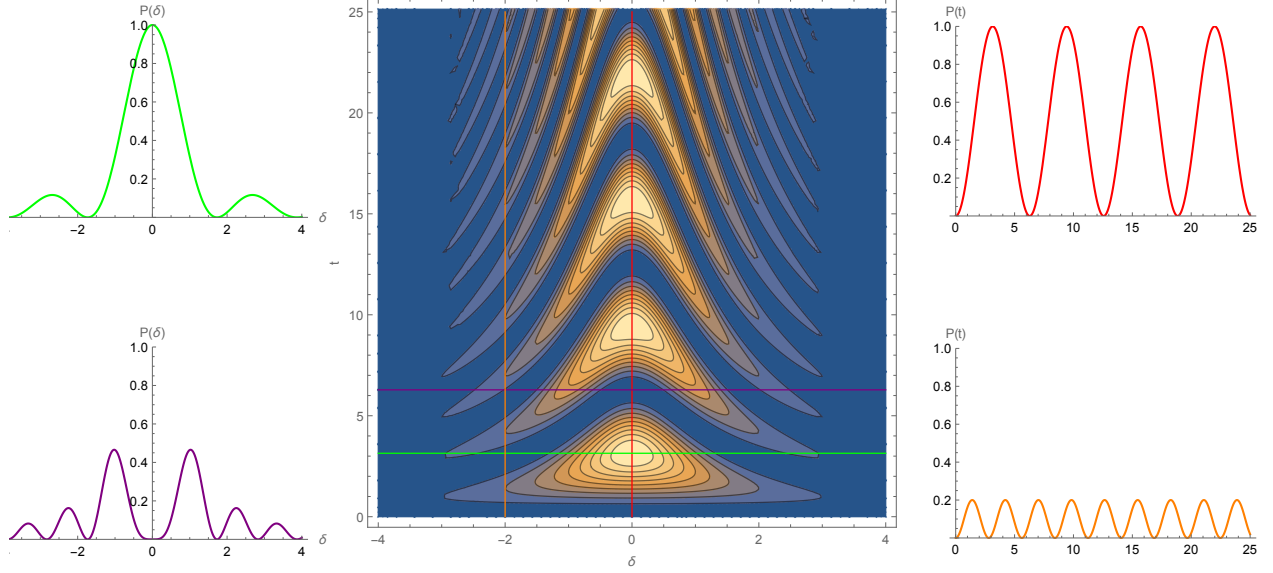


Figure 2.6: A 2D plot of the population in $|1\rangle$ as a function of the microwave detuning δ and the interrogation time t . The four plots along the outside of the central plot represent slices of the 2D plot taken along the lines matching their color. The green curve is the lineshape of an on-resonance π pulse. The purple curve represents the on-resonance lineshape when the interrogation time is mistuned, in this case to a 2π pulse. The red and orange curves show resonant and off-resonant Rabi oscillations. The off-resonant curve oscillates at $\tilde{\Omega} = \sqrt{\delta^2 + \Omega^2}$.

for instance, the 935 nm repump transition $4f^{14}5d\ ^2D_{3/2} \rightarrow 4f^{13}(^2F_{7/2}^o)5d6s(^3D)\ ^3[3/2]_{1/2}^o$. Reading just the term symbols, all we can glean is that $|\Delta J| \leq 1$, which means that it is possible that the transition has some electric dipole character. However, looking at the full configuration lets us get a sense of what is going on and what to expect. We see that the transition can be seen to promote an electron from the core to an excited s orbital. This process involves a large exchange of angular momentum and, therefore, we expect the transition to be fairly weak. Looking at the DREAM database of transition strengths [QP20], we indeed see that the natural linewidth of the transition is roughly 450 kHz, weaker than we would expect for an E1 transition of that frequency.

While such heuristics can tell us something about the nature of transitions, it is important not to take them as hard rules. It is always worth keeping in mind that the states in any atom, be they single-electron excitations or five-electron excitations, are ultimately *collective* phenomena of the electrons, which form a single wavefunction. The only rule that is hard and fast is the conservation of the total angular momentum J .

2.5.5 Off-resonant interactions

2.5.5.1 A Note on naming

The shift in the atomic levels due to a perturbing light field is conventionally called the *AC Stark shift*, the *light shift*, or occasionally the *Autler-Townes shift*. Due to Johannes Stark's antisemitic legacy and enthusiastic collaboration with the Nazi party, we will adopt the term *light shift* throughout this work to describe the phenomenon. In the case where an oscillating magnetic field is causing the shift, we will follow convention and call the phenomenon the *AC Zeeman shift*.

2.5.5.2 Light shift

In addition to driving transitions, the presence of an oscillating electromagnetic field also shifts the eigenstates of the atom. In general, we may write the Hamiltonian for the interaction with radiation in the rotating frame as

$$H = \frac{-\delta}{2}(-|g\rangle\langle g| + |e\rangle\langle e|) + \frac{\Omega}{2}(|g\rangle\langle e| + |e\rangle\langle g|) \quad (2.63)$$

When we write this as a matrix and diagonalize to obtain the new eigenvalues, we obtain the energy of the new state to be

$$E_{\pm} = \frac{\delta}{2} \pm \frac{\sqrt{\Omega^2 + \delta^2}}{2} \quad (2.64)$$

In other words, our new eigenstates $|+\rangle$ and $|-\rangle$ are shifted from their unperturbed values by $\frac{1}{2}\sqrt{\Omega^2 + \delta^2}$

The light shift is discussed in much more detail in 8.1. For a very thorough discussion of light shifts and the general concept of dynamic polarizability, see [LSR13].

2.6 Nuclear Structure

In addition to the electronic structure, the nuclear structure is also of some importance to this thesis. Although a detailed derivation of the particulars is out of the scope of this thesis, a brief overview is in order. For an excellent phenomenological introduction to the nuclear theory relevant to this work, see Krane & Halliday [KH87].

2.6.1 Nuclear Shell Model

Just like an atom, a nucleus is simply a bound state of subatomic particles - protons and neutrons. What significantly complicates the mathematics of the nuclear system are the following three factors:

1. The nucleus has no extremely dense central mass, preventing us from using the reduced mass approximation
2. The nucleus is composed of two distinct types of particle
3. The nucleus is an equilibrium between the strong, short-ranged and attractive strong nuclear force, and the weak, repulsive, and long-ranged electromagnetic force.

While these effects significantly complicate the ab-initio computation of nuclear structure, some things are true about both the nucleus and the atom as a whole. For instance, the nucleons still exhibit a well-defined angular momentum quantum number ℓ and parity π . In ^{173}Yb , for instance the nuclear ground state is written as $5/2-$, indicating $\ell = 5/2$ and negative parity.

The nucleus also has a shell structure, in which nucleons will form spherically symmetric shells. However, the numbers of nucleons in a nuclear shell is not the same as the number of electrons in an atomic shell. Specifically, for protons and neutrons separately, stable shells form at 2, 8, 20, 28, 50, 82, 126, and 184 nucleons.

These shells allow us to talk about *valence nucleons* and to roughly infer a number of nuclear properties without considering the nucleus as a whole. For instance, ^{173}Yb has a fully-filled proton shell, and a single valence neutron.

2.6.2 Nuclear Deformation

In addition to a simple shell structure, the nucleons in large nuclei can exhibit strong collective effects. Rather than forming spherically symmetric states, the nucleus can exhibit collective effects such as vibration, or can assume a non-spherical shape altogether. ^{173}Yb is such a nucleus and exhibits a strong quadrupole deformation. This distribution can be modeled as an ellipsoidal nucleus with added quantum numbers to express the rotational and vibrational states. This is the so-called Nilsson model of the nucleus.

Of course, there is no reason to expect that any nucleus has only a quadrupole deformation, and in general nuclei have multipole moments extending to all orders. However, in most nuclei, these deformations become very small. Exceptional nuclei with large octopole deformations may include $^{222,224,225,226}\text{Ra}$, $^{223,224,226}\text{Th}$, and ^{226}U [But20].

CHAPTER 3

Trapped Ions

3.1 Ion trapping theory

The core of any trapped ion experiment is a trapped ion. As such, we would like to create a potential that can trap an ion. However, Gauss' law states that $\nabla \cdot \mathbf{E} = 0$, while at the center of any trapping potential we must have $\nabla \cdot \mathbf{E} < 0$. As such, we seek another way to create a potential that can trap an ion. There are two approaches: we can use a magnetic field to generate effectively trapped orbits, called a *Penning trap*, or we can introduce time dependence into our potential to generate an effective trapping force, which is called a *Paul trap*. In this work, we employ a Paul trap, and that will be the focus of this section.

3.1.1 Ponderomotive force

In general, we wish to consider the *effective* force experienced by a charged particle of mass m in an oscillating potential. Given that the electric field oscillates at Ω_{RF} , we may write that

$$\ddot{\mathbf{r}} = \frac{e}{m} \mathbf{E}(\mathbf{r}) \cos \Omega_{RF} t \quad (3.1)$$

If we assume that the motion of the particle can be separated into a slow component $r_s(t)$ and a fast component $r_f(t)$, and Taylor expand the electric field we can express this as

$$\ddot{\mathbf{r}}_s + \ddot{\mathbf{r}}_f = \frac{e}{m} \left[\mathbf{E}(\mathbf{r}_s) + x_f \frac{\partial E_x}{\partial x} \Big|_{x=x_s} + y_f \frac{\partial E_y}{\partial y} \Big|_{y=y_s} \right] \cos \Omega_{RF} t \quad (3.2)$$

Making the reasonable approximation that $|\ddot{\mathbf{r}}_f| \gg |\ddot{\mathbf{r}}_s|$ (i.e. that the fast motion is varying much faster than the slow motion) and $|\mathbf{r}_s| \gg |\mathbf{r}_f|$ (i.e. that the slow motion has a much larger amplitude than the fast motion), we get that

$$\ddot{\mathbf{r}}_f = \frac{e}{m} \mathbf{E}(\mathbf{r}_s) \cos \Omega_{RF} t \quad (3.3)$$

$$\mathbf{r}_f = -\frac{e}{m\Omega_{RF}^2} \mathbf{E}(\mathbf{r}_s) \cos \Omega_{RF} t \quad (3.4)$$

Plugging this expression for $\mathbf{r}_f$ back into equation 3.2, we get

$$\ddot{\mathbf{r}}_s = -\frac{e^2}{m^2\Omega_{RF}^2} \left[E_x(x_s) \frac{\partial E_x}{\partial x} \Big|_{x=x_s} + E_y(y_s) \frac{\partial E_y}{\partial y} \Big|_{y=y_s} \right] \cos^2(\Omega_{RF} t) \quad (3.5)$$

$$= -\frac{e^2}{2m^2\Omega_{RF}^2} \left[\frac{\partial}{\partial x} E_x(x_s)^2 + \frac{\partial}{\partial y} E_y(y_s)^2 \right] \cos^2(\Omega_{RF} t) \quad (3.6)$$

Integrating over a period of the motion, we can obtain the net effective force:

$$\ddot{\mathbf{r}}_s = -\frac{e^2}{m^2\Omega_{RF}^2} \int_0^T \nabla E^2 \cos^2(\Omega_{RF} t) dt \quad (3.7)$$

$$= -\frac{e^2}{4m^2\Omega_{RF}^2} \nabla E^2 \quad (3.8)$$

$$= -\frac{e}{m} \nabla \tilde{\phi} \quad (3.9)$$

Where we have defined the pseudopotential as $\tilde{\phi} \equiv \frac{eE^2}{4m\Omega_{RF}^2}$.

We can now make statements about our trap geometry and derive the effective potential. If we assume a trap geometry with four hyperbolic electrodes, two of which are grounded, and two of which oscillate at a frequency Ω_{RF} , we may write the potential as

$$\phi = \frac{V_0}{2} \cos(\Omega_{RF} t) \left(1 + \frac{x^2 - y^2}{r_0^2} \right) \quad (3.10)$$

Where r_0 sets the length scale of the problem, and can be interpreted as the distance from the center of the trap to the electrode. Therefore, we can state that

$$E = -\nabla \phi = -\frac{V_0}{r_0^2} (x\hat{x} - y\hat{y}) \cos(\Omega_{RF} t) \quad (3.11)$$

$$\tilde{\phi} = \frac{eV_0^2}{4m\Omega_{RF}^2 r_0^4} (x^2 + y^2) \quad (3.12)$$

substituting this into equation 3.9, we get the equation of motion:

$$\ddot{\mathbf{r}} = -\frac{e^2 V_0^2}{2m^2 \Omega_{RF}^2 r_0^4} (x\hat{x} + y\hat{y}) \quad (3.13)$$

So we see that the effective confining potential is parabolic, and that the particle is trapped. The frequency of the particle's motion is called the secular frequency

$$\omega_{sec} = \frac{eV_0}{\sqrt{2}m\Omega_{RF}r_0^2} \quad (3.14)$$

3.1.2 Solving the Mathieu equation

In the previous derivation of the effective trapping potential, we have averaged out the fast-oscillating term. If we want to add it back in, we will have to be a bit more careful with our derivation. We will start with the equation of motion for the ion in an oscillating RF field again. In general, we will write our potential as the sum of a static and a dynamic term:

$$\phi = \frac{V_0}{2} \left(1 + \frac{x^2 - y^2}{r_0^2} \right) \cos(\Omega_{RF}t) + U(\mathbf{r}) \quad (3.15)$$

If we assume that we have end caps at the ends of our trap to provide confinement, we can write that near the trap center

$$U(\mathbf{r}) = \frac{U_0}{z_0^2} \left(z^2 - \frac{x^2 + y^2}{2} \right) \quad (3.16)$$

Writing down our equation of motion, we have

$$m\ddot{\mathbf{r}} = -eV_0 \frac{x\hat{x} - y\hat{y}}{r_0^2} \cos(\Omega_{RF}t) - eU_0 \frac{2z\hat{z} - x\hat{x} - y\hat{y}}{z_0^2} \quad (3.17)$$

If we write our equation of motion in the canonical form of the Mathieu equation [BMB98, Gho95]

$$\frac{d^2 f}{dv^2} + (a - 2q \cos(2v))v = 0 \quad (3.18)$$

we can obtain solutions. We can do this for all three dimensions separately. We make the substitution $\xi = \Omega_{RF}t/2$, and we obtain for the x-direction. (In the y- and z-directions things are comparable)

$$\frac{d^2 x}{d\xi^2} = \frac{4eU_0}{mz_0^2 \Omega_{RF}^2} - \frac{4eV_0}{m\Omega_{RF}^2 r_0^2} x \cos(2\xi) \quad (3.19)$$

We can now match terms to the canonical form of the Mathieu equation and identify the parameters a_i and q_i for each direction:

$$a_x = -\frac{4eU_0}{mz_0^2\Omega_{RF}^2} \quad q_x = \frac{2eV_0}{mr_0^2\Omega_{RF}^2} \quad (3.20)$$

$$a_y = -\frac{4eU_0}{mz_0^2\Omega_{RF}^2} \quad q_y = -\frac{2eV_0}{mr_0^2\Omega_{RF}^2} \quad (3.21)$$

$$a_z = \frac{2eU_0}{mz_0^2\Omega_{RF}^2} \quad q_z = 0 \quad (3.22)$$

With the Mathieu equations in canonical form, we can write down the first-order approximate solution [BMB98]:

$$r_i(t) \approx R_i \cos(\omega_{sec,i}t) \left(1 + \frac{q_i}{2} \cos(\Omega_{RF}t)\right) \quad (3.23)$$

Where $\omega_{sec,i} = \frac{\Omega_{RF}}{2} \sqrt{a_i + \frac{1}{2}q_i^2}$. We see that the motion consists of two components: one large-amplitude component that oscillates at ω_{sec} , superimposed on a component of frequency Ω_{RF} , whose amplitude is smaller by a factor of $q/2$. The first term is referred to as the secular motion, and the second is referred to as the micromotion. Generally, for an ion trap to be stable (setting the a parameters to 0), q must be less than 0.908, and generally we prefer q to be a fairly small value, around 0.1 or less. A more precise explanation of the stability regions of an ion trap is given in [Gho95].

3.2 Micromotion Compensation

Regardless of what ion trap is being used, it is important that the ions are near their micromotion minima. Adequate micromotion compensation improves heating rates, reduces dark ion lifetimes, and in some cases helps with readout fidelity [BMB98]. There are several ways in which the relative of micromotion the ion experiences can be determined: ion position, time-tagging, and modulation spectroscopy. Many of these techniques, plus some others are mentioned in the ever-useful paper [BMB98], but they are generally quite intuitive as diagnostics.

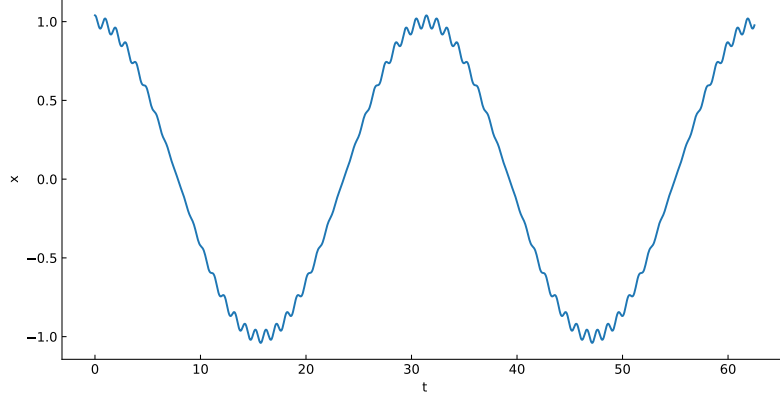


Figure 3.1: An example of the 1-D trajectory of an ion in an ion trap (units are arbitrary). The fast oscillation are the micromotion, and occur at Ω_{RF} . The slow motion is the secular motion ω_{sec} of the ion in the trap's pseudopotential.

3.2.1 Ion position

The crudest way to evaluate the quality of the ion's micromotion is to evaluate how spatially close it is to the center of the trapping potential. The most straightforward way to test this is by adjusting the voltage of the trap's driving RF. As the voltage is increased, the ion will be more tightly confined and will move towards the center of the potential. DC shims can be adjusted to minimize this movement as the trap RF power is changed, placing us near the micromotion minimum.

3.2.2 Modulation spectroscopy

The most common way we compensate for micromotion is to look for phase modulation of our Doppler cooling light. When the ion experiences excess micromotion, its secular motion is modulated by a high-frequency micromotion term at Ω_{RF} . If the ion is moving along the axis of the Doppler cooling beam of frequency ω_L , then the electric field experienced by the ion is of the form $\sin(\omega_L t + \beta \sin \Omega_{RF})$, where β parametrizes the degree of modulation

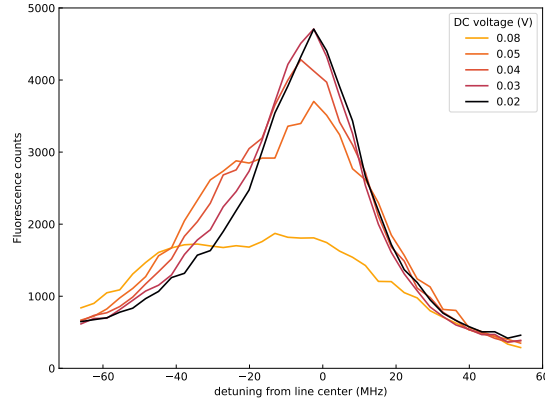


Figure 3.2: Demonstration of the Doppler cooling line shape of the ion with varying amounts of micromotion. Micromotion is well compensated with a 0.02 V DC offset. Increasing the DC voltage displaces the ion vertically away from the micromotion null, and phase modulation sidebands are observed 32 MHz red from the line. No sidebands are observed to the right of the line due to the double pass AOM’s decreasing efficiency, combined with ion heating effects due to probing blue of the atomic resonance.

and is linearly dependent on the micromotion amplitude. In frequency space, this phase modulation is visible in a line scan as a sideband occurring at the trap frequency. By adjusting DC voltages, we can minimize the height of the sideband, and thus minimize the amount of excess micromotion along the axis of the beam.

3.2.3 Time-tagging

In cases where the micromotion does not project onto the axis of a laser with which we can perform spectroscopy, we have another tool at our disposal. We can time-tag the photons hitting our PMT and correlate them to the RF phase. In order to do this, we use a PicoQuant TimeHarp time-tagging chip triggered off of our trap RF. For a more detailed discussion, see [Rom21].

CHAPTER 4

Working with ytterbium

4.1 Photoionization

We produce the ytterbium ions that we trap from a thermal oven source. While this is very convenient, it does not produce ions directly, and we instead have to photoionize the atoms within the trapping region. Although this may seem inconvenient, it gives us a great deal of control when loading ions.

- It allows us to tune a photoionization laser to have isotope selectivity in which species we trap
- It allows us to load with our trap on. Since the trapping potential is conservative, ions that have enough energy to enter the trapping region also have enough energy to *exit* the trapping region, unless they are sufficiently cooled during their time in the trap. This is the case, for example, in the ablation loading of barium [Chr20].

The ionization energy of neutral ytterbium is 50443.2cm^{-1} [AYJ23]. This corresponds to a wavelength of 198 nm. Needless to say, we do not employ such a vacuum-ultraviolet laser in our system. Instead, we turn to a $1 + 1$ REMPI (Resonance-Enhanced Multiphoton Ionization) scheme. We begin by driving the $^1S_0 \rightarrow ^1P_1$ transition at 399 nm. From there, the ionization energy is sufficiently reduced that our 369.5 nm Doppler cooling light can perform the second step of the photoionization.

4.2 Cooling Ytterbium

Fundamentally, trapping ytterbium requires only two lasers: a Doppler cooling laser and a single repump. The 369 nm laser performs the Doppler cooling on the $^2S_{1/2}$ to $^2P_{1/2}^o$, and is typically tuned to be roughly 20 MHz red of the ion's transition frequency. However, this cycle is not closed, since the $^2P_{1/2}^o$ has a roughly 0.5% chance to decay to the $^2D_{3/2}$ state. In order to rescue the ion from this state, which lives for 61.8 ms [SDR15], an additional 935 nm tone is needed. This tone moves population from the $^2D_{3/2}$ state to the $^1[3/2]_{1/2}^o$ state. This state strongly decays back into the $^2S_{1/2}$, returning the ion to the cooling cycle.

4.2.1 Cooling $^{171}\text{Yb}^+$

Although it is sufficient to have two tones in the absence of a nuclear spin, the spin-1/2 nucleus of $^{171}\text{Yb}^+$ introduces hyperfine structure that complicates the Doppler cooling process. Specifically, if tones are not added to the 369 and 935 nm lasers, the ion will quickly be pumped into a dark state that is not addressed by the frequencies applied. To counteract this, we modulate both lasers using fiber electro-optic modulators (EOMs).

4.2.1.1 Doppler cooling sidebands

The 369 nm light is tuned to drive the $^2S_{1/2} F = 1 \leftrightarrow ^2P_{1/2}^o F = 0$ transition. Since the $^2P_{1/2}^o F = 0$ cannot decay into the $^2S_{1/2} F = 0$ state, this forms a nearly closed cycle, but off-resonant scattering off of the $^2P_{1/2}^o F = 1$ state can occasionally populate it. In order to recover population that enters the $F=0$ state, a 14.379 GHz tone is needed, as this is the sum of the hyperfine splitting of the ground state (12.642 GHz) and of the excited state (2.105 GHz).

The 935 nm light needs to be treated similarly. While most decays from the cooling light are from the $^2P_{1/2}^o F = 0 \rightarrow ^2D_{3/2} F = 1$ state, and are rescued by driving the

$^2D_{3/2} F = 1 \rightarrow ^3[3/2]_{1/2}^o F = 0$ transition, scattering off of the $^2P_{1/2}^o F = 1$ state allows the $^2D_{3/2} F = 2$ state to occasionally be populated. To rescue the ion from this state, we apply a weak 3.06 GHz tone to the laser to drive from the $^2D_{3/2} F = 2$ state to the $^3[3/2]_{1/2}^o F = 1$ state.

4.2.1.2 Coherent population trapping

It is worth mentioning that the addition of the above sidebands is not on its own sufficient to successfully cool an ion with hyperfine structure. This is due to the effect of coherent population trapping [BB02]. In short, the EOM tones we apply destabilize the individual angular momentum states in which the ion can become trapped, but they cannot prevent the ion from being pumped into coherent superpositions of degenerate states that do not fluoresce. These states are called *coherent dark states*, and the exact superposition formed depends on the polarization of the applied laser light. Specifically for the 369nm transition in $^{171}\text{Yb}^+$, the dark states are [BB02]

$$\begin{bmatrix} E_- \\ 0 \\ E_+ \end{bmatrix} \text{ and } \begin{bmatrix} E_0 E_+ \\ -(E_-^2 + E_+^2) \\ E_0 E_- \end{bmatrix} \quad (4.1)$$

where $E_{-,0,+}$ are the σ^- , π , and σ^+ -polarized components of the incident light respectively, and the components of the vector.

In order to get around the formation of these dark states, they need to be destabilized by introducing some kind of time dependence to the system. One way to do this is by rapidly modulating the polarization of the incident light, which directly modulates the terms in the above expression. Although doable and elegant, implementing this is not simple. A much more straightforward way to achieve the same effect is by lifting the degeneracy of the three states via the Zeeman effect. By introducing a magnetic field of a few Gauss, (in our experiment, typically around 5), we effectively cause these dark states to oscillate at the Zeeman splitting of several MHz. Solving the atomic Hamiltonian with the dark states

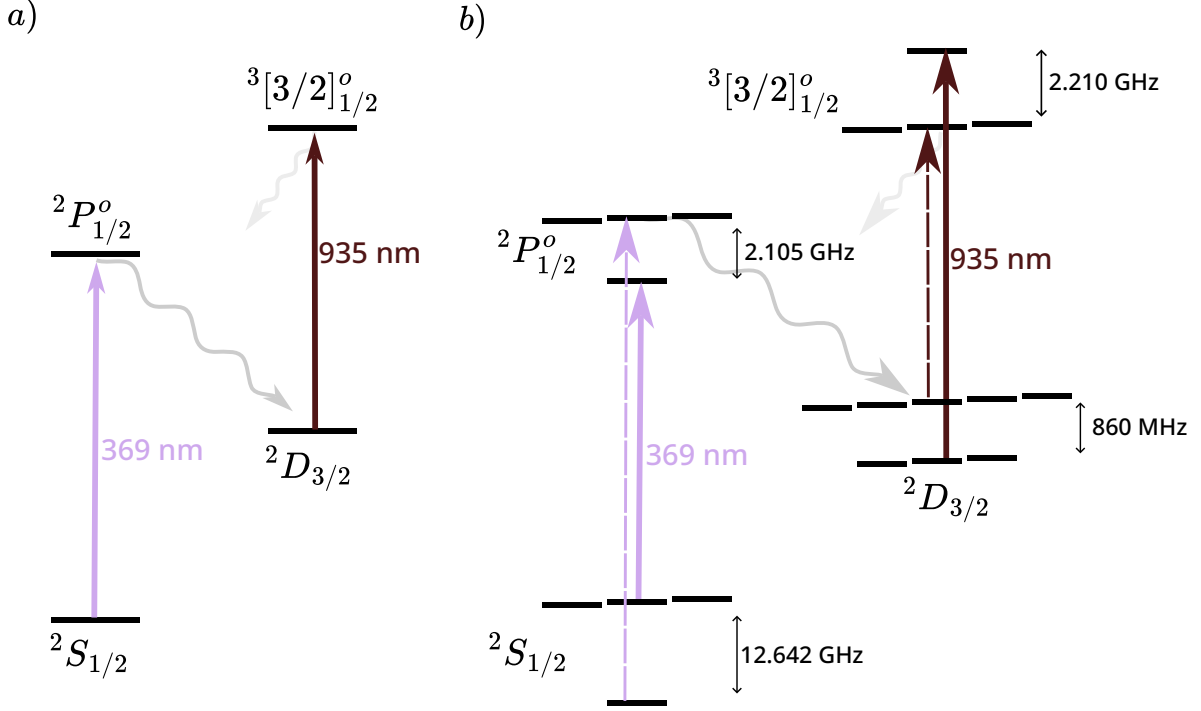


Figure 4.1: The cooling schemes for $^{174}\text{Yb}^+$ and $^{171}\text{Yb}^+$. a) depicts the cooling scheme in $^{174}\text{Yb}^+$, where hyperfine structure is absent. 369nm light Doppler cools the ion, and the 935 nm light repumps the 0.5% decay to the $^2D_{3/2}$, returning it to the cooling cycle. b) depicts the cooling scheme in $^{171}\text{Yb}^+$, which is broadly similar, but EOM sidebands are added to address the hyperfine structure. A 14.374 GHz blue sideband on the 369 nm laser drives $^2S_{1/2}F = 0 \leftrightarrow ^2P_{1/2}^oF = 1$, and a 3.07 GHz red sideband on the 935 nm laser addresses the $^2D_{3/2}F = 2 \leftrightarrow ^3[3/2]_{1/2}^oF = 1$.

destabilized in this way yields an optimal laser polarization that we should use (see [BB02] for a more detailed explanation) of $\theta_{pol} = \arccos(1/\sqrt{3}) \approx 54.7^\circ$.

4.2.2 Cooling $^{173}\text{Yb}^+$

In $^{173}\text{Yb}^+$, there are more hyperfine sublevels than in $^{171}\text{Yb}^+$, and thus more and different laser tones need to be applied.

4.2.2.1 Doppler cooling schemes

Initially when we were loading $^{173}\text{Yb}^+$, we relied on a bulk EOM to perform the modulation. As bulk EOMs are resonant devices, they are built to operate at a specific frequency. This forced us to adopt a scheme in which the 10.491 GHz hyperfine structure of the ground state was addressed by parking the carrier in the middle of the splitting, and using the red sideband to address the $F = 2$ state, while the blue sideband addresses the $F = 3$ state. Both states are coupled to only one of the two F -manifolds in the excited state.

While the above scheme worked, it affords limited flexibility, as there is no way to selectively turn off one of the two cooling tones. Once we acquired a prototype 369 nm fiber EOM from ADVR, we were able to move our cooling laser to directly drive the $^2S_{1/2} F = 3 \leftrightarrow ^2P_{1/2}^o F = 3$ transition, and use a 10.491 GHz red sideband to address the $^2S_{1/2} F = 2 \leftrightarrow ^2P_{1/2}^o F = 3$ transition. This allows us to optically pump into the $F=2$ manifold by turning off the sideband.

4.2.2.2 Repump

The $^2D_{3/2}$ state in $^{173}\text{Yb}^+$ has sublevels $F = 1, 2, 3, 4$, and this large amount of structure largely informs how we choose to cool the $^{173}\text{Yb}^+$ ion. For instance, this structure informs our selection of the $F = 3$ state as the upper state of our Doppler cooling. While the $^2P_{1/2}^o F = 2$ state can decay into the relatively spread-out $^2D_{3/2} F = 1, 2$, and 3 states, the $^2P_{1/2} F = 3$

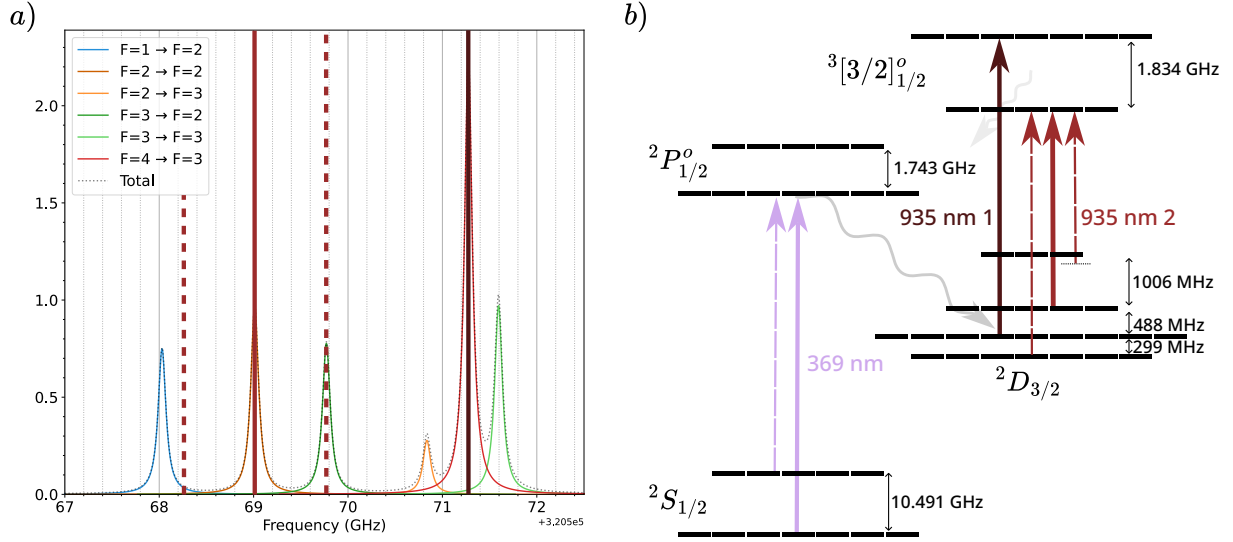


Figure 4.2: The cooling scheme in $^{173}\text{Yb}^+$. b) depicts the level structure of $^{173}\text{Yb}^+$, showing the sidebands needed to cool the ion. On the 369 nm transition, we use a red 10.491 GHz EOM sideband to drive the $^2S_{1/2}, F = 2 \leftrightarrow ^2P_{1/2}^o, F = 3$ transition. To repump population that decays from the $^2P_{1/2}^o, F = 3$ manifold, two repumps are used. A strong repump ("935 nm 1" in the figure) repumps the $^2D_{3/2}, F = 4$ state, and off-resonantly helps to repump the $F = 2$ and $F = 3$ levels. A second repump with a 786 MHz sideband explicitly repumps the $F = 2$ and $F = 3$ states, while off-resonantly repumping the $F = 1$ sub-level, which is only populated due to off-resonant scattering from the $^2P_{1/2}^o, F = 2$ state. a) depicts this repump scheme superimposed on the spectrum of the 935 nm transition. The heights of the peaks correspond to the relative strengths of the transitions after angular momentum is taken into account.

state decays into the $^2D_{3/2}F = 2, 3$, and 4 states. Transitions that clear out these states are relatively close together, and a broad and high-power repump can address all three. This allows us to use relatively little light to address the transition that clears out the $F=1$ state. The spectroscopy of these transitions is further discussed in section 9.4. Here, I simply report the repump tones that we apply, which are also shown in Figure 4.2. We place a high-power unmodulated repump tone on the $^2D_{3/2}F = 4 \rightarrow ^3[3/2]_{1/2}^o F = 3$ transition, which also weakly drives the transitions $^2D_{3/2}F = 2, 3 \rightarrow ^3[3/2]_{1/2}^o F = 3$. To supplement this, we use a second 935 nm repump laser on the $^2D_{3/2}F = 2 \rightarrow ^3[3/2]_{1/2}^o F = 2$ transition, with a 784 MHz tone whose blue sideband addresses the $^2D_{3/2}F = 3 \rightarrow ^3[3/2]_{1/2}^o F = 2$ transition, and whose red sideband off-resonantly rescues any residual population that decays into the $^2D_{3/2}F = 1$ state.

4.3 Common pulse sequences

In essentially all experiments performed in this work, several common steps come up, as they are used in the state preparation and measurement of the qubit. It is worth going into some extra detail about these schemes.

4.3.1 Optical pumping

In $^{171}\text{Yb}^+$, optical pumping is our method of state preparation. By turning off the EOM sideband that drives $^2S_{1/2} F = 0 \rightarrow ^2P_{1/2}^o F = 1$, and turning on a sideband that drives $^2S_{1/2} F = 1 \rightarrow ^2P_{1/2}^o F = 1$, we create a situation in which the ion is optically pumped into the $^2S_{1/2} F = 0$ state. Thus, after several microseconds, our $|0\rangle$ state is prepared. If we prepare the $|1\rangle$ qubit, we follow the optical pumping step with a microwave pi-pulse.

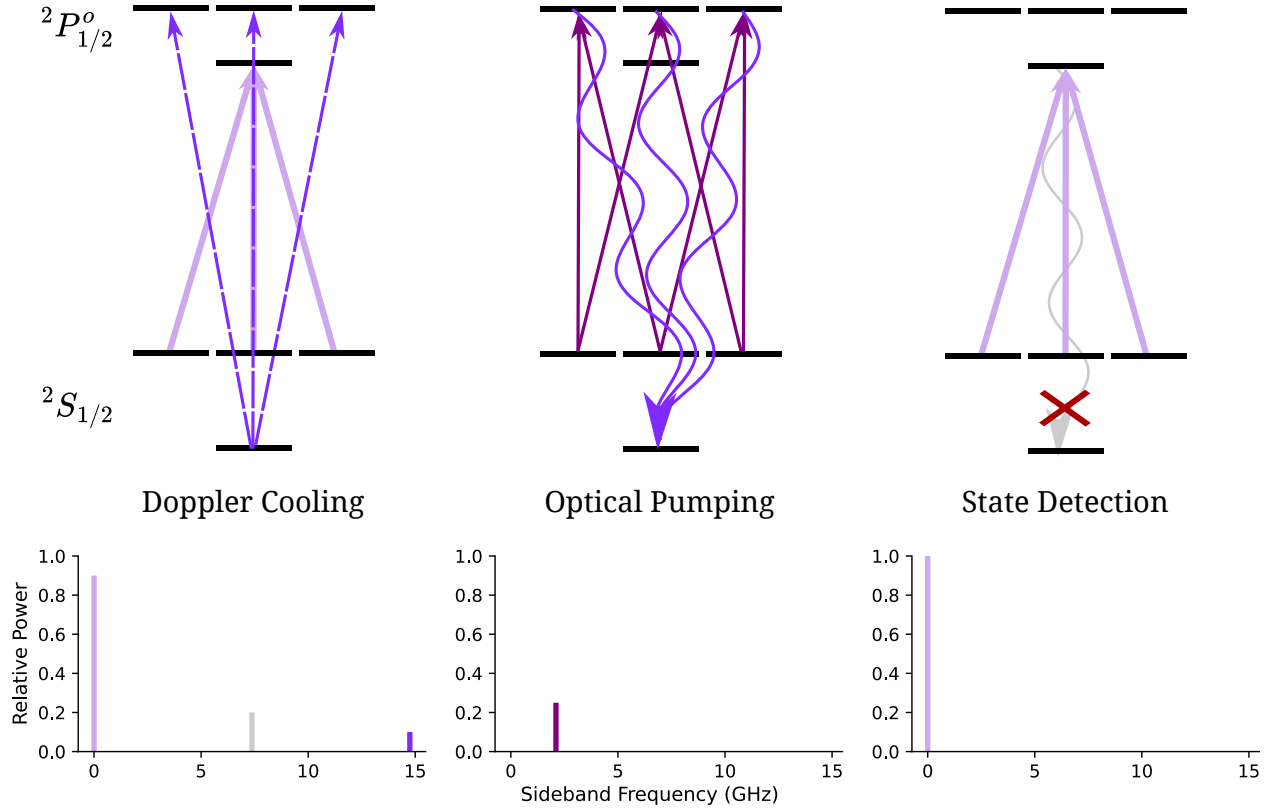


Figure 4.3: Standard Doppler cooling, optical pumping, and state detection schemes in $^{171}\text{Yb}^+$, showing what transitions are being driven during each operation. Below each Grotrian diagram is a plot indicating the EOM sidebands being applied and their relative powers. During Doppler cooling, a 7.374 GHz tone is applied, whose second-order sideband drives the $F = 0 \rightarrow F = 1$ transition. During optical pumping, a 2.105 GHz sideband is driven, depleting the carrier and driving the $F = 1 \rightarrow F = 1$ transition.

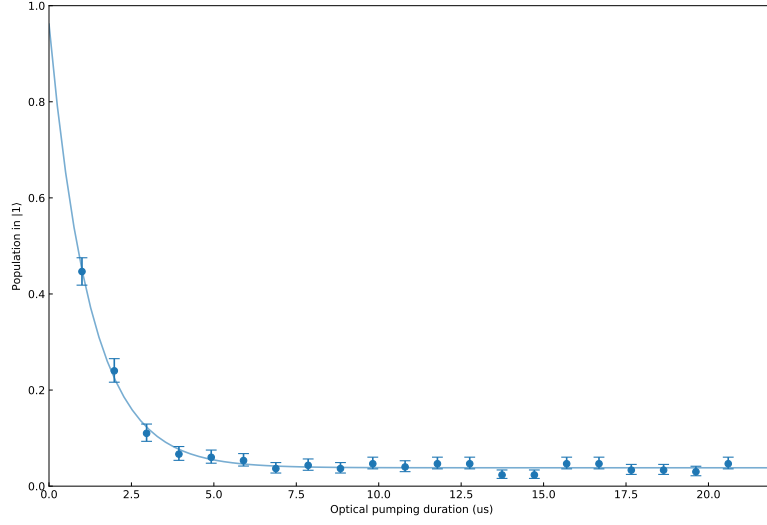


Figure 4.4: A scan of the population in $|1\rangle$ as a function of optical pumping duration. We can see that the population is pumped into the $|0\rangle$ state with a time constant $\tau = 1.25(3) \mu\text{s}$.

4.3.2 State detection

When detecting the state of the $^{171}\text{Yb}^+$ atom, we turn off all EOM sidebands so that we are only driving the $^2S_{1/2} F=1 \rightarrow ^2P_{1/2}^o F=0$, transition. Since by selection rules, the $^2P_{1/2}^o F=0$ cannot decay to the $^2S_{1/2} F=0$, the ion will only fluoresce if it is in the $|1\rangle$ state, leaving the $|0\rangle$ state untouched. By thresholding the photons collected on the PMT during the state detection, we can determine the ion's state with above 90% fidelity. The state-of-the-art for state detection fidelity in the ytterbium qubit is 99.931(6) [CCV19], but given the light collection efficiency of our system, we can at best achieve roughly 99%, though we generally tweak up to a 95% fidelity for day-to-day operation.

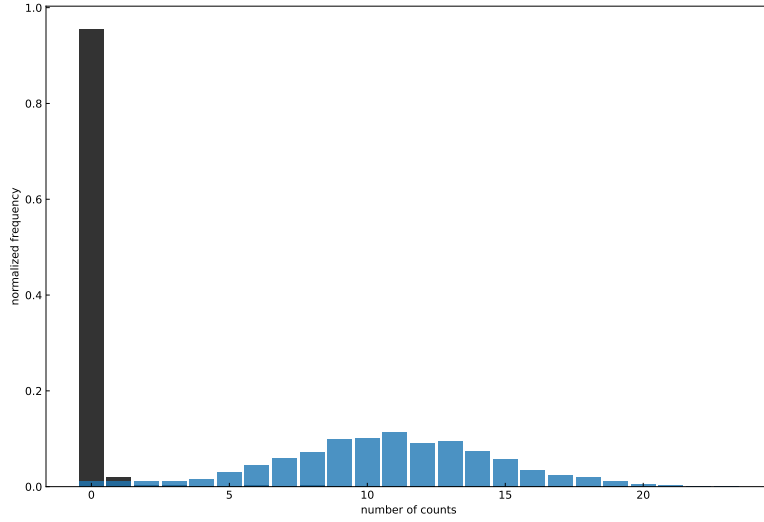


Figure 4.5: An example of state detection using the standard scheme in $^{171}\text{Yb}^+$. The blue histogram represents the fluorescence counts collected by the ion when prepared in the $|1\rangle$ state over 3000 repeated experiments, while the black histogram represents the fluorescence counts collected by the ion when prepared in the $|0\rangle$ state. To determine the state of the ion, we place a threshold on our photon counts. Above the threshold, we consider the ion to be in the $|1\rangle$ state. This gives us a fidelity of 95%.

4.3.2.1 State detection error

While it is common to use a Gaussian approximation when estimating the error of a measurement of the ion's state, this is not strictly correct. A good way to see this is to consider the error interval when an ion is measured to be in the $|0\rangle$ state in 99 of 100 trials. Using a normal Gaussian 95% confidence interval, this gives a fidelity of 0.99 ± 0.019 , reported as $0.99(2)$. Immediately it is apparent that this is unphysical, as the probability that we measure the ion to be in the $|0\rangle$ state cannot exceed 1.

Instead, we should use a confidence interval that captures the nature of the process we are performing. Specifically, we are trying to estimate the *binomial proportion* of a set of trials: given n trials and a measured success rate $\hat{p}$, we would like to estimate the true p value of the underlying binomial distribution. Generally, the most widely used confidence interval for the binomial proportion is the Wilson score [Wil27]. It reads

$$p_{\pm} = \frac{\hat{p}n + \frac{z_{\alpha}^2}{2}}{n + z_{\alpha}^2} \pm \frac{z_{\alpha}}{n + z_{\alpha}^2} \sqrt{n\hat{p}(1 - \hat{p}) + \frac{z_{\alpha}^2}{4}} \quad (4.2)$$

where z_{α} is the quantile of the normal distribution associated with the target error rate α . Testing this error interval with the above example, we get $p_{+} = 0.998, p_{-} = 0.946$, reported as $0.99^{+0.008}_{-0.044}$.

4.3.3 Microwave interrogation

During our experiments, we often apply microwaves to drive the ground state qubit. In $^{171}\text{Yb}^{+}$, we generate the 12.6 GHz microwaves by using a fixed source and a mixer. The hardware is discussed in more detail in Section 5.4.3.

Generally, when we apply microwave interrogation pulses, we are aiming to control the qubit's state via either a π pulse or a $\pi/2$ pulse. The frequency of the microwaves is tuned using microwave Ramsey experiments in combination with scans of the microwave lines, and the Rabi frequency is determined by driving Rabi oscillations in the ion once the frequency

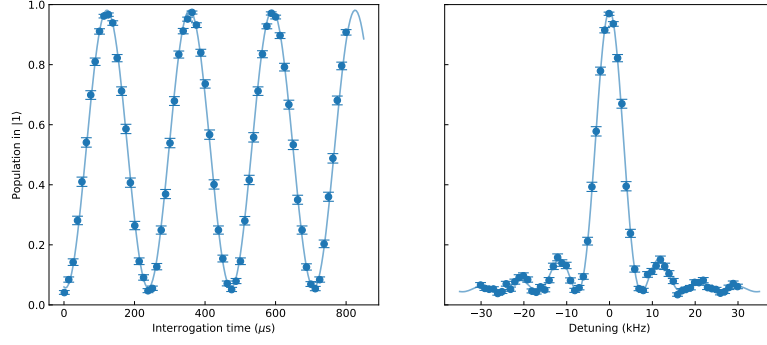


Figure 4.6: Examples of a typical Rabi oscillation and a microwave line scan on our ion. The Rabi oscillations represent a fairly un-optimized example, and the best π times we achieve are approximately $30\mu\text{s}$.

is set.

CHAPTER 5

Our Experiment

5.1 Oblate Ring Trap

Before November 2023, we employed an oblate ring trap as our primary ion trap. The design of this trap is specified in [YSD15], and further elaborated on in Michael Ip's thesis [Ip18], but I will summarize the relevant parts of the trap's design here. The trap was made of gold plated onto titanium, on a 1mm-thick fused silica substrate. The ion trap itself was created by a 500 μm hole in this trap, with an equatorial ring electrode on which was placed the trap RF. This created an oblate ellipsoidal trapping potential. Around the RF ring were placed eight DC electrodes, each controlled by an AD660 DAC chip conditioned with a multistage LC filter. These electrodes allowed for fine control of the ions' position. Through the edge of the trap's body, 7 tunnels were bored to allow optical access through the side of the ion trap for lasers and our ytterbium oven. These tunnels tapered down from roughly 500 μm at the edge of the trap body, to 100 μm diameter at the RF ring, meaning that any laser light essentially needed to be fiber coupled into the trap.

The RF drive of the ion trap was at 48 MHz. Depending on the applied RF voltage, the radial secular frequencies were generally 200-400 kHz, while the axial secular frequency was around 1 MHz.

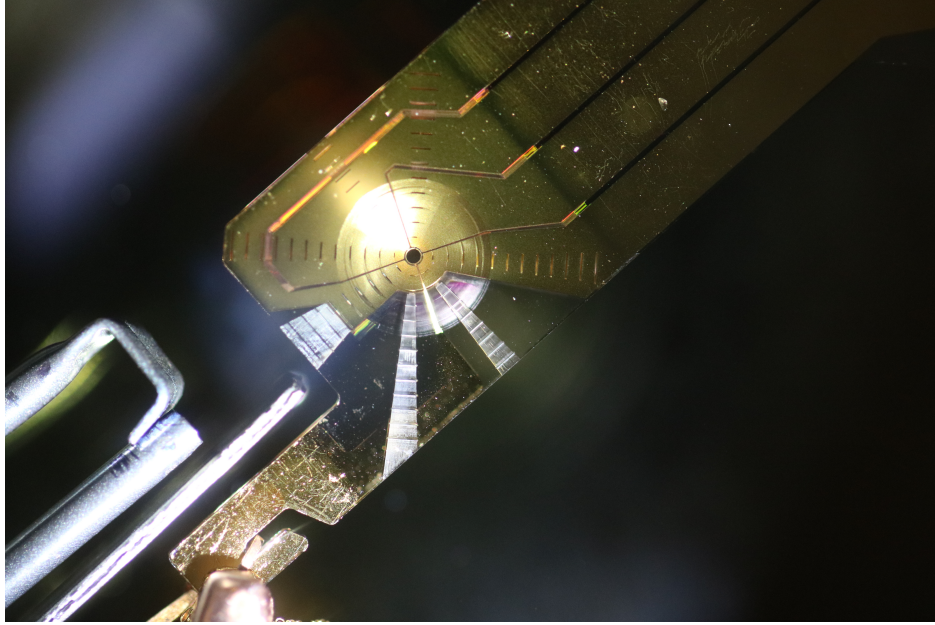


Figure 5.1: The oblate ring trap used during the first several years of this work before being ultimately replaced. The top four wedge-shaped DC electrodes are visible surrounding the trapping region, as are the seven tunnels drilled into the side of the fused silica substrate. RF is applied through the hook-shaped electrode on the bottom left, while DC is applied on the top left. The tube on the bottom left is the ytterbium oven, which emits ytterbium atoms through a tunnel into the trapping region where they are ionized.

5.2 Four Rod Trap and Accessories

The oblate ring trap is not ideally suited for studies of trapped ion quantum computation. Most obviously, it is unable to trap linear chains of ions, which precludes the possibility of using the collective motion of the ions to effect entanglement. Perhaps more subtly, the shape of the potential and its inherent 3D-ness means that the micromotion null in the trap is a point, making it more time-consuming to shim out excess micromotion in the trap, and meaning that in any multi-ion crystal, no ion can be at the micromotion minimum. In contrast, a standard four-rod Paul trap has a degree of translational symmetry that means that the micromotion null is located along a line, rather than at a point. This makes it easier to find the null, and allows for linear chains of ions which are all at their micromotion minimum.

In light of these difficulties, and in response to anomalous ion motion in the oblate ring trap that seemed indicative of a subtle electrical fault, we decided to design a drop-in replacement four-rod ion trap that could be installed with a minimum of effort. The dimensions of the trap followed a proven design from [Olm09], though the physical implementation is different. The trap employs four pure tungsten rods 0.50mm in diameter, and two needle-shaped end cap electrodes 0.40mm in diameter. The needle ends were designed to be placed 2mm apart, though the distance was left variable, in case we wanted to increase our axial secular frequency.

5.2.1 Design and simulation of the trap

Since we wanted to use this trap to explore motional entanglement, we needed the axial secular frequency of the trap to be resolvable. To confirm the predicted secular frequencies of the trap, I performed a numerical analysis of the trapping potential.

The trap RF was modeled in 2 dimensions, with an axial potential added on when computing the secular frequencies. Unlike the oblate ring trap, the four rod trap is a linear

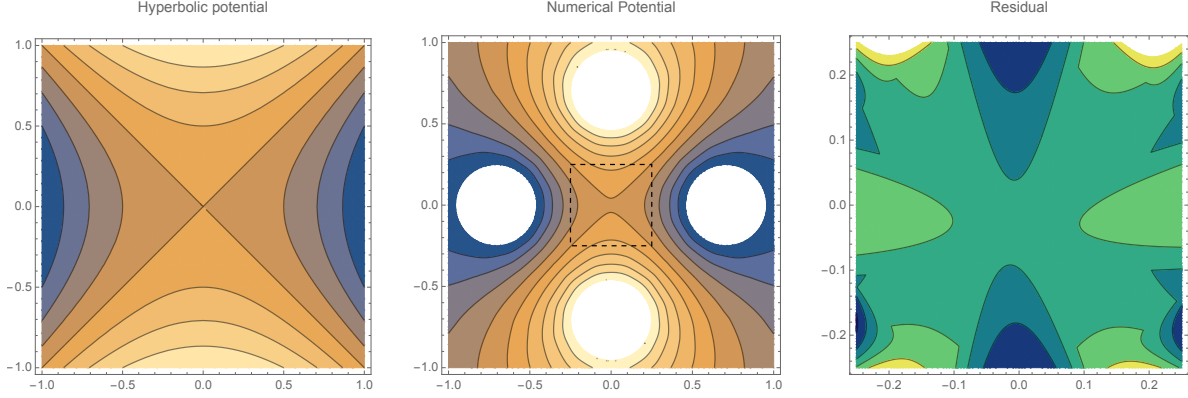


Figure 5.2: An example of the process for evaluating the geometric factor associated with a given trap geometry. Axes are in mm. On the left is an ideal hyperbolic trap potential, and in the center is a modeled trap potential for a four rod trap. The region within the dotted square is fitted such that the deviation between the simulated trap parameter and the ideal trap parameter is minimized. On the right is the residual of the fit, showing primarily an octupole character. This is indicative of the quality of the fit, as the quadrupole component has been completely subtracted out.

Paul trap of the same geometry as described in section 3.1. However, unlike in that section, we cannot assume that our rods are perfect hyperboloids. This adds a correction factor κ to equation 3.15, which now reads:

$$\phi = \frac{\kappa V_0}{2} \cos(\Omega_{Rf} t) \left(1 + \frac{x^2 - y^2}{r_0^2} \right) \quad (5.1)$$

In order to obtain a value for this geometric correction factor, I numerically solved Laplace's equation using the cylindrical rod voltages as boundary conditions. Then a least-squares fit was performed in a small area (0.5x0.5mm) near the trap center to determine the value of κ that yielded the closest approximation to a pure hyperbolic potential given in equation 5.1.

Various rod spacings and diameters were evaluated for numerical aperture (NA), secular frequency in the axial and radial directions, and Mathieu q parameter. The simulation using our final trap parameters is shown in Figure 5.3, with an example potential fit being

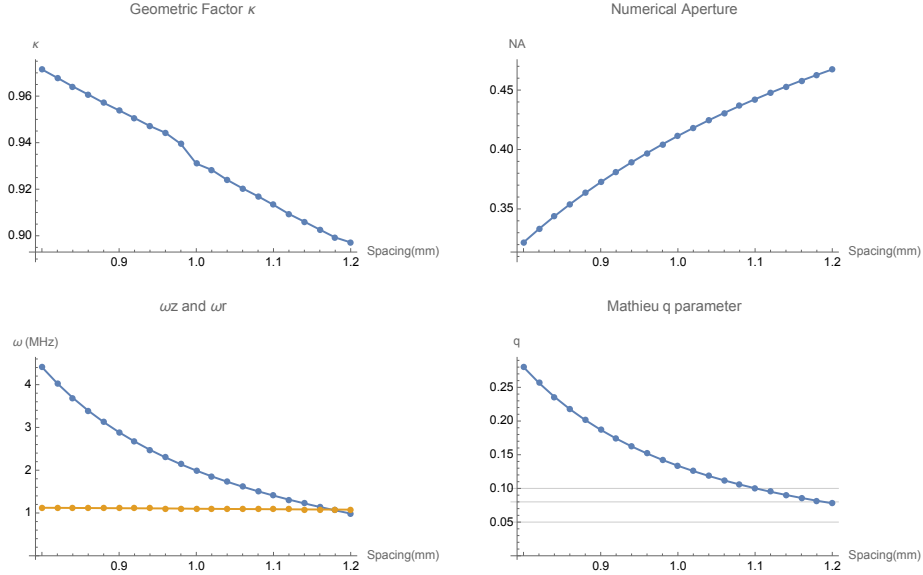


Figure 5.3: Example analysis of the secular frequencies of our four-rod trap as we vary the rod spacing. In this example, $V_0 = 1\text{kV}$, $U_0 = 200\text{V}$, the endcaps are spaced 2.1mm apart ($z_0 = 1.05\text{mm}$), and the rod diameter is 0.5 mm.

demonstrated in Figure 5.2.

Ultimately, we decided on a rod spacing of 1 mm and a rod diameter of 0.5 mm, as this provided a good balance between numerical aperture and secular frequency. It was also in some sense a proven option, as a comparable rod spacing was employed in [Olm09]. In addition, attempting to reduce the trap size below 1 mm would have risked dangerously high electric field gradients in vacuum, and would have made the alumina spacers significantly more difficult to machine.

5.2.2 Construction of the trap

5.2.2.1 The original four-rod trap

The trap design explained above was eventually turned into a new experiment, and was ultimately constructed by Patrick McMillin and Hassan Farhat. However, spare parts were

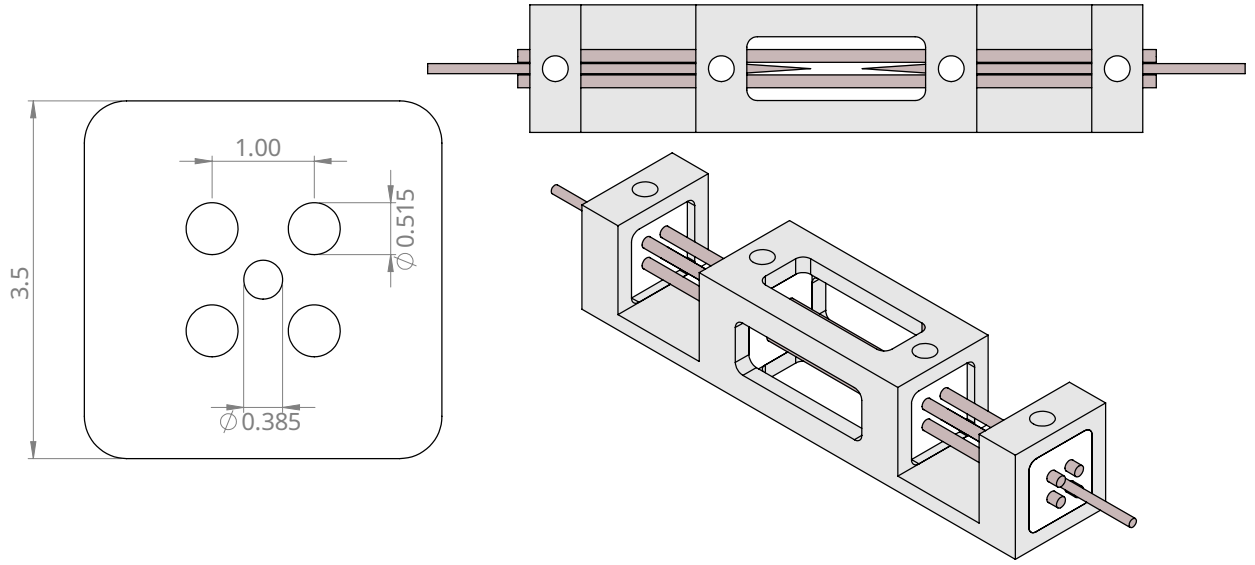


Figure 5.4: Diagram of the four-rod trap we constructed after the failure of the oblate ring trap. Units are in millimeters. The alumina spacers are shown in white, the stainless steel cage is shown in gray, and the tungsten rods are shown in brown.

ordered in case the oblate ring trap ever needed to be replaced.

In December of 2023, the oblate ring trap suffered an electrical failure in which one of the DC electrodes (the one connected to DAC 19) became disconnected from the vacuum feedthrough. After a month of attempting to compensate for the loss of this DAC with no success, it was decided that it was time to perform a drop-in replacement with a near-exact replica of the four rod trap that we had designed, with minor adjustments to address lessons learned from the first trap construction: the alumina spacers were thickened, minor modifications were made to the trap cage, and the end cap diameter was slightly reduced.

5.2.2.2 Electropolishing the rods

In order to get our rods down to the dimension required by the spacers that we had manufactured for the ion trap, we utilized electropolishing, loosely following the methods employed by [Kot19]. We electropolished our rods in a 2M solution of sodium hydroxide in distilled

water, with a graphite cathode attached to the negative terminal of a power supply, while the polished part is used as the anode and is attached to the positive terminal. A steel tube was used to help distribute the voltage on the rod, in the hopes of helping the anode see a cylindrically symmetrical electric potential.

The rods of the trap are produced from the 0.020" tungsten rod stock from McMaster-Carr. They were polished until they had a snug sliding fit through the 0.5mm-diameter holes of our alumina spacers, while being smooth and of uniform diameter. Polishing with too high a voltage yielded inconsistent diameter, while polishing with a voltage that was too low, or with mechanical stirring, yielded a poor surface finish. Ultimately, electropolishing at 12V for 1.25 minutes gave good consistent results. *It's worth noting, however, that the source of the tungsten greatly affected polishing times, with polishing rates being much lower with tungsten from Midwest Tungsten Services than from McMaster, for example.*

5.2.2.3 Electropolishing the needles

Electropolishing well-shaped tungsten needles is a considerably more difficult task than making the rods. Since there is no detent mechanism built into our trap design, the taper of the needles themselves is what holds them in place in the ion trap assembly. This means that the exact amount of polishing of the needles determines the ultimate spacing of the end caps in the trap. Furthermore, the sharp nature of the needles makes them very susceptible to bending and misalignment.

We began our needle polishing with pre-made 0.020" tungsten probe needles used in micro-dissection, which needed to be polished down to 0.4mm. The needles were first laid in a corrugated mesh holder and ultrasonically cleaned in a dilute solution of Simple Green for 20 minutes at the lowest power setting to remove surface contaminants, and then taped using electrical tape to a sacrificial tungsten welding electrode, such that about 25mm of the needle was exposed. This was then dipped into the NaOH solution such that the tape was about half-submerged. Polishing then proceeded as follows:

1. Polish the needle for 3 minutes at 0.35A constant current
2. Rotate the needle 180 degrees to counteract any bending that is developing due to asymmetry in the polishing
3. Polish the needle for another 3 minutes at the same current
4. Dip the needle in distilled water to clean it, taking care to prevent it touching anything, as this would immediately cause the tip to bend

After about 15 needles were made, they were taken into the clean tent and pairs of needles were tried in the assembled trap under a microscope. After a pair of needles was found that rested the right distance apart, the needles were aligned to one another by lightly nudging the tips with a third (reject) tungsten needle. We considered using a vacuum-compatible epoxy to maintain the needle position, but ultimately decided to rely on friction rather than introduce a potential outgassing source so close to the ion.

5.2.3 Ion trap performance

After installation, the ion trap performed well, and is able to retain ions significantly longer than the previous oblate ring trap. The axial secular frequencies scale strongly with the power applied to the trap, and a plot of secular frequency versus applied trap power is given in Figure 5.5. As end cap voltage is increased, the secular frequency should be able to be further increased, should the desire to resolve ion motion in the trap ever arise.

5.2.4 Ovens

Two ovens were constructed when the ion trap was built: one for ytterbium, and one for thulium. Each oven is constructed out of 0.056" stainless steel tubing, which was crimped at the center using round-nose pliers. One side of this crimp was used to hold a small amount of the metal in question, while the other end was crimped onto copper wire, which connected

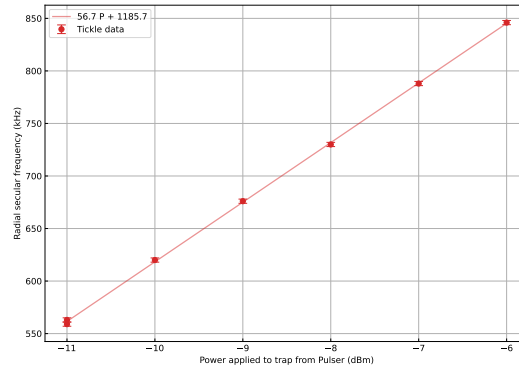


Figure 5.5: Trap radial secular frequencies as a function of applied trap power, at an end-cap voltage of 10.0 V. Data was taken using “Tickle scans”, in which a small AC voltage is applied to a DC rod, exciting parametric resonances in the trap. While equation 3.14 suggests that the secular frequency should be linear with applied voltage, the data suggest a more logarithmic trend (note that the x axis is in dBm). This could be due to saturation of the trap amplifier. Above -6 dBm, the trap stability begins to become less reliable.

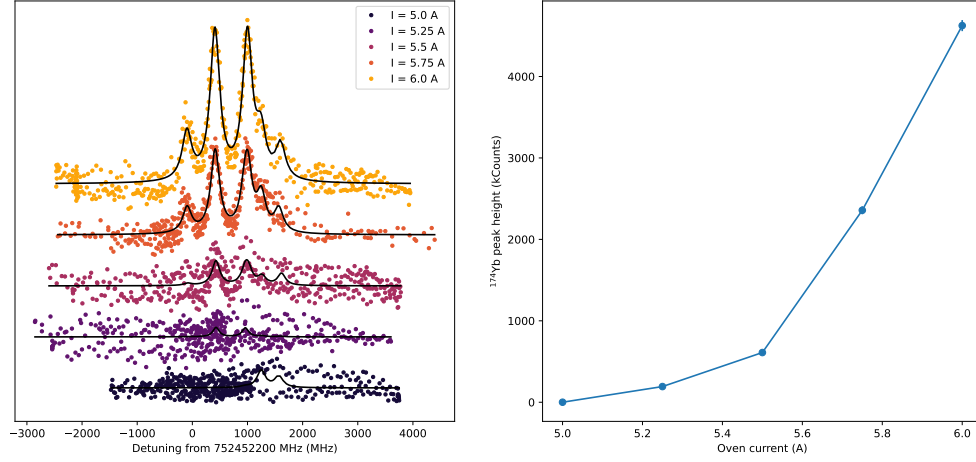


Figure 5.6: A demonstration of the neutral fluorescence we observe in out trap as a function of applied oven current. The left plot displays several spectra of the 399nm transition, demonstrating the peaks associated with different isotopes. The relevant spectroscopic reference is [KGB16]. The curves are offset vertically from each other for clarity. The right curve displays the height of the central ^{174}Yb peak as a function of the applied oven current. In reality, the point where neutral fluorescence is visible on the PMT represents far too high a flux of ions for daily loading, and we load our chamber using 4.75A of oven current.

the ovens to the external feedthrough. The ovens are supported from the front by a bracket, which also electrically connects them to ground via the chamber. Thus, when current is applied to the oven feedthrough, it passes through the oven on its way to ground, resistively heating the stainless steel walls of the tube.

Prior to pumping down our chamber, we tested both ovens by running current through them under vacuum while they pointed at a clean microscope slide. After 30 minutes at 6A, we observed a visible mark on the microscope slide due to the deposited metal. This deposit appeared as a hazy blue film when illuminated by a flashlight, and was not enough to form a mirror.

After the final bake-out, we tested the ytterbium oven by looking for fluorescence at 399nm in neutral ytterbium on our PMT. We find that sending 5A is more than sufficient to load ions, and generally run the ytterbium oven at 4.75A in daily use. The thulium oven has not been tested to date, but we expect the thermal performance of the two ovens to be quite similar.

5.2.5 Helical resonator

The new trap required a helical resonator that operated at a higher target frequency of 35 MHz. We constructed the resonator based on a design in use in the Monroe group at Duke, which was also in use at UCLA. The design was lengthened to accommodate a lower RF drive frequency, making sure to remain within the bounds of good helical resonator design as described in [SSW12] and [MS59]. The primary coil consists of 4 turns and is mounted to an 80 TPI screw adjuster so that it can be moved to adjust the q-factor of the resonator, and is supported by a 3D-printed mount.

The primary coil is inductively coupled to two intertwined secondary coils, each of which is electrically connected to a different trap rod, making this a *bifilar* helical resonator. These each consist of 8 turns of wire supported by four 3D-printed combs, and connect directly

to their pin on the trap RF feedthrough using barrel connectors. The other ends of the secondary coils are attached via SMA ports integrated into the resonator body to bias tees. The RF side of the bias tees is shorted to ground, and the DC side is connected to our DAC board, allowing us to superimpose DC onto our trap rods. The resonator body itself is screwed into a mounting ring with tapped holes that is attached to the feedthrough flange. It is cantilevered off of the flange and uses no secondary support.

After construction, the resonator-trap system was found to resonate with a q factor of roughly 150 at a frequency of roughly 32 MHz.

5.2.5.1 Thermal effects

As the amount of RF power delivered to the trap increases, the resonance of the helical resonator must be re-tuned. This is because the trap itself changes in temperature, causing changes in the trap's resistance and capacitance. We believe that this change in capacitance occurs due to thermal expansion of the alumina spacers that support the trap rods, as we observe that the rods physically move by 10 μm or more as the trap power is increased. While this makes it difficult to rapidly change the trap power, the quality of the helical resonator's resonance is highly stable as long as the trap power is maintained at a consistent value.

5.3 Vacuum Chamber

The vacuum chamber in which these experiments were performed consisted of an 8-inch octagon, two re-entrant viewports, and several attached pumps:

- a 40 L/s ion pump
- a non-evaporable getter (NEG) pump
- a titanium sublimation pump

Two electrical feedthroughs were attached to the octagon, one on either side of the ion trap. The first feedthrough is used to attach the RF resonator, and has two pins which are connected to the RF rods of the ion trap. The second feedthrough is a 2x Dsub 9 feedthrough, and contains all other electrical connections: the DC rods, the end caps, the oven, and four antennas that we added since the feedthrough had extra pins. The antennas that we included were:

- an 8-inch dipole antenna
- a 4-inch dipole antenna
- a loop antenna with a roughly 3-inch diameter, oriented horizontally below the trap
- a loop antenna with a roughly 1.5-inch diameter, oriented vertically around a viewport

5.4 Electronics

5.4.1 LabRAD experimental control

5.4.1.1 The Pulser

Experimental control in our experiment is primarily handled by the "Pulser", a home-built pulse sequencer based on an XEM6010 FPGA by OpalKelly. The details of the pulser are described in [PK15]. In short, the Pulser has 32 TTL outputs and 16 programmable DDS outputs, whose power, phase, frequency, and sweep can be adjusted with a resolution of roughly 60 ns. It is also connected to our PMT, and can receive photon counts up to a 10 ns resolution (which is sufficient, given the 10 nm pulse width of our PMT).

An abstract framework has been constructed inside LabRAD to program the Pulser in a convenient and modular way. The key to this abstraction is the `PulseSequence` class, which encodes pulse sequences in an extensible manner, and contains the functionality to compile

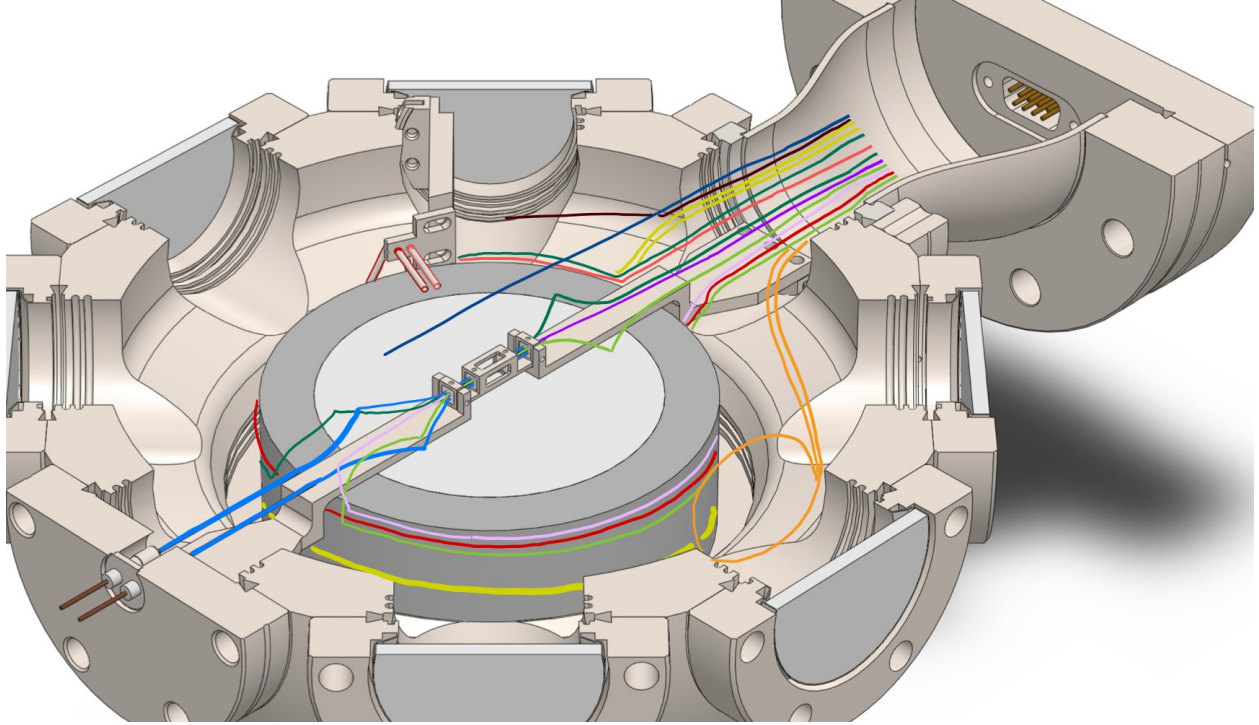


Figure 5.7: CAD diagram of the vacuum assembly surrounding the trap, with a wiring diagram superimposed. In red are the wires for the oven. Orange and yellow represent in-vacuum loop antennas. Purple shows the end caps, and green shows the DC rods, and blue shows the RF rods. A detailed pinout is given in Figure 5.8.

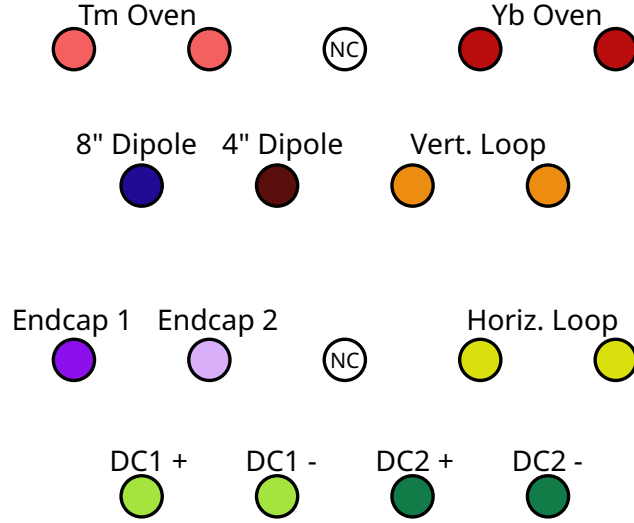


Figure 5.8: The pinout of the DS9 feedthroughs of the vacuum chamber. The pinout of the connectors used is mirrored. For the oven feedthroughs, two pins were used to avoid nearing the 5A current limit of the feedthrough pins. Colors are matched to those used in Figure 5.7.

itself into the bytecode that is used to program the pulse sequence into the Pulser FPGA. The class has three primary methods of importance: `addDDS`, `addTTL`, and `addSequence`. `addDDS` and `addTTL` are used to directly toggle DDS's and TTLs on the pulser, with specifiable timings and parameters. We confine the use of these methods to sub-sequences, which we then compose. This adds a useful level of abstraction. `addSequence` allows the user to concatenate and combine different pulse sequences, allowing for complex sequences to be built out of simple and reusable subunits. Examples of such sub-sequences are `DopplerCooling` and `StateDetection`.

5.4.2 Trap DC

Our trap DC electrodes are powered by an XEM6010 FPGA board, which controls up to 8 AD660 16-bit DAC chips. The voltages produced on this board are passed through a four-stage low-pass filter, and then transmitted to the feedthrough on the vacuum chamber

using SMA cables. These DACs can be accurately controlled to produce any voltage from -10 to 10 V, to a resolution of 0.0003 V, and are referenced to the chamber ground.

5.4.3 Microwave sources

5.4.3.1 EOM sidebands

In order to generate EOM sidebands for cooling $^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$, we use two oscillators from Windfreak Technologies: a Windfreak SynthHD pro generates the 7.374 GHz tone used in Doppler cooling, and a Windfreak SynthNV pro generates the 2.105 GHz tone used for optical pumping. These two signals are triggered on and off using TTLs from the Pulser. They are combined on a ZDSS-3G4G-S+ diplexer, which allows for lossless combining of signals, as long as one is below 3 GHz, and the other is above 4 GHz. The signal is then passed through a low-noise amplifier before being sent to the EOM. When Doppler cooling $^{173}\text{Yb}^+$, the SynthHD is set to output 10.491 GHz, and the SynthNV is not used.

To generate the EOM sidebands on our 935 nm lasers, a National Instruments PXIe 1073 signal generator is used.

5.4.3.2 $^{171}\text{Yb}^+$ qubit microwaves

The 12.642 GHz microwaves that drive the $^{171}\text{Yb}^+$ qubit need to be phase- and frequency-stable to a level not needed in the EOM tones, where the linewidth of the cooling laser renders the stability of those tones relatively unimportant. Instead, we employ an always-on HP 8672A oscillator referenced to a rubidium clock. This oscillator constantly produces a 13.000 GHz tone, which is passed through an absorptive TTL switch connected to the Pulser.

After the TTL switch, the 12.642 GHz tone is created using a frequency mixer. When the 13.000 GHz tone from the HP 8672A is mixed with a 358 MHz tone generated by the Pulser, a sum and difference tone are produced. The resultant is amplified and passed through a 12.2-12.7 GHz band pass filter to remove unwanted tones created by the mixer. This final

signal is passed through a high-power amplifier and sent into the chamber using a horn antenna pointed through a viewport. This is shown in Figure 5.9.

The feedthrough of our vacuum chamber also contains several simple in-vacuum antennas, as well as the potential ability to superimpose microwaves onto the DC rods of the ion trap. While impedance matching from a 50 Ohm transmission line onto the trap rods has proven challenging, work to superimpose microwaves onto the DC rods is ongoing in our lab, and pi times comparable (within a factor of three) to those achieved with the horn antenna have been achieved.

5.4.3.3 $^{173}\text{Yb}^+$ qubit microwaves

During initial experiments, microwaves were delivered to the $^{173}\text{Yb}^+$ ion using a 3 GHz horn. The microwaves were synthesized using a Windfreak SynthHD pro.

5.5 Lasers

5.5.1 369 nm diode laser

Our 369 nm laser is a Moglabs ECD004 ECDL, controlled by a Moglabs DLC102 controller. Inside the ECDL cavity, we use a Nichia NDU1113E laser diode. *It is worth mentioning that these diodes do seem to have a finite lifetime, and that we generally have to replace them every one to three years.* This system generally produces at least 2 mW of 369nm light. Of this, 1.8 mW is sent to our double-pass AOM system, which provides fine frequency control. Following the double pass, roughly 200 uW of 369nm light is sent through our ADVR fiber EOM (see section 4.3), where modulation tones can be applied to address the hyperfine structure of $^{171}\text{Yb}^+$ and $^{173}\text{Yb}^+$ (see section 4.2).

Before reaching the ion trap viewport, the mode of the beam is cleaned using a 50 um pinhole, and the polarization is cleaned using a PBS and a wave plate. Finally, the beam is

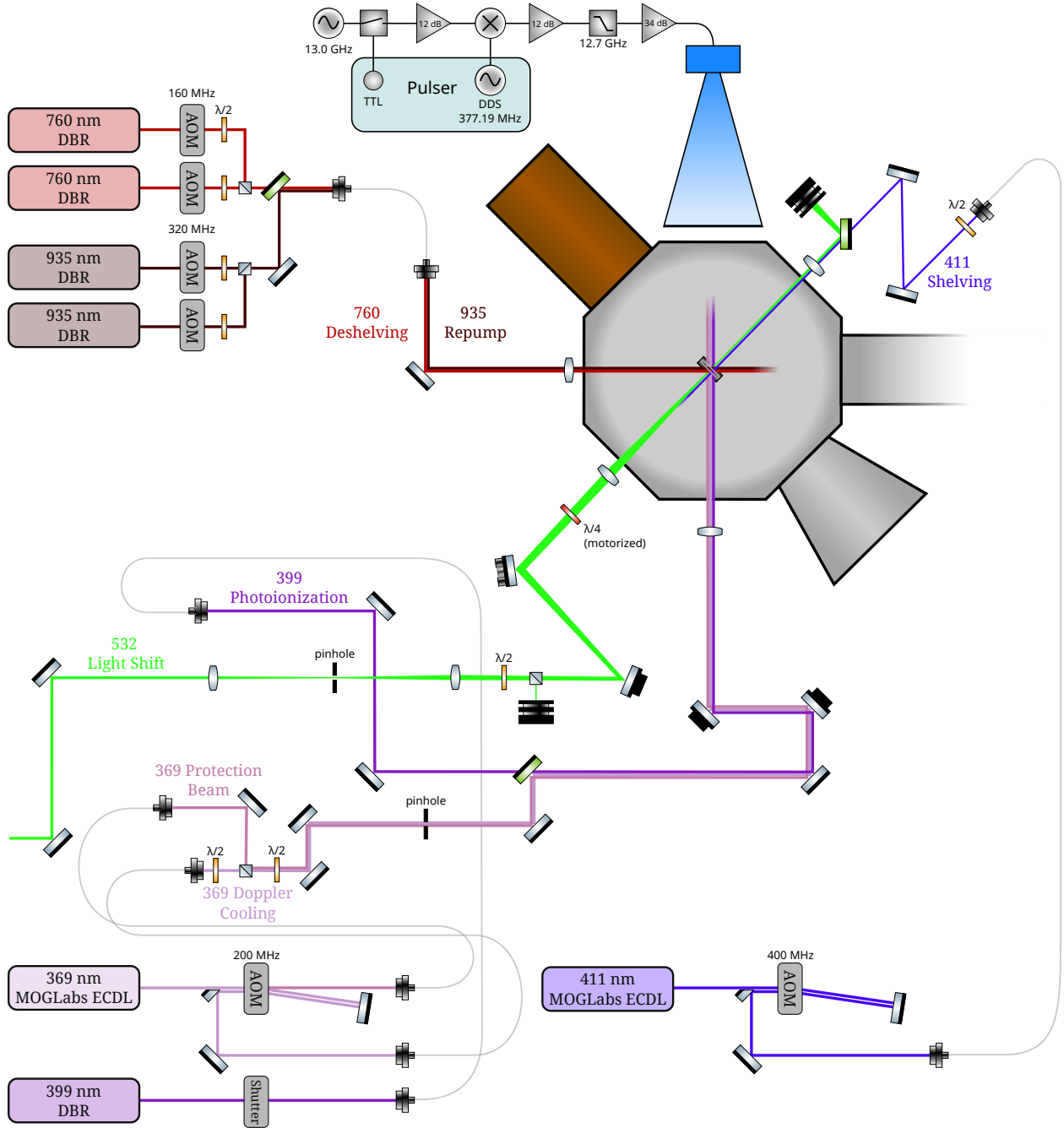


Figure 5.9: The beam paths addressed onto the ion. All optic locations immediately surrounding the trap are accurate. Outside of the immediate vicinity of the trap, beam paths are simplified. In addition is shown the pipeline by which the 12.642 GHz microwaves are generated and delivered to the trap using a horn antenna.

focused onto the ion using a 100mm focal length lens. The overall beam path is shown in Figure 5.9.

During some of our experiments, we also implemented a 369 nm protection beam. The idea of a protection beam is to add a far-red-detuned Doppler cooling beam to the system, such that any ions that are too hot to be cooled by the main Doppler beam are kept in the trap and cooled to a point where the main Doppler beam can finish cooling them. We make our protection beam by simply sampling the zeroth diffraction order from our double-pass AOM and overlaying it with the Doppler cooling beam (see Figure 5.9). With the protection beam in place, we see improved loading times and ion retention in ^{174}Yb . However, it is inconclusive whether the protection beam helps us when loading $^{171}\text{Yb}^+$ or $^{173}\text{Yb}^+$, as the protection beam lacks the necessary sidebands and polarization to effectively cool the ions given their hyperfine structure.

5.5.2 399 nm laser

The 399 nm laser is a Toptica D100 pro DBR laser. Since this laser is used only intermittently during photoionization, we take no special care to stabilize the wavelength, and switch the laser using a shutter. The laser passed through a pinhole and overlaid onto the 369 nm beam path using a long-pass dichroic beam splitter, after which it is focused onto the ion using a 100 mm lens.

5.5.3 935 nm lasers

5.5.3.1 935nm bulk DBR

One of our 935 nm lasers is a DBR controlled by an Arroyo 6305 laser diode controller. This light is fiber coupled, sent through a 320 MHz AOM for switching, overlaid with the three other red lasers (the other 935 and the two 760s), coupled through a single-mode fiber, and finally focused onto the ion using a 100mm focal length lens.

5.5.3.2 935nm pigtailed DBR

Our second 935nm laser is a pigtailed design, controlled by a Thorlabs LDC205C diode controller. It is butt-coupled to a fiber EOM to allow us to address hyperfine structure, and then outcoupled into free space, where it passes through essentially identical optics to the other 935nm DBR.

5.5.4 760 nm lasers

Both of our 760nm lasers employ Photodigm PH760DBR080T8 laser diodes and follow essentially the same beam path. They are shaped using anamorphic prism pairs, passed through switching AOMs, and combined on a polarizing beam splitter, before being overlaid onto our 935nm lasers using a dichroic mirror. From there, they are coupled into a single mode fiber, after which they are focused onto the ion using a 100mm focal length lens.

5.5.5 411 nm laser

The 411 nm laser used in this experiment is a MogLabs ECDL, which is double-passed using a 450 MHz AOM, and coupled to an external ULE cavity. During many experiments, this cavity was the same one that was used for our 369 nm laser, and the two were locked simultaneously with the double pass AOMs used for final frequency adjustment. Throughout various experiments it has taken several different paths to the ion.

5.5.6 976 nm laser

During our shelving experiments, a 976 nm laser was used during the quadrupole optical pumping step. This laser was a Thorlabs DBR976PN pigtailed DBR laser, which was switched using an AOM and overlaid onto the 935 nm lasers using a dichroic beam splitter.

5.5.7 532 nm laser

The 532 nm laser that we employ is a 15 W Millennia eV Pump laser, which was originally used as the pump for the mode-locked laser used in [IRJ18, Ip18]. This laser is passed through a high-power single-pass AOM, and is then focused through a pinhole in order to clean up the mode of the beam. The beam is then passed through a PBS in order to clean up the polarization, and focused onto the ion using a 100 mm focal length lens. Just before this lens, we place a computer-controlled quarter wave plate to ensure repeatability in our light shift measurements. Ultimately, roughly 2W of 532 nm light shines on the ion, at a spot size of roughly 15 μm .

CHAPTER 6

Shelving

6.1 Motivation - Shelving for SPAM

6.1.1 Fault Tolerance in Quantum Computing

One of the key limitation to quantum computation is the fairly low fidelity of quantum operations compared to their classical counterparts. As an illustrative example, assume that we can perform all quantum operations necessary for quantum computing with a fidelity of 99.99%. Even at these state-of-the-art levels of fidelity, after 10000 operation we only possess a 36% chance that no errors occurred during the operation. Given these limitations, it is desirable to design error correction codes that can correct errors, allowing *logical* qubits composed of several physical qubits, with error rates better than those of individual qubits.

Modern error correcting codes nominally have a fault tolerance threshold of approximately 1% error rate in qubits [RH07]. However, due to the large computational overhead when operating near the error tolerance threshold, conventional wisdom suggests that the threshold for *practical* quantum error correction is around an error rate of 10^{-4} [BCM19]. Our goal in implementing the shelved readout scheme in this chapter is to provide a technique that allows one to reduce state preparation and measurement (SPAM) infidelity to a value at or below this threshold.

6.1.2 Limits on Conventional SPAM

Given the splitting of the $^2P_{1/2}^o$ state in $^{171}\text{Yb}^+$, there exists a fundamental limit on the fidelity of qubit SPAM, set by the off-resonant scattering rate due to state detection. State detection in the standard scheme is based on the fact that the $^2P_{1/2}^o$ $F = 0$ state cannot decay to the $^2S_{1/2}$ $F = 0$ state due to the conservation of angular momentum. We exploit this fact to cycle between the $^2S_{1/2}$ $F = 1$ state and the $^2P_{1/2}^o$ $F = 0$ state, allowing for photon collection without mixing the qubit. However, given the natural linewidth of the 369 nm transition, there is a chance when driving this transition that we instead drive a transition to the $^2P_{1/2}^o$ $F = 1$, which is 2.105 Gz blue-detuned. This state can decay to the $^2S_{1/2}$ $F = 0$, causing qubit mixing. The rate for this scattering process is [NVG13]

$$R_d = \left(\frac{2}{3}\right) \left(\frac{1}{3}\right) \left(\frac{\Gamma}{2}\right) \left(\frac{2\Omega^2}{\Gamma^2}\right) \left(\frac{\Gamma}{2\Delta_{HF}}\right)^2 \quad (6.1)$$

The factor of $3/2$ accounts for the presence of coherent dark states, and the factor of $1/3$ accounts for the branching ratio of the decays from the $F = 1$ state. Putting in the appropriate values for $^{171}\text{Yb}^+$, this formula yields a scattering error rate of approximately $1/20000$. This means that on average, the ion can scatter 20000 photons before the qubit state is mixed, and puts a ceiling on detection time. One way to improve fidelity therefore is to increase collection efficiency, either by improving the detector or by increasing the NA. This allows for a larger number of photons to be collected and therefore for the interrogation time to be reduced. The current state-of-the-art SPAM fidelity for a $^{171}\text{Yb}^+$ qubit read out in this way is 99.931(6)[CCV19], which utilized state-of-the-art superconducting photon detectors and very high-NA collection optics.

6.1.3 Shelving

The other way to increase SPAM fidelity is to try to completely sidestep this fundamental limit on readout time. The way we propose to do this is by employing a *shelved* readout

scheme, in which the qubit population in one of our two states is moved to a long-lived shelving state while readout is performed. In this way, we do not need to worry if the readout light scatters off of the $^2P_{1/2}^o$ $F = 1$ state, since the qubit information is protected from decoherence. We can therefore perform state detection for a duration that is only fundamentally limited by the lifetime of the state we are using to shelve.

Ytterbium is an excellent candidate for this shelving procedure as the lifetime of $^2F_{7/2}^o$ is 1.58 (8) years [LPH21] is far longer than we could ever wish to detect for. While increasing detection time may seem an unattractive prospect in a quantum computing world where gates speeds are constantly being improved, we argue that detection is a relatively rare part of most quantum circuits, and that a very low rate of SPAM error can help minimize the need to repeat potentially time-intensive computations.

6.2 Shelving Theory

6.2.1 Fidelity vs Detection Time

Ultimately, the trick of shelving is that it allows us to make a trade-off between detection time and readout fidelity. Where the standard state detection scheme in ytterbium (see section 4.3) is limited in the possible detection time by off-resonant scattering due to the $^2P_{1/2}^o$ $F = 1$ state, shelving allows us to perform state detection for an arbitrary amount of time, letting us collect as many photons as we want without undesirable state mixing.

6.2.2 Shelving Scheme

While the $^2F_{7/2}^o$ state in ytterbium is extremely long lived, it is also extremely difficult to directly populate. The 467nm transition's linewidth is $1/(1.58 \text{ years}) = 20\text{nHz}$. While this transition has been directly driven in optical clock experiments [HSL16], doing so is a technically difficult process requiring high stability and high power. Instead, the scheme

we employ for shelving our qubit to the F state uses optical pumping via the $^2D_{5/2}$ state. Specifically, we shelve the population of the $^2S_{1/2}$ $F = 1$ manifold to the $^2D_{5/2}$ $F = 3$ manifold. The $^2D_{5/2}$ state has an 83% chance to decay into the $^2F_{7/2}^o$ state, and a 17% chance to decay back to the ground state, though such decays do not mix the qubit since selection rules prevent the decay to $|0\rangle$. While this transition is narrow (20 Hz), we can significantly broaden it by increasing the intensity of the laser. By a lucky coincidence, the g-factors g_F of the two states are also identical, allowing us to sidestep the problem of addressing the Zeeman structure of the transition.

To evaluate the rate of this optical pumping process, we can make a few assumptions. If we assume that we are significantly broadening the transition, then we can say that the transition is saturated. This means that at any given time, the population in the $^2S_{1/2}$ and $^2D_{5/2}$ states are equal. Taking into account the branching ratio ζ of the transition, we then have an optical pumping rate of

$$\tau = \frac{2\tau_D}{\zeta} = 17.3 \text{ ms} \quad (6.2)$$

where τ_D is the total lifetime of the $^2D_{5/2}$ state, and the factor of two comes from the fact that the un-shelved population at any given time is split between the S and D states. Implicit in this calculation is also the assumption that the Rabi frequency of the transition is fast compared to the relevant lifetimes.

6.2.3 Shelving Rate Calculations

6.2.3.1 Three-level System

In order to more precisely evaluate the shelving rate and ultimate fidelities we can achieve, we can employ the optical Bloch equations to set up an n-level system which we can solve numerically. Starting from the optical Bloch equations and assuming that the 411 nm light is on resonance, we can write down rate equations for the overall three-level system of the S

state, D state, and F state as:

$$\begin{aligned}
\dot{\rho}_s &= -\frac{\Omega}{2}\rho_s + (1 - \zeta)\Gamma_d\rho_d + \frac{\Omega}{2}\rho_d + \Gamma_f\rho_f \\
\dot{\rho}_d &= \frac{\Omega}{2}\rho_s - \Gamma_d\rho_d - \frac{\Omega}{2}\rho_d \\
\dot{\rho}_f &= \zeta\Gamma_d\rho_d - \Gamma_f\rho_f
\end{aligned} \tag{6.3}$$

In this simple model, we can solve for the equilibrium fidelity by taking two steps. First, we assume that we can saturate the transition. In this limit, we can state that $\rho_s = \rho_d$, allowing us to simplify our system into two equations. Then, using the fact that the total probability must equal one, we obtain the equilibrium value for the F state population (which we will call the fidelity $\mathcal{F}$ of the operation):

$$\mathcal{F} = \frac{\Gamma_d\zeta/2}{\Gamma_d\zeta/2 + \Gamma_f} \tag{6.4}$$

The dynamics of 6.3 are shown in Figure 6.1, and the decay to this equilibrium value is clearly visible.

6.2.3.2 Hyperfine Structure

If we now consider the slightly more complicated problem in which we include the hyperfine structure, the population dynamics become somewhat more interesting, as now population that started out in the $|1\rangle$ state is able to leak into the $|0\rangle$ state, potentially limiting the fidelity of the transfer. If we set up analogous equations, we obtain the following system:

$$\begin{aligned}
\dot{\rho}_{s0} &= \frac{1}{2}\Gamma_{f3}\rho_{f3} \\
\dot{\rho}_{s1} &= -\frac{\Omega}{2}\rho_{s1} + \left(\frac{\Omega}{2} + (1 - \zeta)\Gamma_d\right)\rho_d + \Gamma_f\rho_{f4} + \frac{3}{7}\Gamma_f\rho_{f3} \\
\dot{\rho}_d &= \frac{\Omega}{2}\rho_{s1} - \left(\frac{\Omega}{2} + \Gamma_d\right)\rho_d \\
\dot{\rho}_{f3} &= \frac{1}{28}\zeta\Gamma_d\rho_d - \Gamma_f\rho_f \\
\dot{\rho}_{f4} &= \frac{27}{28}\zeta\Gamma_d\rho_d - \Gamma_f\rho_f
\end{aligned} \tag{6.5}$$

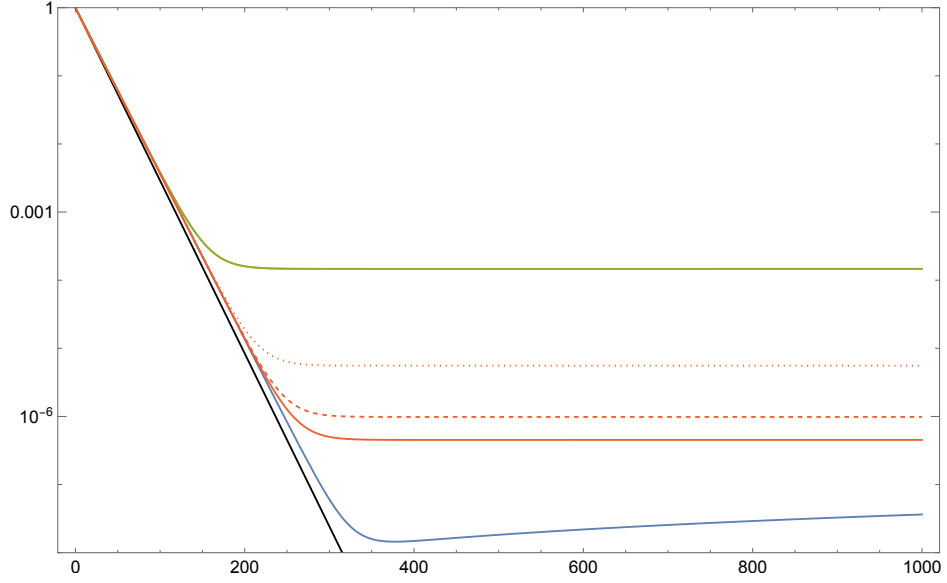


Figure 6.1: The rate of shelving into the ${}^2F_{7/2}^o$ state as a function of the shelving time in ms. The black curve shows the shelving rate when hyperfine structure is not considered. The blue curve includes the possibility of the F state decaying into the ${}^2S_{1/2}$ $F = 0$. The green curve shows the shelving rate when the M1 decay ${}^2D_{5/2} \rightarrow {}^2D_{3/2}$ is taken into account, limiting the ultimate fidelity. Finally, the red curves represent the effect of quenching the M1 decay with various strengths of quenching. The dotted line represents $\Omega = 1\text{kHz}$, the dashed line represents $\Omega = 10\text{kHz}$, and the solid line represents the limit of large Ω .

Here, we have ignored any terms due to off-resonant excitation of the atomic transitions – a safe approximation, as we can estimate the 411 nm scattering rate to be roughly 10^{-17} for the off-resonant state. The prefactors for the different hyperfine components of the decays are obtained by applying the repeated reduction formula (A.8) to the hyperfine-free decay rates using Fermi’s golden rule. The details of how these prefactors are derived can be instructive and are given in Appendix D.1.

When we solve this system numerically and examine the dynamics, we can see that we no longer have a stable fidelity equilibrium. Where once our F state lifetime reached an equilibrium with the shelving rate, we now have a situation in which we are *incredibly slowly* optically pumping into our $|0\rangle$ state. We see that a minimum appears at 379 ms, where this optical pumping curve intersects with our shelving curve. This yields a transfer infidelity of 1.3×10^{-8} . The dynamics of the five-level system discussed here are shown in Figure 6.1.

6.3 Deshelving

In order to remove population from the F state after it has been shelved, we employ a 760 nm transition to the $^1[3/2]_{3/2}$ state. This state decays strongly back to the $^2S_{1/2}$ state. Once again, as this process is one of optical pumping, it is incoherent. Figure 6.2 shows an example of this deshelling process, which in our experiment generally takes roughly 1-2 ms. This rate is largely limited by the narrow linewidth of the 760 nm transition, and could be considerably improved by better stabilizing our 760 nm lasers, for instance on an optical cavity.

6.4 SPAM Measurement

In order to test the projected fidelity of the F-state shelving SPAM protocol, an experiment must be performed that carries out the protocol. Due to the extremely small infidelities

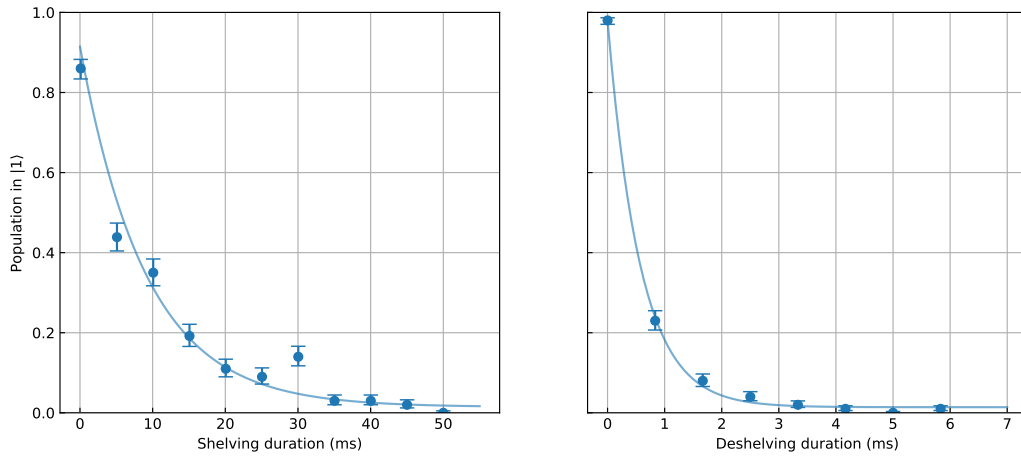


Figure 6.2: Examples of shelving rate and deshelling rate experiments. During the shelving rate experiments, narrowband optical pumping using the 411 nm light was performed for various amounts of time, and then population in the ground state was measured. During deshelling, the ion is optically pumped into the $^2F_{7/2}^o$ state, and then 760 nm deshelling time is varied.

we are trying to measure, the experimental design becomes somewhat complicated, as small drifts during the experiment can significantly affect the final results.

6.4.1 Experimental Procedure

6.4.1.1 State Preparation

This first part of any SPAM protocol is SP – the state preparation. In our case, we exploit the matched g-factors of the $^2S_{1/2}$ $F = 1$ state and the $^2D_{5/2}$ $F = 3$ state, meaning that we need a high-fidelity way to prepare the $|1\rangle$ state. The canonical way to perform this operation in ytterbium is to use microwaves to perform the transfer, and this has been reported in calcium to an infidelity of 10^{-7} in the literature [HAB14, SLM24]. In order to place limits on our measurement fidelity, thorough characterization of our state preparation fidelity is required.

Optical pumping Before using microwaves to transfer our qubit to the $|1\rangle$ state, we perform the standard optical pumping described in Section 4.3.1. The dynamics and limits of the conventional optical pumping that we employ are discussed in great detail in [Rom21], and the theoretical minimum infidelity that we can achieve through conventional optical pumping is approximately 10^{-5} . However, experimental sources of error, such as imperfect carrier suppression in the fiber EOM and various sources of RF pickup, likely limit our optical pumping fidelity to well below this.

After conventional optical pumping, we performed additional optical pumping steps to ensure the quality of our preparation of the $|0\rangle$ state. This *quadrupole optical pumping* step was performed following normal optical pumping, serving as an extra measure against state preparation errors. The basic principle relies on the fact that the error of the optical pumping process is due to off-resonant couplings of the 369 nm laser to the $|0\rangle$ state:

$$1 - \mathcal{F}_{OP} = \frac{2}{1 + \left(\frac{2\Delta}{\Gamma}\right)^2} \quad (6.6)$$

where Γ is the linewidth of the optical pumping transition and Δ is the ground state hyperfine splitting. Consequently, if we use a narrower-linewidth laser to perform optical pumping, we can achieve a lower ultimate optical pumping infidelity, although we require an additional repump laser to quickly return the atom to the ground state.

A prime candidate for this ground state narrowband optical pumping is the 411 nm laser that is already being used for shelving, combined with 976 nm laser that drives the $^2D_{5/2} \rightarrow ^3[3/2]_{3/2}$ transition, which rapidly decays back down to the ground state. As this excited state also has a small branching fraction to the $^2F_{7/2}^o$ state, the 760 nm repump lasers are kept on during quadrupole optical pumping to suppress this decay. The 435 nm transition to the $^2D_{3/2}$ state, repumped to the $^3[3/2]_{1/2}$ state provides an additional option with fewer unwanted decay channels, at the cost of the added complexity of another narrow-linewidth laser.

During the shelving experiments, this quadrupole optical pumping step was performed for an additional 10 ms following conventional optical pumping. Given the measured QOP time constant of roughly 0.2 ms [Rom21], this should significantly suppress optical pumping error to roughly the 10^{-8} level, leaving the remaining errors largely caused by experimental imperfections such as RF pickup affecting AOMs and EOMs.

Randomized Benchmarking In order to gain a sense of the fidelity of our single-qubit gates, we employed a randomized benchmarking (RB) protocol [KLR08]. Such a protocol allows one to isolate the effect of SPAM error from error due to single-qubit gates, and it is this single-qubit gate error that we are interested in measuring. An RB sequence consists of an alternating sequence of Clifford gates (i.e. 90 degree rotations on the Bloch sphere) and Pauli gates (i.e. 180 degree rotations on the Bloch sphere). The axes of the gates are selected at random, and the final gate in the sequence is selected such that it deterministically populates either the $|0\rangle$ and $|1\rangle$ state. Examining how the fidelity of the overall operation scales with sequence length allows for the separation of SPAM error and single qubit gate

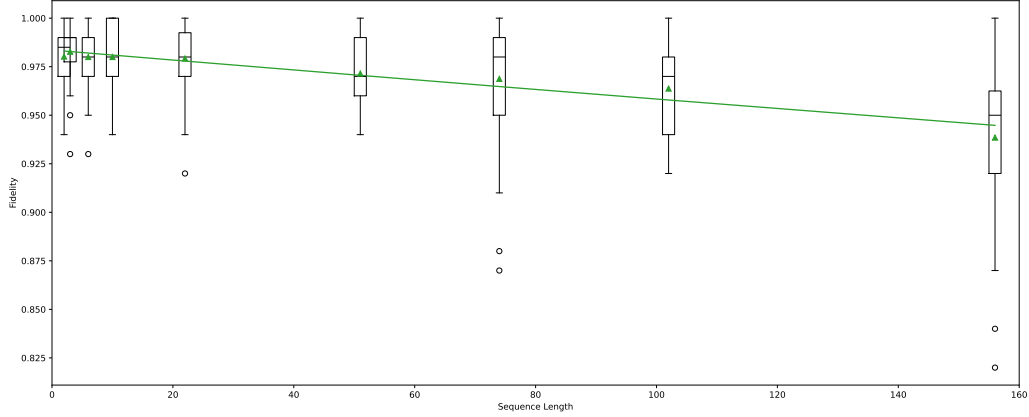


Figure 6.3: An example result of a randomized benchmarking experiment. 100 randomized gate sequences were used for each sequence length, and each sequence was run 100 times. Sequence lengths are Fibonacci numbers. The green triangles represent the mean values at each sequence length, and the line represents an exponential fit.

error.

Figure 6.3 shows the randomized benchmarking experiment we performed. The x-intercept corresponds to the SPAM during the experiment, and the rate of the exponential decay of the fidelity as a function of sequence length corresponds to the gate fidelity. The ultimate single-qubit gate infidelity that we measure using this technique is $8(2) \times 10^{-5}$, with a SPAM infidelity of $9.1(7) \times 10^{-3}$.

6.4.1.2 Shelving

Following state preparation, shelving is performed on the $^2S_{1/2}F = 1 \rightarrow ^2D_{5/2}F = 3$ transition in order to selectively outcouple the $F = 1$ qubit manifold into the F state. The equal g factors for these two states mean that the Zeeman structure is not resolved, relaxing the linewidth requirements of our 411 nm laser. During the fidelity experiment, shelving was performed for 200 ms.

6.4.1.3 State Detection

Once the qubit is shelved into the F state, state detection is performed in the ground state to check for remaining population. Since the $|1\rangle$ state is shelved, there is no risk of mixing the qubit, and so the full Doppler cooling set of tones is used during state detection. This allows for arbitrary state detection durations, and during the shelving experiment, state detection was performed for 17 ms. State detection should generally be sufficiently long to ensure that the $^2D_{5/2}$ state has time to decay.

6.4.1.4 Deshelving

Following state detection the qubit was deshelled using the 760 nm lasers in order to reinitialize for the next experiment. This process was performed for 35 ms, after which the ion was Doppler cooled and the experimental sequence could begin again.

6.4.1.5 Calibrations

Due to the long spans of time required in order to gather sufficient statistics during the shelving experiment, it is important to perform regular calibrations to maintain the correct values of the experimental parameters.

Wavemeter calibration As all of our red lasers are software-locked to the wavemeter, it is important that the wavemeter accuracy is maintained. To do this, the wavemeter was automatically recalibrated to our 780 nm rubidium saturated absorption setup. This setup consists of a 780 nm diode laser which is PDH locked to one of the 780 nm hyperfine transitions in the ^{87}Rb D2 line.

This wavemeter calibration is of special importance to the lock of the 822 nm Ti:Sapph laser that was used to generate the 411 nm laser throughout this experiment. As the shelving transition is narrow, small drifts in this laser lock could affect shelving rates and cause

experimental errors.

Microwave calibration Another source of error due to the long duration of the experiment is the small variations in the amplitude and frequency of our applied microwaves. These occur largely due to slight changes in room temperature and humidity. In order to compensate for these drifts, the experiment was paused every 10 minutes and a microwave calibration was performed. This calibration consisted of a Rabi flopping experiment to calibrate the π time, followed by a Ramsey experiment to calibrate the qubit frequency to the Hz level.

6.4.2 Data Analysis

6.4.2.1 Error processing

In order to minimize the effect of slow drifts in the data, the bright state and dark state data were interleaved every 100 experiments, such that any drifts occurring between the two were common mode.

Doppler Count Thresholding In addition to accounting for slow drifts, loss of ion integrity during the experiments often posed a problem. Occasionally while the ion is in the dark state, background collisions can cause the ion to be heated, causing one of two errors to occur:

- The collision affects the internal state of the ion, potentially deshelving and causing a bright ion during a dark state experiment
- The collision causes the ion to be heated to a point where it cannot be effectively Doppler cooled

In order to account for both of these effects, we performed measurements of the Doppler

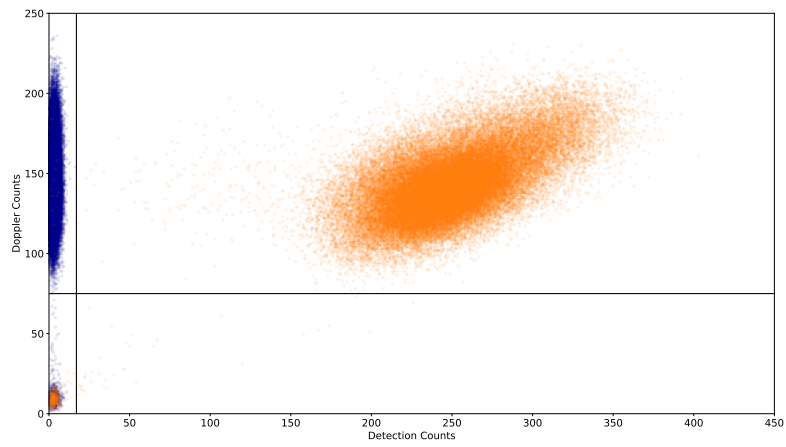


Figure 6.4: Doppler cooling and state detection photon counts plotted together on the same plot. The bright state ($|0\rangle$) histogram is shown in orange, while the dark state ($|1\rangle$) is shown in blue. The y-axis shows Doppler counts, with the 75-count threshold shown as the horizontal black line. Experiments whose associated Doppler counts below this threshold are discarded as recoiling or storage errors. Overall, 119000 experiments are plotted.

cooling fluorescence before and after each shelving experiment and conditioned our data on the success of the Doppler cooling process. If the Doppler cooling photon counts are anomalous, we discard the associated shelving experiment in our data analysis. Figure 6.4 shows the distribution of these Doppler cooling errors in our data, and indicates which experiments were kept and which were discarded.

In order to confirm that these Doppler cooling-related errors are due to events that occur during the experiment, rather than an intrinsic part of our shelving protocol, we performed an auxiliary experiment in which the ion was left in the trap for the same duration as the shelving experiment, but without 411 nm light applied. The Doppler error rates with and without the 411 nm light similar, allowing us to conclude that these errors are not intrinsic to the shelving process.

6.4.2.2 Blinding

Following data collection, the discriminator used to distinguish the bright state and the dark state needed to be set. In order to prevent bias from entering the decision (e.g. setting the threshold to maximize the fidelity for our specific experiment), we employed a blinding process to the data. While deciding on the threshold position, only 10% of the data was available to us.

The Doppler threshold was set using the blinded data, and the final state detection threshold were set using the blinded data. In each case, the Poisson parameter λ of the bright- and dark-state histograms was estimated by computing the average number of photons collected in each histogram. Due to the skewed shape of the bright state counts and the highly Poissonian nature of the bright state counts, the threshold was set to 5σ above the dark state mean, which in our case placed it at 17. 5σ was selected, as it would place the probability of dark state errors occurring during an ideal experiment below 5% given our number of trials.

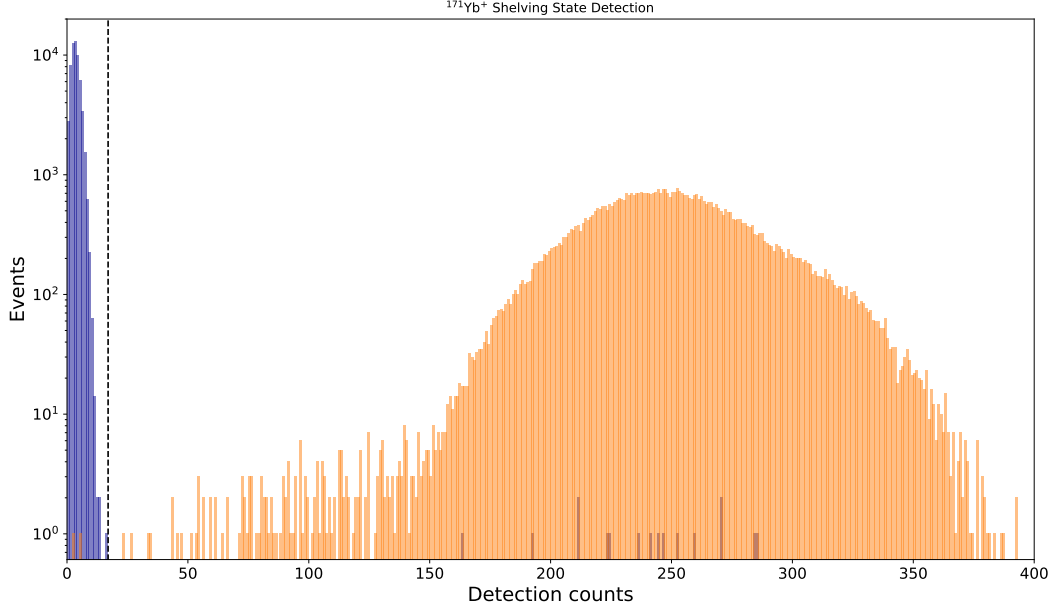


Figure 6.5: The bright and dark state state detection histograms of the shelving experiment following Doppler thresholding. The bright state ($|0\rangle$) histogram is shown in orange, while the dark state ($|1\rangle$) is shown in blue. The large separation between the two states shows that we can clearly distinguish the bright and dark states. The vertical dotted line sets the state detection threshold, set 5σ above the mean of the data in the blinded dark state.

6.4.3 Results of the Shelving Experiment

When the dataset was unblinded, the final SPAM infidelity for the experiment was $1.5^{+1.2}_{-0.7} \times 10^{-4}$. The full range of fidelities that we examined can be found in Table 6.4.3. The bright state infidelity is consistent with the infidelity of our preparation of the $|1\rangle$ state via the microwave state transfer detailed in Section 6.4.1.1. The source of the dark state errors can be largely attributed to the uncompensated M1 decay, which is further detailed in the following sections. More detail on the attribution of the errors is found in Table 6.4.3.

Processing	Bright State	Dark State	Overall Error ($\times 10^{-4}$)
Doppler filtering	$0.3^{+0.3}_{-0.2}$	$2.7^{+0.8}_{-0.6}$	$1.5^{+0.3}_{-0.4}$
No filtering	$2.8^{+0.8}_{-0.6}$	$88.3^{+3.9}_{-3.7}$	$45.6^{+2.0}_{-1.9}$

Table 6.1: The ultimate infidelity of the shelving experiments. Dark state and bright state error are given separately. After filtering out ion storage errors by thresholding the data based on Doppler counts, remaining error rates in the bright state are consistent with the infidelity of our $|0\rangle \rightarrow |1\rangle$ microwave transfer error.

Error Source	Dark State ($ 1\rangle$)	Bright State ($ 0\rangle$)
	Error ($\times 10^{-4}$)	Error ($\times 10^{-4}$)
$ 0\rangle$ state preparation	< 0.02	< 0.02
Unflagged ion loss error		$0.1^{+0.2}_{-0.01}$
$ 0\rangle \rightarrow 1\rangle$ transfer error	$0.74(1)$	
Finite shelving duration	$0.06(3)$	
$^2D_{5/2} \rightarrow ^2D_{3/2}$ M1 decay	$0.82(3)$	
Predicted average inaccuracy		$0.9^{+0.4}_{-0.1}$
Flagged ion loss		$2.9^{+0.6}_{-0.5}$
Predicted average infidelity		$2.4^{+0.4}_{-0.3}$

Table 6.2: Error budget for the shelving process, as predicted via various auxiliary diagnostic experiments. Here, we use *inaccuracy* to refer to the infidelity of the shelving process taken alone, while we use *infidelity* to refer to the overall infidelity of our shelving experiment. The final predicted inaccuracy is consistent with the inaccuracy we measured, shown in Table 6.4.3.

6.5 M1 Decay Rate

6.5.1 Observation

Inspecting Table 6.4.3, it can be observed that the predicted transfer infidelity alone is not sufficient to account for the error that we observe due to our shelving experiment. After further close inspection of the shelving process, we observe that one weak decay channel is not taken into account: the 7.28-micron M1 decay $^2D_{5/2} \rightarrow ^2D_{3/2}$. We can use equation 2.51 to estimate the rate of this decay.

The value of the square of the reduced matrix element for the transition then becomes (see Eq. 2.50)

$$\begin{aligned}
& \mu_B^2 \left| \langle ^2D_{3/2} || \mathbf{L} + g_s \mathbf{S} || ^2D_{5/2} \rangle \right|^2 = \\
& \mu_B^2 (2F' + 1)(2F + 1)(2J' + 1)(2J + 1) \left\{ \begin{matrix} J' & I & F' \\ F & 1 & J \end{matrix} \right\}^2 \\
& \times \left[\left\{ \begin{matrix} L & J & S \\ J' & L & 1 \end{matrix} \right\} \sqrt{(2L + 1)(L + 1)L} + g_s (-)^{J-J'} \left\{ \begin{matrix} S & J & L \\ J' & S & 1 \end{matrix} \right\} \sqrt{(2S + 1)(S + 1)S} \right]^2 \\
& = \frac{14}{5} (g_s - 1)^2 \mu_B^2
\end{aligned} \tag{6.7}$$

Plugging into equation 2.51, this yields a decay rate of $A_{M1} = 2\pi \times 4.5\text{mHz}$. Figure 6.6 shows the infidelity of the shelving as a function of shelving duration. The horizontal plateau that is apparent in the plot is consistent with an M1 decay rate of $A_{M1} = 2\pi \times 4.1_{-1.5}^{+2.3}\text{mHz}$, which is consistent with the predicted decay rate.

6.5.2 Suppression

6.5.2.1 Computation of M1 Decay Infidelity

We can model the effect of this M1 decay by adding the level to the five-level rate equation 6.5. If we add in this decay, we are left with the following set of rate equations:

$$\dot{\rho}_{s0} = \frac{2}{5}\Gamma_m\rho_m + \frac{4}{7}\Gamma_f\rho_{f3} \quad (6.8)$$

$$\dot{\rho}_{s1} = -\frac{\Omega}{2}\rho_{s1} + \frac{3}{5}\Gamma_m\rho_m + \left(\frac{\Omega}{2} + \zeta_s\Gamma_d\right)\rho_d + \Gamma_f\rho_{f4} + \frac{3}{7}\Gamma_f\rho_{f3} \quad (6.9)$$

$$\dot{\rho}_m = -\Gamma_m\rho_m + \epsilon\Gamma_d\rho_d \quad (6.10)$$

$$\dot{\rho}_d = \frac{\Omega}{2}\rho_{s1} - \left(\frac{\Omega}{2} + \Gamma_d\right)\rho_d \quad (6.11)$$

$$\dot{\rho}_{f3} = \frac{1}{28}\zeta_f\Gamma_d\rho_d - \Gamma_f\rho_{f3} \quad (6.12)$$

$$\dot{\rho}_{f4} = \frac{27}{28}\zeta_f\Gamma_d\rho_d - \Gamma_f\rho_{f4} \quad (6.13)$$

where ρ_m represents the population of the $^2D_{3/2}$ state, and ϵ is the branching fraction of the M1 decay from the $^2D_{5/2}$. Evaluating this system for a new equilibrium, we see that our infidelity is greatly increased, and our transfer fidelity is now limited to $\sim 7 \times 10^{-5}$.

If we choose to repump our $^2D_{5/2}$ state by applying 935 nm light, the story is much the same. Since we are repumping to a state with $J = \frac{1}{2}$, the ground state qubit still becomes mixed, and all we have accomplished is to reduce the effective lifetime of the $^2D_{3/2}$ state.

6.5.2.2 Computation of Quenching

We may, however, be able to suppress the M1 decay by repumping the $^2D_{3/2}$ state to a state with higher J , which cannot decay into the $^2S_{1/2}$ $F = 0$ state. The 861 nm transition to the $^1[3/2]_{3/2}$ $F = 2$ state provides us with such a repump. This state decays strongly (97.4%) to the $^2S_{1/2}$ $F = 1$ state via an E1 transition and is forbidden from decaying into the $F = 0$ by E1 selection rules.

If we wish to evaluate the rate equations including this state, we are forced to include several more states in our calculation. This is because the $^1[3/2]_{3/2} F = 2$ can decay into two states that could not previously be populated: the $^2D_{3/2} F = 1$ and the $^2D_{5/2} F = 2$. Since both of these states are able to decay into the $F = 0$ state, we are forced to include their effect. Our rate equations in the presence of this 861 nm light thus become:

$$\begin{pmatrix} \dot{\rho}_{s0} \\ \dot{\rho}_{s1} \\ \dot{\rho}_{f3} \\ \dot{\rho}_{f4} \\ \dot{\rho}_{m1} \\ \dot{\rho}_{m2} \\ \dot{\rho}_{d2} \\ \dot{\rho}_{d3} \\ \dot{\rho}_{b2} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{4}{7}\Gamma_f & 0 & 0 & \frac{2}{5}\Gamma_m & \frac{3}{5}\zeta_s\Gamma_d & 0 & 0 \\ 0 & -\frac{\Omega_{sd}}{2} & \frac{3}{7}\Gamma_f & \Gamma_f & \Gamma_m & \frac{3}{5}\Gamma_m & \frac{2}{5}\zeta_s\Gamma_d & \zeta_s\Gamma_d + \frac{\Omega_{sd}}{2} & \eta_s\Gamma_b \\ 0 & 0 & -\Gamma_f & 0 & 0 & 0 & \zeta_f\Gamma_d & \frac{1}{28}\zeta_f\Gamma_d & 0 \\ 0 & 0 & 0 & -\Gamma_f & 0 & 0 & 0 & \frac{27}{28}\zeta_f\Gamma_d & 0 \\ 0 & 0 & 0 & 0 & -\Gamma_m & 0 & \frac{9}{10}\zeta_m\Gamma_d & 0 & \frac{3}{10}\eta_m\Gamma_b \\ 0 & 0 & 0 & 0 & 0 & -\Gamma_m - \frac{\Omega_{mb}}{2} & \frac{1}{10}\zeta_m\Gamma_d & \zeta_m\Gamma_d & \frac{7}{10}\eta_m\Gamma_b + \frac{\Omega_{mb}}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\Gamma_d & 0 & \frac{1}{5}\eta_d\Gamma_b \\ 0 & \frac{\Omega_{sd}}{2} & 0 & 0 & 0 & 0 & 0 & -\Gamma_d - \frac{\Omega_{sd}}{2} & \frac{4}{5}\eta_d\Gamma_b \\ 0 & 0 & 0 & 0 & 0 & \frac{\Omega_{mb}}{2} & 0 & 0 & -\Gamma_b - \frac{\Omega_{mb}}{2} \end{pmatrix} \begin{pmatrix} \rho_{s0} \\ \rho_{s1} \\ \rho_{f3} \\ \rho_{f4} \\ \rho_{m1} \\ \rho_{m2} \\ \rho_{d2} \\ \rho_{d3} \\ \rho_{b2} \end{pmatrix} \quad (6.14)$$

Where we have rewritten our system of equations as a matrix equation to be slightly more legible. Numerically solving this system is tricky due to the presence of the two different Rabi frequencies of our 369 and 861 beams, but by making the same saturation assumption as we did in equation 6.3, we can eliminate the 369 Rabi rate and simplify the problem. The solutions for this set of rate equations are plotted in Figure 6.1. With the quenching applied with enough power to saturate the transition, we can suppress the infidelity induced by this M1 decay to roughly 4.5×10^{-7} , an improvement of two orders of magnitude.

6.5.2.3 Experimental Observation of Quenching

Following the shelving experiment, we sought to test the hypothesis that our limited SPAM fidelity was caused at least in part by this weak M1 decay channel. In order to do this, we borrowed light from a Ti:Sapphire laser from a neighboring experiment tuned to the 860 nm transition to the $^1[3/2]_{3/2}^0 F = 2$ state discussed above. Due to the extremely time-consuming

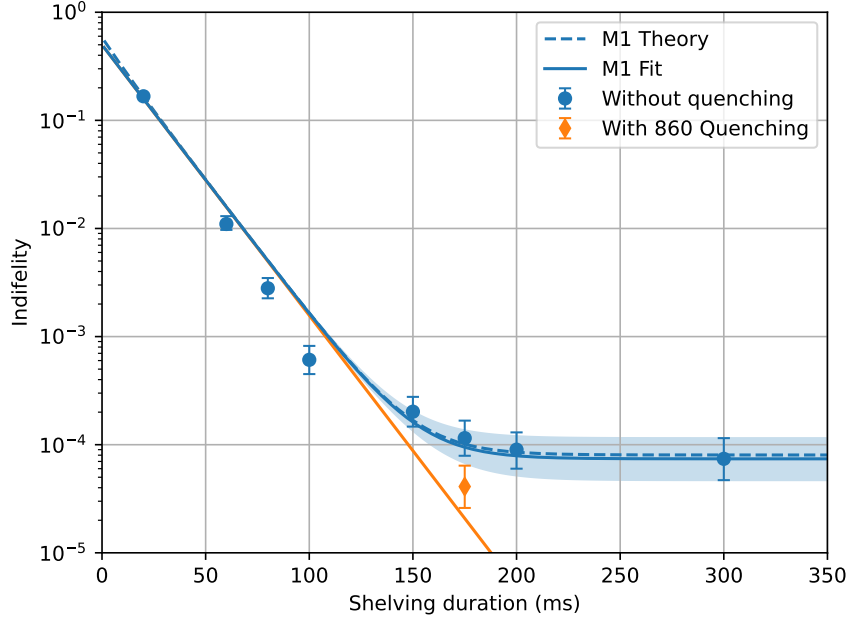


Figure 6.6: Shelving infidelity as a function of the shelving duration. In the absence of any repump, the error at long timescales is dominated by the M1 decay $^2D_{5/2} \rightarrow ^2D_{3/2}$. These data are shown in blue. With the addition of an 860 nm laser that drives $^2D_{3/2} \rightarrow ^1[3/2]_{3/2}$ we observe significant quenching of the M1 decay channel, and observe infidelities below what was achievable without it.

nature of gathering sufficient data to measure the extremely small infidelity of a shelving operation and the uncooperative nature of the borrowed laser, we were only able to make a single measurement of the shelving rate with the quenching applied. Further exploration of this quenching effect using a purpose-built 860 nm diode laser would be a logical avenue for further exploration, although the amount of data required to characterize the infidelity would be immense.

Nonetheless, the data we took show a significant improvement in the fidelity compared to data taken in the absence of shelving. Figure 6.6 shows the infidelity of shelving as a function of shelving time in the absence of M1 quenching, as well as the results of our M1 quenching experiment. The infidelity of the shelving is clearly seen to level off at a value of $-40.8(2)$ dB (8.3×10^{-5}).

The measured infidelity corresponds to an M1 decay rate of $4.1^{+2.3}_{-1.5}$ mHz, which is consistent with the decay rate of 4.5 mHz predicted by equation 2.51.

When we apply the 860 nm light, the infidelity decreases to a value below any which was observed in the absence of the light, demonstrating a clear improvement in the infidelity. However, without further experimentation, the ultimate fidelity of shelving with a quenched M1 decay has not been experimentally demonstrated, although we can safely say that the infidelity will be less than -45 dB.

6.6 Conclusion

Ultimately, the shelving scheme in ytterbium has proven to be very successful at suppressing the SPAM error of the $^{171}\text{Yb}^+$ ground state qubit, and the ability to trade detection time for increased fidelity is a very useful tool.

6.6.1 Further Improvements to Transfer Fidelity

A different way to improve the fidelity of the shelving operation is to use multiple π pulses to effect the transition from $|0\rangle$ to $|1\rangle$. Utilizing the fact that there exist three Zeeman sublevels in the $F = 1$ manifold, we can use three π pulses – one to each sublevel – to transfer population into the $F = 1$ prior to shelving. Because these sublevels are distinct, this process can be performed without mixing the qubit, allowing us to suppress our state transfer infidelity by a factor of the cube of the infidelity.

6.6.2 Speedup

Ultimately, one of the main drawbacks of the narrowband optical pumping scheme is that it is very slow compared to essentially every other readout scheme employed in trapped-ion experiments. Moreover, this speed is fundamentally limited by the structure of the ion: as long as optical pumping is occurring, the rate-limiting step of the process will be the decay of the $^2D_{5/2}$ state. However, there do exist schemes to transfer the ion into the F state without relying on optical pumping. Although the fidelity of these schemes is likely limited by technical limitations, a combination of these schemes with optical pumping can potentially lead to a significant reduction in state transfer time without compromising state transfer fidelity.

If we want to get around the dissipative speed limit set by the decay rate of the $^2D_{5/2}$, we will have to rely on coherent operations in order to populate the F state. The most obvious way to achieve this goal is to directly drive the E3 transition between the S state and the F state. While this 467nm laser has been used in atomic clocks [HSL16] and in searches for King plot nonlinearities [CHA20], the extremely narrow linewidth of the transition makes driving it very difficult, generally requiring exotic laser stabilization methods such as cryogenic cavities.

Another more technologically accessible method for coherently populating the F state

is to use a combination of 411 nm light and 3.4 micron light. While a 3.4 micron laser is technologically difficult to create, it is possible to generate using four-wave mixing of two high-power lasers, one at 812 nm, and one at 1064 nm [TRB98]. With both of these lasers available and sufficiently stabilized to perform coherent operations, two schemes are possible.

The first two-laser method leverages the relatively long lifetime of the $^2D_{5/2}$ state and performs a double-pulsed state transfer, in which a π pulse to the D state is performed using the 411 nm laser, followed immediately by a 3.4 micron π pulse to the F state. This two-stage transfer has been experimentally demonstrated to a fidelity of 0.97 [YMW22]. Alternatively, a two-laser Raman scheme could be employed, which has the advantage of never populating the $^2D_{5/2}$ state. However, the difficulty of phase-stabilizing two lasers at such different frequencies may make this scheme difficult. In either case, successful state transfer can be heralded if being used for state preparation of the metastable qubit.

In the two-laser schemes, fast deshelling can also be performed, using a combination of the 3.4 micron and 976 nm lasers to deshelve via two electric dipole transitions, rather than using the 760 nm electric quadrupole transition. This is the scheme employed in [YMW22].

6.6.3 Use in a register

One of the great advantages of electron shelving is that the quantum information in shelved ions is insensitive to the light used for state readout. Because of this, it is possible to utilize shelving for efficient readout of a qubit register. In this scheme, ions that are not being read out have their quantum information coherently shelved into the F state, insulating them from the readout. Even incoherently, ions can be shelved one at a time at the end of a long computation to reduce the crosstalk caused by the limited fidelity of imaging.

One potential hazard for this scheme, however, is that a 3.4 micron photon emitted during a shelving event could in principle deshelve nearby qubits. Since the spacing of qubits is roughly on the order of the wavelength of the light, superradiant effects could occur that

enhance the probability of such absorption. Should such an effect turn out to be problematic, it could likely be mitigated by utilizing light shifts or adjusting the spacing of the ions.

6.6.4 Related SPAM work

While this work was being performed, we were by no means the only group exploring the possibility of using the F state in ytterbium as a quantum computing resource. Edmunds et al. at the University of Sydney performed very similar experiments to ours, using the F state to suppress the infidelity of trapped ion state detection to $1.8(2) \times 10^{-3}$ [ETM21]. Yang et al. at Tsinghua University have performed great work using a 3.4 micron laser to implement coherent two-state population transfer between the ground state and the metastable state [YMW22].

After leaving our group, Anthony Ransford continued onward to Quantinuum, where he, along with An et al, employed a similar shelving technique to that discussed here to $^{137}\text{Ba}^+$, using narrowband optical pumping and cabinet shelving to obtain a SPAM infidelity of $9.0 \pm 1.3 \times 10^{-5}$ [ARS22]. The techniques used in that work are generally applicable to qubit species with $I > 1/2$, and could potentially be employed in $^{173}\text{Yb}^+$ to obtain extremely high SPAM fidelity.

Finally, Sotirova et al. at Oxford University employed a combination of the techniques described in this chapter and an inventive heralding protocol to obtain heralded SPAM infidelities of $8(4) \times 10^{-6}$ in the ground state qubit and $5(4) \times 10^{-6}$ in the metastable qubit [SLV24].

CHAPTER 7

The Metastable Qubit in $^{171}\text{Yb}^+$

Utilizing the hyperfine structure in the F state, we can also define a 3.6 GHz hyperfine clock qubit between the $F = 3, m_F = 0$ state and the $F = 4, m_F = 0$ state. The ability to define such a metastable qubit has many advantages, which are explained in great detail in [ACC21].

7.1 Metastable qubit SPAM

The metastable qubit can be dissipatively prepared using narrowband optical pumping, exactly as was performed for the shelving experiment detailed in Section 6.4. Using the $^2D_{5/2} F = 2$ state as the intermediate this process incoherently populates the $F = 3$ manifold in the $^2F_{7/2}^o$ state. A microwave π pulse can then be applied resonant with the $F = 3, m_F = 0 \leftrightarrow F = 4, m_F = 0$ transition, followed by 760 nm light which depopulates the $F = 2$ manifold. After this process, the ion is either in the $^2F_{7/2}^o F = 3 m_F = 0$ state or the ground state, and can be projected into the metastable state by performing state detection in the ground state manifold.

This heralding process necessarily discards a large number of experimental shots, as only a third of the experiments will result in successful state preparation. a Magnetically-Assisted Optical Pumping scheme [ARS22] could be performed prior to heralding to reduce this overhead, although the experimental complexity would be somewhat increased.

Since the readout of the metastable qubit occurs by outcoupling the qubit to the ground

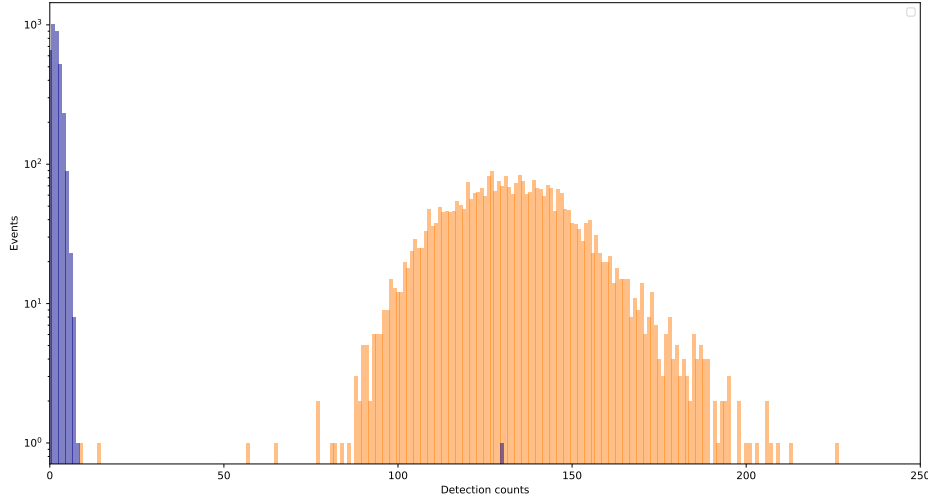


Figure 7.1: Results of a SPAM experiment in the metastable qubit showing high SPAM fidelity after successful heralding of the $|1\rangle_m$ state, followed by successful heralding of the $|0\rangle_m$ state. The first heralding step eliminates infidelity due to imperfect transfer to the metastable manifold, and the second eliminates infidelity due to pulse area error in the π pulse. The ultimate SPAM infidelity is $1.5^{+2.4}_{-0.9} \times 10^{-4}$

state via a narrow channel, the SPAM fidelity that can be achieved is very high. By heralding the preparation of the $|1\rangle_m$ state, we achieve an overall SPAM infidelity of $6^{+2.0}_{-1.5} \times 10^{-3}$. By adding an additional heralding step following the π pulse to prepare the $|0\rangle_m$ state, we see a further improvement in the fidelity of the metastable SPAM, giving us an infidelity of $1.5^{+2.4}_{-0.9} \times 10^{-4}$. Figure 7.1 shows the results of a SPAM experiment performed in this way in the metastable state.

7.1.1 Dissipative operations

The metastable qubit has the unique ability to be read out without destroying the quantum information stored within. We can consider schemes in which we use this property to

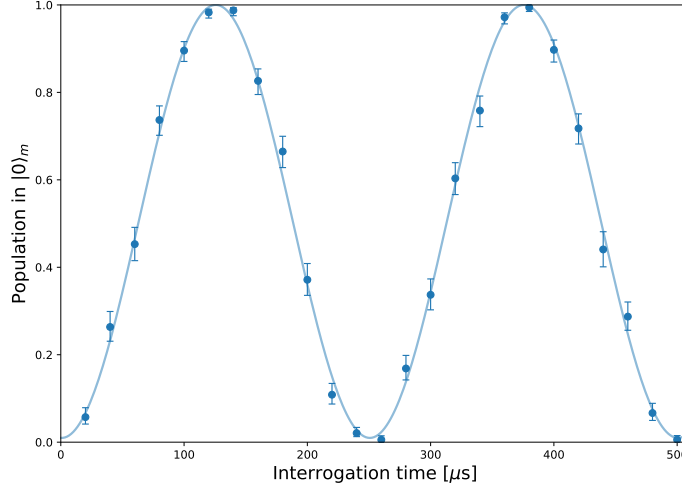


Figure 7.2: Rabi oscillations in the metastable manifold. Oscillations were driven using an ex-vacuo microwave horn antenna, and SPAM was performed using heralding as described in the text

our advantage. For instance, it had been proposed in reference [Cam20] to use this weak outcoupling as a method to flag errors during a quantum computation, and in general non-destructive mid-circuit measurement is an important part of many error correction protocols [NMM14, HRW07].

7.1.2 Measurement-driven gates

One interesting demonstration of our ability to perform non-destructive projective measurements of the qubit is in using the act of measurement to drive a single-qubit rotation in the metastable state. Fundamentally, the concept is straightforward: performing a projective measurement in a given basis projects the qubit into an eigenstate of that basis. We may, therefore, by rotating our measurement basis in many small increments, effectively cause the qubit state to rotate. This is essentially the same classic demonstration as that of crossed polarizers, in which adding more polarizers between the crossed polarizers causes the propor-

tion of transmitted light to increase. The protocol we employ to perform this demonstration is as follows:

1. Prepare the $|1\rangle_m$ state
2. Measure the qubit in a basis that is rotated by θ/N with respect to the σ_z eigenbasis
3. Repeat the measurement N times, further rotating the measurement basis by θ/N each time
4. Measure the final state

As the number of measurement steps N increases, the success rate of the measurement-driven gate scales as $\cos^{2N}(\frac{\pi}{2N})$, approaching 1 as $N \rightarrow \infty$. Figure 7.3 shows the success rate of a measurement-driven π rotation, as well as an additional experiment showing the protocol executed with the same microwave rotations, but without the intermediate measurement steps.

7.2 Two-Qubit Gates

In order to consider performing entangling gates in the metastable qubit, we would like to examine potential candidate states for light shift gates, as demonstrated in reference [BBF21]. Such states should be connected to the metastable qubit via a quadrupole transition, and ideally one in which scattering errors will not leak into the ground state manifold. A number of candidate transitions are depicted in Figure 7.4. Work to probe the 860 nm transition to the $^3[7/2]_{9/2}^o$ state is being performed in our group by Patrick McMillin, Hassan Farhat, and Will Liu at the time of this thesis' submission.

An alternative scheme for two qubit gates is to use a Raman transition that scatters off of an even-parity $J_1 J_2$ coupled state. While many of these transitions are deep into the

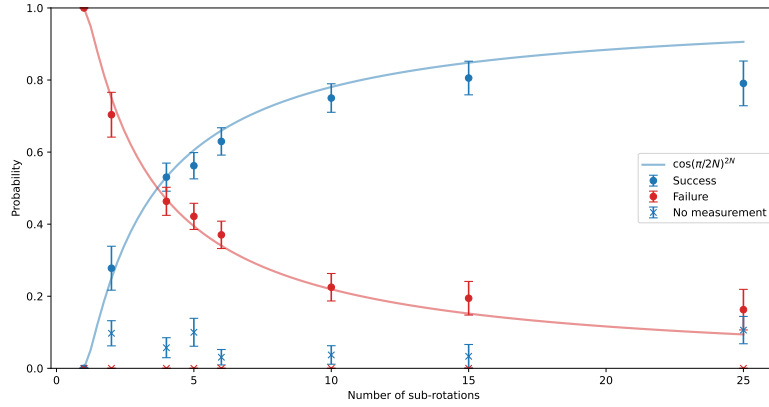


Figure 7.3: Success probability of a measurement-driven π rotation as a function of the number of steps taken. As the number of steps grows, the probability of success increases. In addition, the x markers show that in the absence of the measurement step, the rotation does not occur.

ultraviolet, a few transitions appear to have more reasonable wavelengths, namely those to the $(7/2, 1)$ fins structure manifold.

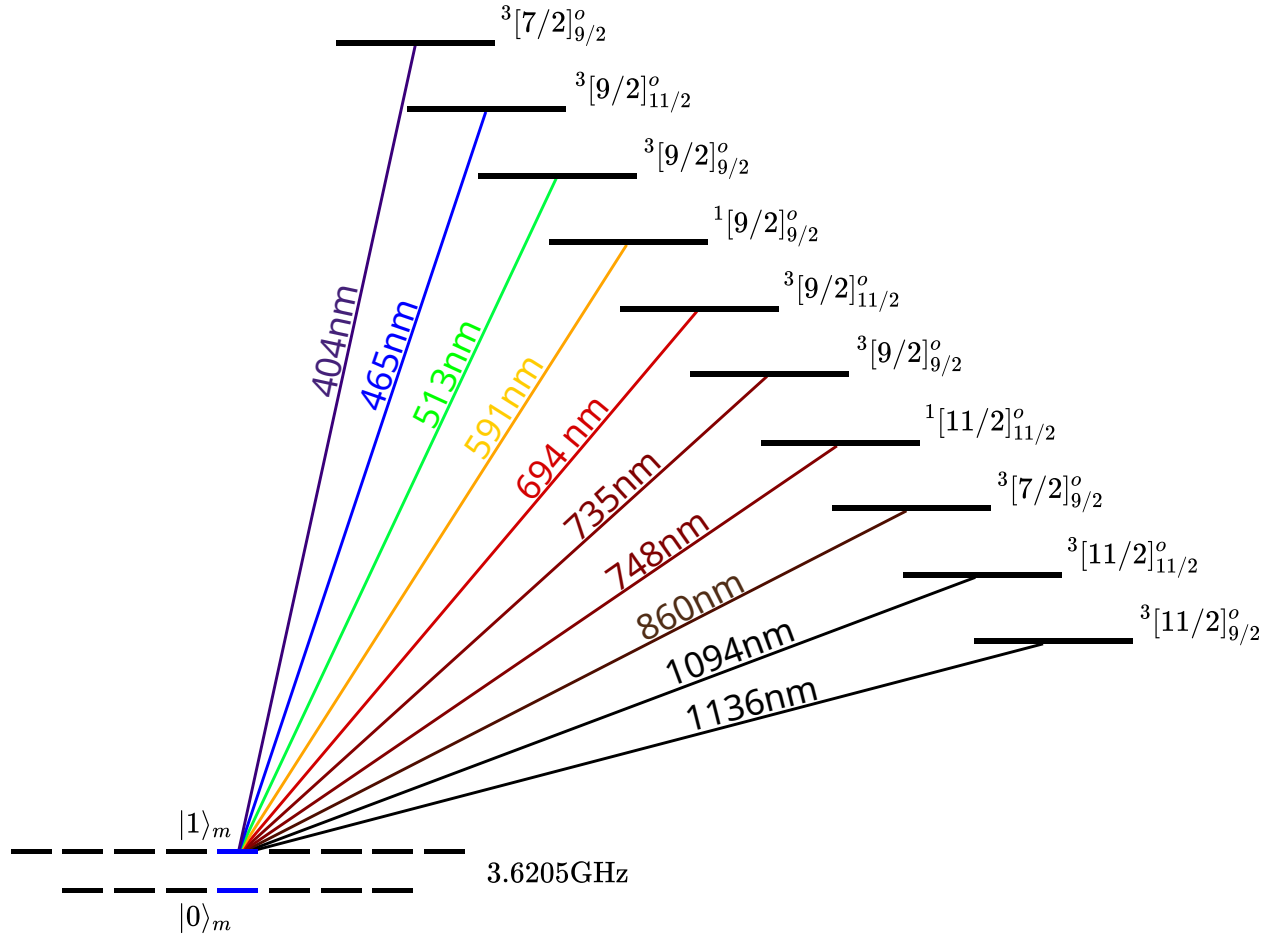


Figure 7.4: Candidates for light shift gate wavelengths from the metastable qubit in $^{171}\text{Yb}^+$. A number of quadrupole transitions to excited J_1K coupled states are possible.

CHAPTER 8

Magic Polarization

8.1 Theory of Magic Polarization

8.1.1 The Incorrect, Approximate Approach

Without appealing to any kind of fully-hyperfine Hamiltonian, we can still estimate the approximate value of the critical magnetic field. In addition, I find that this rough estimation of the magic polarization lends a degree of intuition without requiring that we diagonalize the hyperfine and light shift Hamiltonians at the same order.

8.1.1.1 No magnetic field

In the absence of magnetic field, we can write the differential light shift as originating due to this slight difference in detuning between the $|^2S_{1/2}F=1, m_F=0\rangle \equiv |1\rangle$ and the $|^2S_{1/2}F=0, m_F=0\rangle \equiv |0\rangle$ states:

$$\Delta E_{diff} = -\frac{|\langle 1|\mathbf{d}|^2P_{1/2}\rangle|^2 E^2}{4\Delta} - \left(-\frac{|\langle 0|\mathbf{d}|^2P_{1/2}\rangle|^2 E^2}{4(\Delta + \omega_{HF})} \right) \quad (8.1)$$

where ω_{HF} is the hyperfine splitting between the two states, Δ is the detuning of the light shift laser from the $|1\rangle \rightarrow |^2P_{1/2}\rangle$ transition. We can simplify this expression by factoring

out the angular momentum dependence of the matrix elements:

$$\langle 1|d_q|^2P_{1/2} F=1, m_F=q\rangle = (-)^{1-q}3 \begin{pmatrix} 1 & 1 & 1 \\ 0 & q & -q \end{pmatrix} \begin{Bmatrix} 1/2 & 1/2 & 1 \\ 1 & 1 & 1/2 \end{Bmatrix} \langle ^2S_{1/2}||d||^2P_{1/2}\rangle \quad (8.2)$$

$$= 3\frac{\mp 1}{\sqrt{6}}\frac{-1}{3}\delta_{q,\mp 1} \langle ^2S_{1/2}||d||^2P_{1/2}\rangle \quad (8.3)$$

$$\equiv \pm d \quad (8.4)$$

$$\langle 0|d_q|^2P_{1/2} F=1, m_F=q\rangle = (-)^{1-q}\sqrt{3} \begin{pmatrix} 0 & 1 & 1 \\ 0 & q & -q \end{pmatrix} \begin{Bmatrix} 1/2 & 1/2 & 1 \\ 1 & 0 & 1/2 \end{Bmatrix} \langle ^2S_{1/2}||d||^2P_{1/2}\rangle \quad (8.5)$$

$$= \sqrt{3}\frac{1}{\sqrt{6}}\frac{1}{\sqrt{3}}\delta_{q,\mp 1} \langle ^2S_{1/2}||d||^2P_{1/2}\rangle \quad (8.6)$$

$$\equiv d \quad (8.7)$$

Since the matrix elements are equivalent up to a sign, we can write

$$\Delta E_{diff} = -\frac{d^2 E^2}{4\Delta} - \left(-\frac{d^2 E^2}{4(\Delta + \omega_{HF})} \right) \approx -\frac{d^2 E^2}{4} \frac{\omega_{HF}}{\Delta^2} \quad (8.8)$$

Here, we have made the assumption that the detuning Δ red of the fine structure transition. We see that the differential light shift is always negative. We can get an intuitive grasp of this by noting that the light shift generally manifests itself as a repulsion between the two states being mixed by the perturbing light. Since the upper state lies closer to the $^2P_{1/2}$, it experiences a greater repulsion and so slightly shrinks the splitting between the two hyperfine sublevels.

8.1.1.2 Introducing a magnetic field

If we introduce a magnetic field, the eigenstates of the atomic Hamiltonian split slightly. This is of course well known as the Zeeman effect, but perhaps less appreciated is the fact

that the atomic eigenstates are slightly mixed by the magnetic interaction. For a much more thorough handling of this interaction, see section 2.3, but in short, the atomic Hamiltonian in the presence to a magnetic field for a $^2S_{1/2}$ state is

$$H = \begin{pmatrix} 0 & \mu_B B \\ \mu_B B & \omega_{HF} \end{pmatrix} \quad (8.9)$$

Diagonalizing this Hamiltonian yields new eigenvectors and eigenvalues:

$$|\tilde{0}\rangle \approx |0\rangle - \frac{\mu_B B}{\omega_{HF}} |1\rangle \quad (8.10)$$

$$|\tilde{1}\rangle \approx |1\rangle + \frac{\mu_B B}{\omega_{HF}} |0\rangle \quad (8.11)$$

$$E_0 \approx -\frac{\mu_B^2 B^2}{\omega_{HF}} \quad (8.12)$$

$$E_1 \approx \omega_{HF} + \frac{\mu_B^2 B^2}{\omega_{HF}} \quad (8.13)$$

In order to compute the differential light shift, we can now consider the effect of polarization in the interaction. Where previously there was no quantization axis to lift the degeneracy between σ_+ and σ_- transitions, we can now write down the matrix elements of σ_+ and σ_- transitions separately:

$$\langle ^2P_{1/2} \ F = 1, m_F \pm 1 | d_{\pm} | \tilde{1} \rangle = \langle P, 1, \pm 1 | d_{\pm} | 1 \rangle + \frac{\mu_B B}{\omega_{HF}} \langle P, 1, \pm 1 | d_{\pm} | 0 \rangle \quad (8.14)$$

$$= \pm d + \frac{\mu_B B}{\omega_{HF}} d \quad (8.15)$$

$$= \pm d \left(1 \pm \frac{\mu_B B}{\omega_{HF}} \right) \quad (8.16)$$

$$\langle ^2P_{1/2} \ F = 1, m_F \pm 1 | d_{\pm} | \tilde{0} \rangle = \langle P, 1, \pm 1 | d_{\pm} | 0 \rangle - \frac{\mu_B B}{\omega_{HF}} \langle P, 1, \pm 1 | d_{\pm} | 1 \rangle \quad (8.17)$$

$$= d - \frac{\mu_B B}{\omega_{HF}} (\pm d) \quad (8.18)$$

$$= d \left(1 \mp \frac{\mu_B B}{\omega_{HF}} \right) \quad (8.19)$$

where we have used the results from 8.7 to simplify the matrix elements. We can now compute the differential light shift in the presence of the $\sigma_{\pm}$ polarized light:

$$\Delta E_{B \neq 0, \sigma_{\pm}} = -\frac{|\langle \tilde{1} | d_{\pm} |^2 P_{1/2} \rangle|^2 E^2}{4\Delta} - \left(-\frac{|\langle \tilde{0} | d_{\pm} |^2 P_{1/2} \rangle|^2 E^2}{4(\Delta + \omega_{HF})} \right) \quad (8.20)$$

$$= -\frac{d^2 E^2 \left(1 \pm \frac{\mu_B B}{\omega_{HF}}\right)^2}{4\Delta} - \left(-\frac{d^2 E^2 \left(1 \mp \frac{\mu_B B}{\omega_{HF}}\right)^2}{4(\Delta + \omega_{HF})} \right) \quad (8.21)$$

$$\approx -\frac{d^2 E^2}{4\Delta} \left(1 \pm \frac{2\mu_B B}{\omega_{HF}}\right) + \frac{d^2 E^2}{4(\Delta + \omega_{HF})} \left(1 \mp \frac{2\mu_B B}{\omega_{HF}}\right) \quad (8.22)$$

$$\approx -\frac{d^2 E^2}{4} \frac{\omega_{HF}}{\Delta^2} \mp \frac{2\mu_B B}{\omega_{HF}} \frac{d^2 E^2}{4} \left(\frac{1}{\Delta} + \frac{1}{\Delta + \omega_{HF}} \right) \quad (8.23)$$

$$\approx -\frac{d^2 E^2}{4} \frac{\omega_{HF}}{\Delta^2} \mp \frac{\mu_B B}{\omega_{HF}} \frac{d^2 E^2}{\Delta} \quad (8.24)$$

$$= \Delta E_{B=0} \left(1 \pm \frac{4\mu_B B \Delta}{\omega_{HF}^2} \right) \quad (8.25)$$

From this expression, we can see that there exists a value of B for which the differential light shift completely disappears! Specifically, purely σ_- polarized light perfectly cancels the differential light shift at the critical magnetic field

$$B_c = \frac{\omega_{HF}^2}{4\mu_B \Delta} \quad (8.26)$$

If $B > B_c$, we are able to tune the polarization of the light such that the differential light shift is cancelled. We can additionally define the *helicity projection*, which parametrizes the imbalance between σ_+ and σ_- light, and therefore affects the relative contributions of the σ_+ and σ_- parts of equation 8.25.

8.1.1.3 Considering the spin-spin basis

It can be instructive to consider the magic polarization effect in the uncoupled spin basis of the nuclear spin and the electron spin. In this basis, we have

$$|0\rangle = \frac{1}{\sqrt{2}} (|^2S_{1/2} \downarrow_n \uparrow_e\rangle - |^2S_{1/2} \uparrow_n \downarrow_e\rangle) \quad (8.27)$$

$$|1\rangle = \frac{1}{\sqrt{2}} (|^2S_{1/2} \downarrow_n \uparrow_e\rangle + |^2S_{1/2} \uparrow_n \downarrow_e\rangle) \quad (8.28)$$

In this case, we may write the modified terms in the presence of a magnetic field (equivalent to expressions 8.19) as

$$|\tilde{0}\rangle = \frac{1}{\sqrt{2}} \left[\left(1 + \frac{\mu_B B}{\omega_{HF}}\right) |^2S_{1/2} \downarrow_n \uparrow_e\rangle + \left(1 - \frac{\mu_B B}{\omega_{HF}}\right) |^2S_{1/2} \uparrow_n \downarrow_e\rangle \right] \quad (8.29)$$

$$|\tilde{1}\rangle = \frac{1}{\sqrt{2}} \left[\left(1 - \frac{\mu_B B}{\omega_{HF}}\right) |^2S_{1/2} \downarrow_n \uparrow_e\rangle + \left(1 + \frac{\mu_B B}{\omega_{HF}}\right) |^2S_{1/2} \uparrow_n \downarrow_e\rangle \right] \quad (8.30)$$

Since the excited states that we couple to with $\sigma_{\pm}$ polarized light are $|^2P_{1/2} \uparrow_n \uparrow_e\rangle$ ($|^2P_{1/2} \downarrow_n \downarrow_e\rangle$), we see that only the right (left)-hand terms of equation 8.30 couple to the excited state, since the other term would require flipping the spin of the nucleus. In this picture, it becomes clear that the effect of the magnetic field is to slightly enhance the coupling of one of the states to the $^2P_{1/2}$, while slightly reducing the coupling of the other. Since the degree of this enhancement is tunable, the differential light shift between the two states is therefore also tunable.

8.1.1.4 Multiple excited states

In general, we are summing over terms with different linewidths, Equation 8.25 then becomes a weighted sum over the different states to which the light shift laser couples. While there isn't as neat an expression for the DLS in this case, in general only one term is dominant at a given laser frequency. It is also worth noting that the convenient equality in equation 8.7 is not guaranteed to hold for all manifolds, and that there is, in general, also magnetic field-induced state mixing within the excited state hyperfine manifolds. In this case, the expression for the DLS becomes a sum over many different terms. When $I = 1/2$, we can

assume that the excited state will always have two hyperfine sublevels and write the light shift of a single state with angular momentum F as [LSR13]

$$\Delta E = -\frac{E^2}{4} \sum_n \frac{\langle F|d|F_n\rangle \langle F_n|d|F\rangle}{\Delta_n} \quad (8.31)$$

When we account for field-induced state mixing in both the ground and excited states, we make the substitutions $|\tilde{F}_\pm\rangle = |F\rangle \pm \beta(\omega_{HF}) |F'\rangle$ and $|\tilde{F}_{n\pm}\rangle = |F_n\rangle \pm \beta(\omega_n) |F'_n\rangle$, where $\beta(\omega) = \frac{\mu_J B}{\omega}$ represents the degree of field-induced mixing given a hyperfine splitting ω , with the magnetic sensitivity μ_J defined as

$$\begin{aligned} \mu_J \equiv \mu_B g_J (-)^{2F+J+I+1} \sqrt{J(J+1)(2J+1)} & \begin{pmatrix} F & 1 & F' \\ 0 & 0 & 0 \end{pmatrix} \\ & \times \sqrt{(2F'+1)(2F+1)} \begin{Bmatrix} J & J & 1 \\ F' & F & 1/2 \end{Bmatrix} \end{aligned} \quad (8.32)$$

which is proportional to the off-diagonal matrix element of the Zeeman Hamiltonian, as defined in section 2.4.1.

After applying the substitution and discarding terms of order β^2 , we can now write the light shift of a state under this field-induced mixing as a sum of terms. For the upper state F'_+ :

$$\Delta E_{LS}(F'_+) \approx -\frac{E^2}{4} \sum_n |d_n|^2 \left[\frac{D_{F',F'_n}^{F_n,F_n} + 2\beta(\omega_{HF}) D_{F,F'}^{F_n,F_n} - 2\beta(\omega_n) D_{F',F'_n}^{F_n,F'_n}}{\Delta_{F',F_n}} + \frac{D_{F',F'_n}^{F'_n,F'_n} + 2\beta(\omega_{HF}) D_{F',F'_n}^{F'_n,F'_n} + 2\beta(\omega_n) D_{F',F'_n}^{F_n,F'_n}}{\Delta_{F',F'_n}} \right] \quad (8.33)$$

$$\Delta E_{LS}(F_-) \approx -\frac{E^2}{4} \sum_n |d_n|^2 \left[\frac{D_{F,F}^{F_n,F_n} - 2\beta(\omega_{HF}) D_{F',F}^{F_n,F_n} - 2\beta(\omega_n) D_{F,F}^{F_n,F'_n}}{\Delta_{F,F_n}} + \frac{D_{F,F}^{F'_n,F'_n} - 2\beta(\omega_{HF}) D_{F',F}^{F'_n,F'_n} + 2\beta(\omega_n) D_{F,F}^{F_n,F'_n}}{\Delta_{F,F'_n}} \right] \quad (8.34)$$

where

$$D_{F,F'}^{F_n,F'_n} \equiv (-)^{F+F'} \sqrt{(2F+1)(2F'+1)(2F_n+1)(2F'_n+1)} \times \\ \begin{pmatrix} F & 1 & F_n \\ 0 & q & -q \end{pmatrix} \begin{pmatrix} F' & 1 & F'_n \\ 0 & q & -q \end{pmatrix} \begin{Bmatrix} J_n & F_n & 1/2 \\ F & J & 1 \end{Bmatrix} \begin{Bmatrix} J_n & F'_n & 1/2 \\ F' & J & 1 \end{Bmatrix} \quad (8.35)$$

We may note that when $q = 0$, the magnetic-field-dependent terms cancel, as $D_{F,F'}^{F_n,F'_n}$ will always violate a selection rule.

We can evaluate the differential light shift between states $|F\rangle$ and $|F'\rangle$ by subtracting their light shift. We can note that all terms that depend on the hyperfine structure of the excited state cancel to first order. We then obtain

$$\Delta E_{diff} \approx \Delta E_{diff}(B=0) - \sum_n \frac{E^2 |d_n|^2}{\Delta_n} \frac{\mu_J B}{\omega_{HF}} \left(D_{F,F'}^{F_n,F_n} + D_{F,F'}^{F'_n,F'_n} \right) \quad (8.36)$$

If we only consider one excited state, we can simplify this further:

$$\Delta E_{diff} \approx -\frac{E^2 |d|^2 \omega_{HF}}{4\Delta^2} \left(D_{F',F'}^{F_n,F_n} + D_{F',F'}^{F'_n,F'_n} \right) - \frac{E^2 |d|^2}{\Delta} \frac{\mu_J B}{\omega_{HF}} \left(D_{F',F}^{F_n,F_n} + D_{F',F}^{F'_n,F'_n} \right) \quad (8.37)$$

Allowing us to write the critical field as

$$B_c \approx \frac{\omega_{HF}^2}{4\mu_J \Delta} \frac{D_{F',F'}^{F_n,F_n} + D_{F',F'}^{F'_n,F'_n}}{D_{F',F}^{F_n,F_n} + D_{F',F}^{F'_n,F'_n}} \quad (8.38)$$

We can check our math by applying equation 8.38 to the DLS of the ground state when coupling to the $^2P_{1/2}$ excited state. In this case we have the following nonzero terms:

$$\begin{aligned} D_{0,1}^{1,1} &= \frac{1}{6}q \\ D_{1,1}^{1,1} &= \frac{1}{6}|q| \\ D_{1,1}^{0,0} &= \frac{1}{6}\delta_{q,0} \\ D_{0,0}^{1,1} &= \frac{1}{6} \end{aligned}$$

When $B = 0$, q has no effect on the DLS in the limit that the upper state hyperfine splitting is ignored:

$$\begin{cases} \Delta E_{diff} \propto \left(\frac{1}{6\Delta} - \frac{1}{6(\Delta + \omega_{HF} + \omega_P)} \right) & q = 0 \\ \Delta E_{diff} \propto \left(\frac{1}{6\Delta} - \frac{1}{6(\Delta + \omega_{HF})} \right) & q = \pm 1 \end{cases} \quad (8.39)$$

While when $B \neq 0$, we have:

$$\Delta E_{diff} = \Delta E_{diff}(B = 0) - \frac{E^2 |d_o|^2}{\Delta} \frac{\mu_B B}{\omega_{HF}} \frac{1}{6} q \quad (8.40)$$

$$= \Delta E_{diff}(B = 0) \left(1 - \frac{4\mu_B B \Delta}{\omega_{HF}^2} q \right) \quad (8.41)$$

Which is the same expression as we obtained in equation 8.25.

If we wish to rewrite this in terms of the helicity of the light rather than in terms of q , we may define the helicity projection $A = -i(\hat{\epsilon} \times \hat{\epsilon}^*) \cdot \hat{z}$. We then have

$$\Delta E_{diff} = \Delta E_{diff}(B = 0) \left(1 - \frac{4\mu_B B \Delta}{\omega_{HF}^2} A \right) \quad (8.42)$$

8.1.2 Doubly-magic wavelengths

The magic wavelength effect describes in the previous section exists in principle for any hyperfine clock qubit. In the *omg* architecture [ACC21], quantum information can potentially be stored in both a metastable and a ground state manifold during the course of a quantum computation (even if one of the manifolds is generally left unpopulated). As such, it is worth evaluating whether there exist any *doubly-magic* wavelengths - that is, wavelengths for which both the ground state qubit and the metastable qubit exhibit cancellation of the differential light shift at the same magnetic field and polarization.

While the precise evaluation of such magic wavelengths would require fully hyperfine corrections to the atomic wavefunctions and are therefore outside of my computational ability, we can use the "experimentalist" model discussed in the previous section to gain an idea of where such magic wavelengths will exist and how they will behave. We will evaluate the

presence of double-magic wavelengths in both $^{133}\text{Ba}^+$ and $^{171}\text{Yb}^+$, as both are used in our group to demonstrate metastable qubit operations.

We will begin by examining the final form of the critical magnetic field:

$$B_c \sim \frac{\omega_{HF}^2}{4\mu_B\Delta} \quad (8.43)$$

We see that for a given manifold, B_c scales with the scale of the hyperfine splitting, so in general the critical magnetic fields will be separated by a factor of $(\omega_g/\omega_m)^2$ when the applied radiation is far from any atomic resonances. This means that it is unlikely for the two states to have simultaneous magic conditions in the presence of purely σ -polarized light unless their hyperfine splittings are comparable. In ytterbium, the hyperfine splittings differ by roughly a factor of 3, while in barium they differ by roughly a factor of 10. In both cases, we only really expect to see the critical fields cross when approaching an optical resonance in one of the two states. This is unfavorable, as any gates performed at these wavelengths would result in high scattering errors for quantum gates.

Figure 8.1 shows this effect. It is clearly evident that the mismatched hyperfine splittings largely preclude the possibility of far-off-resonant doubly magic wavelengths, although there is a candidate crossing in ytterbium at $779 \text{ THz} = 385 \text{ nm}$. At this frequency, we are 32 THz red-detuned from the 369 nm cooling transition in the ground state, and 9 THz red-detuned from the $^2F_{7/2}^o \rightarrow (7/2, 0)_{7/2}$ transition. For Raman gates in the ground state, this detuning would come with an associated Raman scattering error rate per π pulse of 2×10^{-6} , while in the metastable state the scattering error rate per π pulse is 2×10^{-8} [OIB07, BWV23].

8.1.3 Correctness

While the expressions for the differential light shift and critical magnetic field derived in the previous sections allow us to estimate the critical field, these calculations are, in general, inaccurate. This is because while they include the energy shifts of the states due to the hyperfine interaction, they do not include the modification of the wavefunctions that is

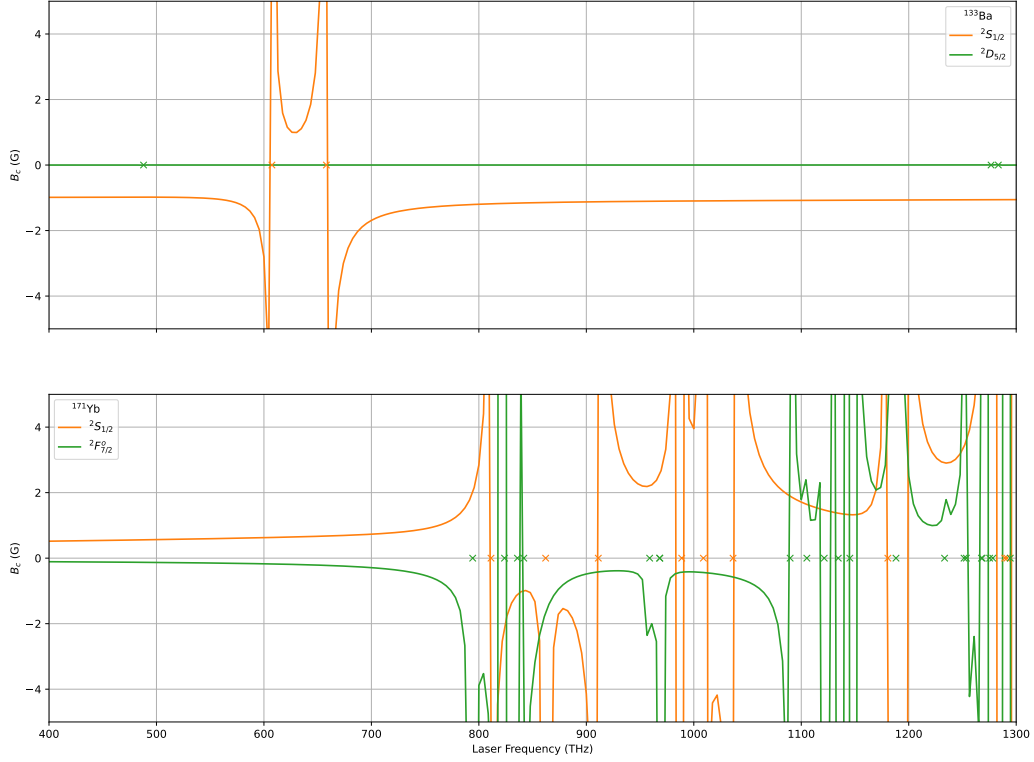


Figure 8.1: The critical magnetic field for the metastable and ground state qubits in both $^{133}\text{Ba}^+$ and $^{171}\text{Yb}^+$. The ω_{HF}^{-2} dependence of the critical field is clearly visible, where the critical magnetic field of the $2D_{5/2}$ state is strongly suppressed relative to the ground state, while in ytterbium the critical fields are comparable in magnitude. Crosses on the x axes represent the locations of transitions from the states in question.

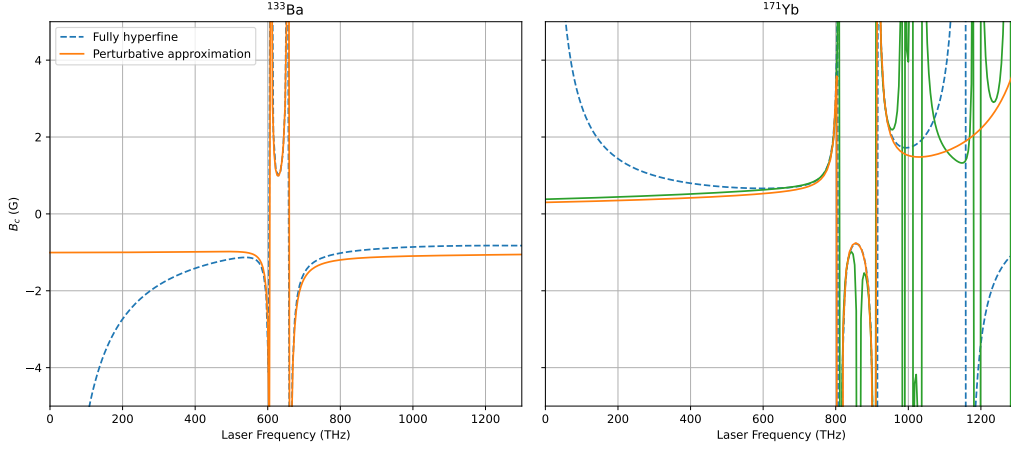


Figure 8.2: A comparison of the critical magnetic field B_c as computed in this thesis, and the differential light shift when the HFI-induced corrections to the wavefunctions are included. While there is good agreement near atomic resonances, the two results diverge in the large-detuning limit. In the green $^{171}\text{Yb}^+$ plot, more transitions were included when using the perturbative approximation than in the fully-hyperfine derivation, leading to additional resonances.

induced by the same interaction. Due to the fact that the hyperfine interaction and the light shift occur at the same order in the perturbation theory, this leads to inaccuracies in the computation of the differential polarizability, especially at large detunings. For a detailed discussion of the source of these inaccuracies, see [Der10], and for a worked out method for computing the differential polarizability using fully-hyperfine wavefunctions, see [RGD09].

In order to get a sense of the scale of the disagreement, we can plot our computed values for B_c against those computed using a formalism including the hyperfine-induced corrections to the wavefunctions. This is plotted for $^{133}\text{Ba}^+$ and $^{171}\text{Yb}^+$ in Figure 8.2. Given that our potential doubly-magic wavelength candidate is located near an atomic resonance, it may not be as badly affected by these inaccuracies compared to DLS occurring in the far-detuned limit.

8.2 Calibration

Prior to attempting to measure the magic angle effect in ytterbium, a number of calibrations needed to be carried out in order to ensure that the magnetic field was aligned to the laser and that we understood the polarization of our input light.

8.2.1 Magnetometry and Field Alignment

8.2.1.1 Magnetometry

The exact magnitude and direction of the magnetic field is required in order to extract the value of the critical field. Misalignment θ between the laser and the magnetic field leads to a reduction in the observed light shift by a factor of $\cos \theta$.

Fortunately, the ion itself is an excellent probe of the magnitude of the magnetic field at its location. By measuring the difference in frequencies between the $m_F = \pm 1$ qubits, one can determine the magnetic field. Measuring both the $+1$ and -1 qubits eliminates the quadratic Zeeman shift of the $|0\rangle$ state as a systematic. By scanning the current through each magnetic field coil, one can determine the properties of each coil. With a total of 9 scans, the full 3D configuration of the magnetic fields produced by each coil can be reproduced. A full description of this procedure is given in Appendix E.

Once the function mapping the coil currents to the magnetic field at the ion is determined, the magnetic field can be aligned to the light shift beam by tuning the magnetic field currents to maximize the magnitude of the light shift.

8.2.2 Polarimetry

After all the magic angle measurements were performed, we performed polarimetry on the 532 nm laser light, a step necessary for converting our wave plate angle to a polarization. Polarimetry was performed using a ThorLabs PAX1000VIS polarimeter, placed both before

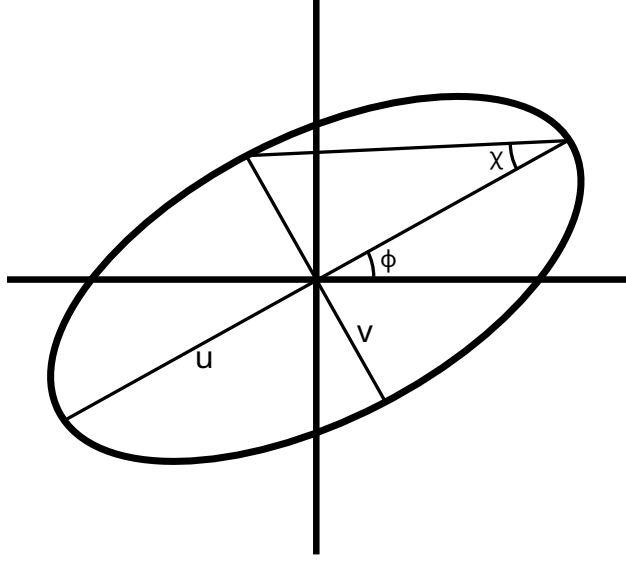


Figure 8.3: The polarization ellipse, with its various relevant parameters labeled.

and after the vacuum chamber to help compensate for any birefringence on the viewports. The two measured values were averaged, with the error taken as the difference between the two.

A note on the polarization ellipse When describing the polarization of a general polarized beam, one usually refers to the polarization ellipse. This ellipse is well-described in the literature, but there exist several different ways of parametrizing just how *elliptical* the ellipse is. Here, I will describe the two main ways one may describe this quantity: The *ellipticity*

$$\chi = \arctan\left(\frac{v}{u}\right) \quad (8.44)$$

And the *helicity*

$$A = -i(\hat{\varepsilon} \times \hat{\varepsilon}^*) \quad (8.45)$$

Specifically when working with magic polarization data, we wish to convert from the ellipticity provided by the polarimeter to the helicity, which is the form used in equation 8.42.

In order to do this, we may begin with the general form for a vector in the Jones calculus:

$$|\psi\rangle = \begin{pmatrix} \sin \theta e^{i\alpha_x} \\ \cos \theta e^{i\alpha_y} \end{pmatrix} \quad (8.46)$$

From [Col05], we may write the ellipticity as

$$\sin(2\chi) = 2|E_x||E_y| \sin(\arg(E_x E_y^*)) \quad (8.47)$$

$$= 2 \sin \theta \cos \theta \sin(\alpha_x - \alpha_y) \quad (8.48)$$

We can similarly write out the helicity projection as

$$A = -i(\varepsilon_x \varepsilon_y^* - \varepsilon_x^* \varepsilon_y) \quad (8.49)$$

$$= -i(\cos \theta \sin \theta e^{i(\alpha_x - \alpha_y)} - \cos \theta \sin \theta e^{-i(\alpha_x - \alpha_y)}) \quad (8.50)$$

$$= 2 \sin \theta \cos \theta \sin(\alpha_x - \alpha_y) \quad (8.51)$$

Since these two statements are identical, we may state that $A = \sin(2\chi)$.

8.3 Experimental Procedure

8.3.1 Light Shift Measurement

In order to measure the light shift, we perform a microwave Ramsey experiment, in which we apply a $\pi/2$ pulse, followed by an interrogation time t , followed by another $\pi/2$ pulse and measurement. If we detune our $\pi/2$ pulses by a detuning δ , the resultant curve traces out the ion's oscillation between the $|1\rangle$ and $|0\rangle$ states, oscillating at a frequency δ . The presence of a differential light shift during the interrogation time shifts the frequency of the qubit, and thus the frequency of the oscillations. Throughout these experiments, δ was chosen to be 1 kHz red of the unperturbed qubit frequency. Prior to any change in the experimental configuration, the light shift was re-calibrated by reverting the magnetic field and quarter wave plate angle to a fixed position. From that setting, the addressing of the 532 nm light

shift beam was adjusted vertically using piezoelectric transducers on the last mirror, and horizontally by applying differential voltage to the trap end caps in order to translate the ion along the trap axis.

During a given light shift measurement, two pairs of measurements are interleaved – one with the light shift beam present, and one with the beam turned off. Interleaving the measurements in this way ensures that any drifts that occur during the execution of the experiment are common to both curves, with the added benefit of decreasing the average fluence of 532 nm light incident on the ion trap, reducing the risk of trap heating due to stray light. Subtracting the frequencies of the resultant Ramsey oscillations yields the light shift, and performing the experiment on the full range of wave plate angles and magnetic field amplitudes allows for extraction of the critical field.

8.3.2 Coherence Measurement

In order to measure the coherence of the ion, the same Ramsey experiment is performed as was described above, except without interleaving. However, the interrogation times are significantly longer. Due to this, experimental durations can exceed ten hours. During the course of these experiments, the cooling line center is automatically re-calibrated every 10 minutes in order to account for cavity drift that would otherwise result in loss of fidelity or in loss of the ion. The frequency of the microwave source was not calibrated with this regularity, as we are more interested in the envelope of the Ramsey oscillations than their specific frequency.

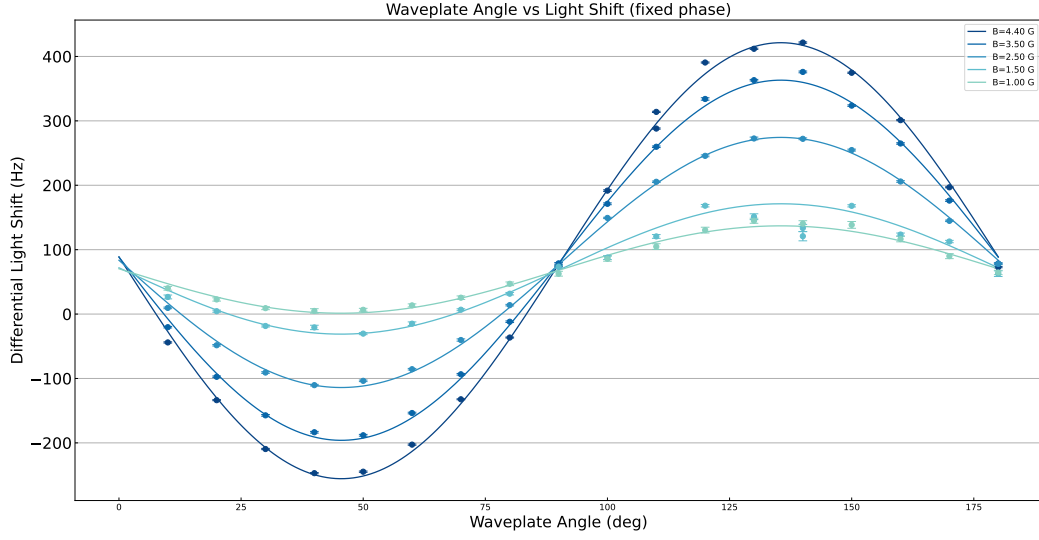


Figure 8.4: The differential light shift as a function of the angle of the quarter wave plate before our trap. As the wave plate is rotated, the light shift oscillates as the helicity of the light moves between σ_+ and σ_- polarization. Where the light shift crosses the x-axis, we have created the magic polarization. Worth noting is that as the magnetic field increases, the sensitivity of the magic polarization on the exact waveplate angle also increases. At $B = B_c$ the first-order sensitivity on the waveplate angle is 0.

8.4 Results

8.4.1 Magic Angle

After running Ramsey cans at the various magnetic fields and waveplate angles, we can plot the differential light shift as a function of the waveplate angle. Doing so reveals the zero crossings that correspond to the magic polarizations. These plots are displayed in Figure 8.4. As evidenced in the figure, there are clear zero crossings in all but one of the curves, indicating that a magic waveplate angle exists given a sufficient magnetic field. However, the shape of these curves alone does not permit us to make claims about the magic polarization. If the waveplate is imperfect, we cannot assume that the light at the maxima and minima of the curves is circularly polarized. As such, we have to plot the DLS against the polarimetry directly.

Another aspect worth appreciating about Figure 8.4 is the fact that with increasing magnetic field, the slope of the crossing also increases. The slope of the zero crossing related to the polarization sensitivity of the magic polarization, which is minimized at B_c . As the field is increased, it becomes more difficult to tune the polarization to the magic polarization. It is therefore desirable to operate the system at the lowest practicable magnetic field in order to make tuning the polarization of the light shift laser straightforward.

Figure 8.5 a shows the DLS plotted as a function of the helicity of the incident light. A strong linear relationship is evident, in which the x-intercept of the line indicates the magic polarization. Plotting the DLS in this way yields us the useful ability to determine the magic polarization at magnetic fields for which no zero crossing exists for any physical polarization. Such fields will have an x-intercept whose magnitude is greater than one.

Having extracted the magic polarization at each magnetic field value, we can find the magic polarizations to the function

$$A_{magic} = \frac{B_c}{B} \quad (8.52)$$

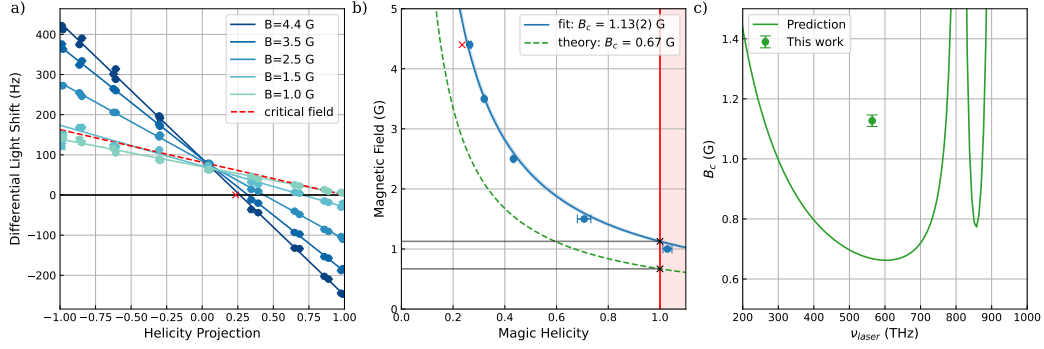


Figure 8.5: The differential light shift of the ground state qubit. a) shows the differential light shift as a function of the helicity projection $A = i(\varepsilon \times \varepsilon^*) \cdot \hat{z}$ of the applied light, as measured with a polarimeter. Where the light shift crosses the x axis, we have the magic polarization. The red dotted line represents the theoretical DLS vs helicity at $B = B_c$. In this case, the DLS is zero for pure σ_+ polarized light. b) shows the magic helicity as a function of the magnetic field. Fitting this curve to $B_c = B/A$ yields the critical magnetic field. The green dashed line represents the atomic structure theory prediction. In both plots a) and b), the red x signifies the configuration of the system during Figure 8.8. c) shows the atomic structure theory prediction as a function of the light shift laser frequency.

Magnetic Field (G)	Misalignment (deg)	Error due to Misalignment
4.41	0	0%
3.50	0.6	0.01%
2.50	1.8	0.05%
1.50	4.6	0.33%
1.02	8.0	0.97%

Table 8.1: Field misalignment error as computed by our magnetometry. As the measured value of the critical field scales as $\cos \theta$, the errors due to the misalignment are quite small.

to extract the critical magnetic field. This fit is plotted in Figure 8.5. Immediately evident is the large disagreement between the value of the critical field that we measured, and that predicted by atomic structure theory [Der10] discussed in section 8.1.3. It is worth noting that B_c represents the *minimum* magnetic field for which the DLS cancellation can be observed, and thus any experimental error will increase the observed value of B_c .

8.4.1.1 Systematics

Alignment of the laser and magnetic field A misalignment angle θ between the light shift beam and the magnetic field would result in an increase in the observed critical field by a factor of $1/\cos \theta$. Our magnetometry (see appendix E) allows us to estimate the magnitude of the misalignment, and these misalignments are given in Table 8.1. The misalignments that we observe are due to our taking the data prior to when we performed thorough magnetometry. While our better understanding of the magnetic fields around the ion could allow us to retake the data, the small error induced by the misalignment we have doesn't warrant this.

Addressing of the light shift beam Figure 8.6 shows the change in the scalar shift extracted from the fits in Figure 8.5 a. While we would expect that the scalar shift stays

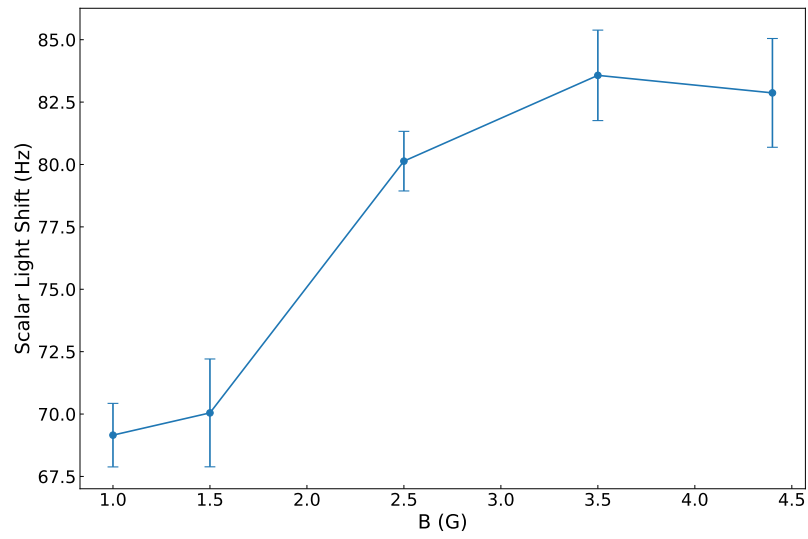


Figure 8.6: The scalar light shift as a function of the applied magnetic field. As the scalar light shift is theoretically independent of the magnetic field, this residual shift points at potential ion movement as we reduce our magnetic field.

constant due to its independence from the magnetic field and polarization, we observe a correlation between it and the magnetic field. When we change the magnetic field, we do observe the ion to move slightly, so it is possible that these small movements affect the addressing of the 532 nm beam. This could be mitigated by re-addressing the 532 nm beam to maximize the DLS for each field value. However, since the addressing of the light shift beam acts as an overall multiplier to the data, the addressing has no impact on the location of the zero crossing. As such, we can extract the critical magnetic field without worrying about this residual effect.

Error in helicity If there is significant birefringence in the vacuum viewports, it is possible that the ion experiences a different polarization than is indicated by our polarimetry. This is unlikely, as the degree in birefringence would have to be large, and would have to be nearly cancelled by the second viewport in order to cause significant error while reproducing the results of our polarimetry.

Additionally, small birefringence or imperfections in the waveplate can limit the purity of the linear polarization the ion experiences. This could result in a small residual shift being observed. The topology of the Poincaré sphere requires that the polarization be linear at **some** waveplate angle if we assume that the polarization is pure. These signs indicate any helicity error is likely small and can be ignored.

8.4.2 Coherence Improvement

Having determined the magic angle, we seek to evaluate the improvement of coherence that magic polarization can lead to. In order to do this, we perform a Ramsey experiment with the aim of observing the characteristic decay rate of the coherence as a function of the polarization of the light shift laser.

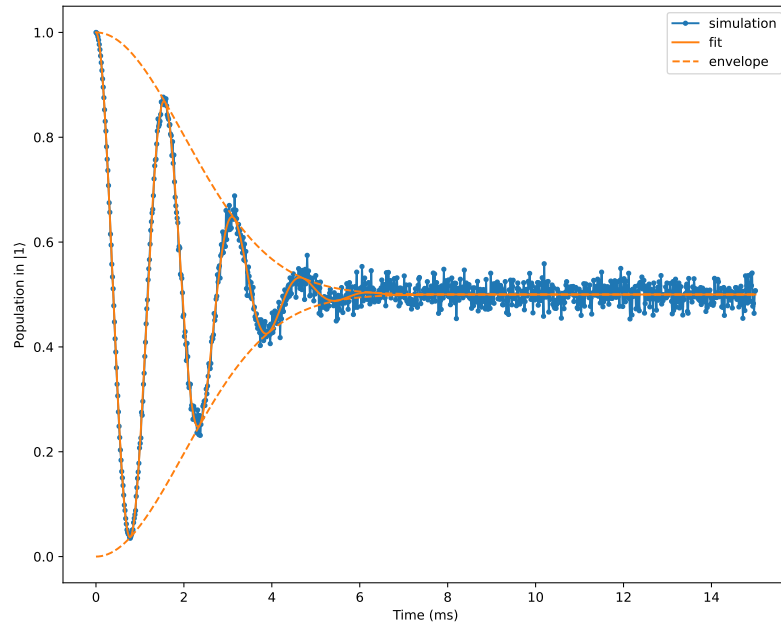


Figure 8.7: Simulated decoherence due to a Gaussian-distributed random light shift. Each simulated data point represents the average of 300 Ramsey experiments with their frequency shifted by a Gaussian random variable. The Gaussian decay formula 8.54 is in good agreement with the simulation.

Decay envelope In the presence of a perturbation to the frequency, the envelope of the decay is not necessarily exponential. We can model the influence of the noise on the light shift beam by assuming that the amplitude of the light varies as a Gaussian random variable. In that case, we expect the frequency of the Ramsey oscillation to vary as $\cos^2 \tilde{\omega}t = \frac{1}{2}(1 + \cos(2\tilde{\omega}t))$, where $\tilde{\omega}$ is a random frequency selected from a Gaussian distribution $\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(\omega-\omega_0)^2/2\sigma^2}$. We wish to evaluate the expectation value $\langle \cos 2\tilde{\omega}t \rangle$, in which case we need to evaluate the integral

$$\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{(\omega-\omega_0)^2}{2\sigma^2}} \cos 2\omega t \, d\omega \quad (8.53)$$

Solving this integral gives the expectation value

$$\langle \cos^2 \tilde{\omega}t \rangle = \frac{1}{2} + \frac{1}{2} \cos(2\omega_0 t) e^{-2t^2\sigma^2} \quad (8.54)$$

The standard deviation σ corresponds to the standard deviation of the light shift, and is therefore a reasonable metric which to evaluate the amplitude stability of the laser. Figure 8.7 shows data which is averaged with random frequencies sampled from a Gaussian distribution, and shows that the expectation value 8.54 is a good fit to the data. This sufficiently motivates us to use a Gaussian envelope when fitting the decays of our Ramsey fringes.

Coherence lifetime The shape of the decay envelope is not necessarily the same as the coherence lifetime of the quantum state. In order to obtain this decay, we wish to track the evolution of the density matrix under the influence of the perturbing laser light. The off-diagonal terms of this density matrix represent the coherence. These terms oscillate as $\sin(2\omega t)$. The decay envelope can therefore be expected to be the same as for the coherences, and numerical simulations support this. It is not necessarily straightforward to assign a coherence lifetime due to this process, as the decay is non-exponential. However, the timescale of the decay is set by the variance of the light shift.

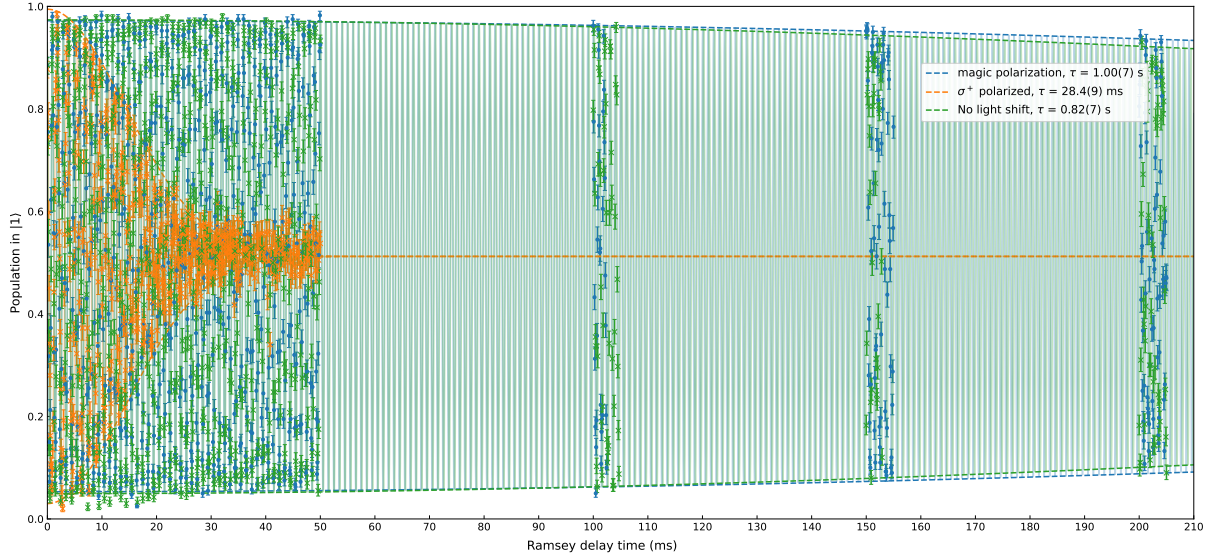


Figure 8.8: The coherence of the ground state qubit under different polarizations of the light shift beam. The green curve represents a control experiment in which no light shift beam is applied. The orange curve represents an experiment in which the light shift beam is tuned to σ_+ polarization, yielding a DLS of 430 Hz. Amplitude noise in the light shift laser causes dephasing of the qubit, resulting in decoherence within 30 ms. The blue curve represents an experiment in which the light shift beam is tuned to the magic polarization, in which no DLS is observed. The coherence of the qubit increases to be indistinguishable with the control.

8.4.2.1 Result

Figure 8.8 shows the decay of three Ramsey oscillations: one with the polarization tuned to be circular, one with the polarization tuned to the magic angle, and one with the 532 nm light turned off as a control. We observe a notable improvement of the characteristic time of the decay, from 28.4(9) ms to 1.00(7) s, which is comparable to the 0.82(7) s lifetime of the control decay. The decay of the control signal is dominated by other sources of decoherence, likely primarily magnetic field noise and the stability of the microwave oscillator across the roughly 12-hour timescale of the experiment.

If we want to probe longer coherence times, several experimental improvements could help. Increasing magnetic stability, either with active compensation or with magnetic shielding is commonly employed to extend coherence lifetimes [XL24]. In addition, disciplining our microwave oscillator to GPS or some other high-stability external reference would be useful, as would be using the same reference for both the Pulser (where the DDS signal is generated) and the HP parent oscillator (which generates the unmixed tone). Currently these two systems use different reference clocks, with the Pulser using a home-built 2 GHz reference clock, and the HP oscillator uses a 10 MHz rubidium reference.

If we wish to constrain the suppression of the differential light shift further without improving our experiment, we have to consider more than just the envelope. It is possible to evaluate the differential light shift by examining the phase difference that accumulates between the magic angle experiment and the control experiment.

In some sense, evaluating the residual light shift by looking at the frequency difference between the control and the magic angle scans is simply a way of evaluating the quality of the tuning we employ to find the magic angle in the first place. Prior to performing the coherence scan, the magic angle was found by tuning the quarter wave plate angle until no differential light shift was visible at a 50 ms interrogation time. The residual DLS was measured by examining the phase shift as a function of the interrogation time. Ultimately, we find that the residual differential light shift is $0.21(4)\text{Hz}$, a value that is likely limited by our polarization stability. This amount of residual DLS corresponds to a suppression of the DLS by a factor of roughly 2000 ± 300 .

8.5 Magic Polarization in the Metastable Qubit

Given the presence of the metastable $^2F_{7/2}^o$ qubit in $^{171}\text{Yb}^+$, there is good reason to investigate the presence of a magic polarization there as well. We can perform a Ramsey experiment in the metastable qubit to look for a differential light shift, analogously to the scheme described

for the ground state qubit. The only difference would be that each Ramsey experiment has to be preceded by a shelving step using 411 nm light and heralded state preparation, and followed by a deshelling step that selectively outcouples the $F = 4$ manifold back to the ground state for readout.

We may predict the critical magnetic field for the $^2F_{7/2}$ state using equation 8.25. When we do this, we predict that the critical magnetic field should be roughly 0.15 Gauss.

Experiments to observe this shift are ongoing in our lab at the time of this thesis' submission.

8.6 Applications

It is useful to consider why we are interested in examining this magic polarization effect in the first place. After all, this technique was first noted in optical lattice traps[LSP10], where trapped atoms are constantly being exposed to high-power beams that form the optical dipole traps. In trapped ion experiments, such constant exposure to high-power light is rare, and the optical power incident on the ion is usually small.

The exception to this case presents itself in the context of quantum computing and Raman entangling gates. Often in such systems, the gate is effected by a Raman transition involving two lasers: a global beam that is experienced by all ions, and an addressed beam that is directed at the specific ions that participate in the gate by a multi-channel AOM [SSL24, LZZ19, MWK24, WBD19]. In this case, whenever a Raman gate is being performed, all ions are subject to high-intensity laser light, regardless of whether they are taking part in the entangling interaction. As such, over the course of a long computation, this exposure to light could represent a significant source of dephasing.

Given state-of-the-art Raman gate durations, we can estimate the amount of dephasing that may be experienced over the course of a lengthy quantum computation due to amplitude noise on the Raman lasers. If we assume a Raman gate duration of roughly 100 μs [BHL16],

then we can infer that after 1000 gates, a spectator ion will be exposed to up to 100 ms of high-intensity laser light at intensities comparable to those demonstrated in this work. While amplitude stability requirements for performing high-fidelity are more stringent than the laser used in this work can meet, the dephasing over such long durations can still be significant. With an amplitude stability 10^4 times greater than used in this work, the overall dephasing error of a spectator qubit reaches 10% after 300 ms of illumination. It is therefore still likely desirable to shield spectator qubits from this dephasing over the course of a long gate set.

There are, of course, other ways to reduce spectator ion dephasing due to the influence of gate lasers, generally by sidestepping the problem altogether. Quantum gates can also be performed in a fully addressed way, in which ions that do not participate in the gate are not exposed to high-intensity laser radiation [SSL24]. Alternatively, gate schemes exist that do not use high-power Raman beams and instead use magnetic gradients to generate the spin-dependent force needed to effect an entangling gate [SBS19].

CHAPTER 9

Hyperfine Spectroscopy of $^{173}\text{Yb}^+$

9.1 Motivation

In the ytterbium ion trapping community, most of the focus has been placed on $^{171}\text{Yb}^+$. This is with good reason. State preparation and detection of the hyperfine clock qubit is straightforward, and the ion is a workhorse among many trapped ion experiments. However, the other spinful isotope of ytterbium, ^{173}Yb , has a great deal of potential in certain types of atomic physics experiments. While its high nuclear spin $I = 5/2$ makes state preparation and measurement more difficult, its hyperfine structure can be turned into an asset.

9.1.1 An Electron Microscope on the Nucleus

The nucleus is very small, and the vast majority of experiments that seek to investigate the structure of the nucleus do so via scattering experiments [KH87]. However, the structure of the atom also shows nuclear effects in the form of hyperfine structure. This perturbation of the electrons due to the nuclear multipole moments allows the electrons to act as a sensitive probe of nuclear structure, with the benefit that all of the spectroscopic techniques used in atomic physics are available to the experimenter.

9.1.2 Estimating the octopole moment

Using nuclear structure theory, we can place rough bounds on the nuclear octopole moment of a nucleus with a single valence nucleon using the following formula from [Sch55]

$$\Omega = \mu_N \langle r^2 \rangle \frac{3}{2} \frac{2I - 1}{(2I + 4)(2I + 2)} \times \begin{cases} (I + 2)[(I - \frac{3}{2})g_l + g_s], & I = l + \frac{1}{2} \\ (I - 1)[(I + \frac{5}{2})g_l - g_s], & I = l - \frac{1}{2} \end{cases} \quad (9.1)$$

For a single free valence neutron, $g_l = 0$ (since the neutron carries no charge), and $g_s = -2.2961127$. However, due to interactions within the nucleus, the gyromagnetic ratio of the neutron is quenched from its free-particle value. We can reasonably expect, however, that the octopole moment will fall within these so-called Schwartz limits. We can estimate the mean-squared nuclear charge distribution by $\langle r^2 \rangle \approx \frac{3}{5}R^2$, where R is the nuclear charge radius of 5.3046fm given in [AM13]. The overall calculation gives an estimate of $5.5b \times \mu_N$ for the nuclear octopole moment.

A more sophisticated calculation that included nuclear deformation was carried out by S. A. Williams, who predicted the nuclear octopole moment to have a much smaller value of $0.003b \times \mu_n$ [Wil62]. The origin of this large reduction in the estimated octopole moment is unclear, but appears to stem from a near cancellation in the angular momentum calculations performed in the work.

9.1.3 Previous experiments

The work that immediately motivates our decision to investigate trapped $^{173}\text{Yb}^+$ stems from an experimental disagreement between nuclear structure experiments that sought to investigate the nuclear magnetic octopole moment of ^{173}Yb .

9.1.3.1 Singh Measurement

The first experiment in question sought to investigate the nuclear structure of ^{173}Yb in great detail was performed by Singh et al [SAN13], who extracted the hyperfine structure of the $^3P_2^o$ state in neutral ytterbium by driving the 770 nm $^3P_2^o \rightarrow ^3S_1$ transition. Via the hyperfine structure of the transition, they extracted a nuclear quadrupole moment of

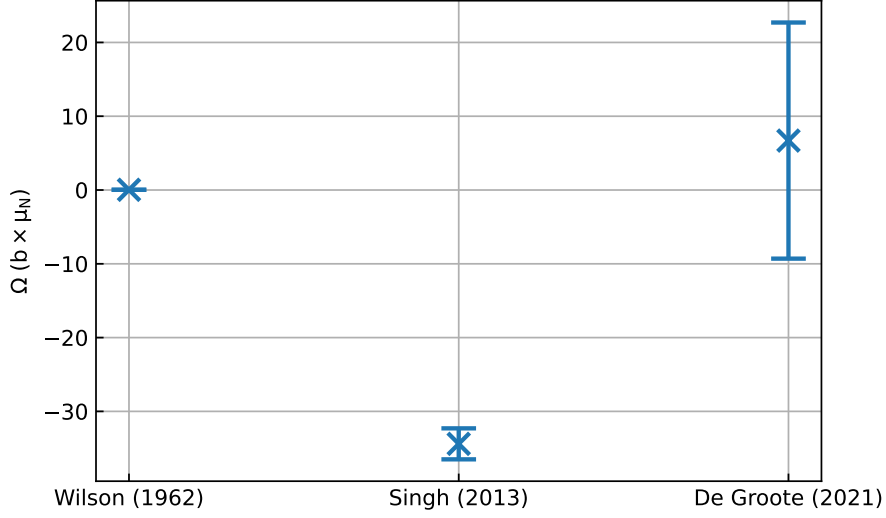


Figure 9.1: Previous determinations of the nuclear octopole moment Ω of ^{173}Yb .

2.46(12)b, and a nuclear octopole moment of $-34.4(21)b \times \mu_N$. The experimental precision of the spectroscopy was roughly 200 kHz, limited by the bandwidth of their laser locking hardware.

The value of the octopole moment was extracted using atomic structure calculations. Specifically, a Fock space Relativistic Coupled Cluster theory [MA10] was used in the computation of the ratio between the hyperfine C coefficient and the nuclear octopole moment.

9.1.3.2 De Groote Measurement

Following the measurement by Singh et al, De Groote et al. performed a similar probe of the nuclear magnetic octopole moment of ^{173}Yb , using collinear laser spectroscopy to investigate the same $^3P_2^o$ state [DKK21]. These transitions were probed: $^3P_2^o \rightarrow ^3P_1^o$ (415 nm), $^3P_2^o \rightarrow ^3P_2^o$ (399 nm), and $^3P_2^o \rightarrow ^2D_{5/2}^o$ (390 nm). All three transitions are forbidden but are power broadened to approximately 75 MHz. The experiment observes a hyperfine C coefficient of 0.03(7)MHz, consistent with zero. This would imply a nuclear octopole moment

of $6.7(160)\text{b} \times \mu_N$, by the calculations performed in the work.

The relationship between nuclear moments and hyperfine coefficients was performed using a Multi-Configuration Dirac Hartree-Fock method, using the GRASP2018 package. This is the same type of computation employed by [XLC20], in which the hyperfine structure of the $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$ is computed, and which is summarized in section 2.2.2.

9.1.3.3 Degree of disagreement

given the experimental uncertainties on their face, there seems to be a significant disagreement between these two experiments, with a 6σ discrepancy between the two results. When atomic structure calculations are taken into account, the picture becomes more muddy, closer to a 2σ disagreement: $-34.4(21)\text{b} \times \mu_N$ versus $6.7(160)\text{b} \times \mu_N$

While the raw measurements of the two experiments both appear sound, the internal consistency between the three separate measurements performed by de Groote et al. especially lend credence to their measurement of the hyperfine structure. However, it seems likely that a large factor in the discrepancy between the two experiments lies in the method by which the atomic structure computations were carried out. This, along with a summary of the measurements discussed here, is shown in Figure 9.1.

9.1.4 The Potential of $^{173}\text{Yb}^+$

The structure of the $^{173}\text{Yb}^+$ ion is very favorable for settling this experimental discrepancy. For a nuclear structure measurement to be performed, the state must have sufficient angular momentum to be sensitive to the nuclear multipole moments in question. For an octopole, moment, J must be greater than $\frac{3}{2}$. In addition, there must be a good scheme for measuring the hyperfine structure. The previous two experiments were both performed in atomic beams, and were limited in the linewidth they could achieve. While the resolutions attained are impressive – at the 1 MHz-100 kHz level – they are insufficient to place significant bounds

on the nuclear octopole interaction, which is expected to be 30 kHz or lower [XLC20].

The $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$ is an ideal candidate for investigating the nuclear structure of ^{173}Yb to a higher level of precision. The high angular momentum ($J = \frac{7}{2}$) of the state, means that the state will be sensitive to all higher-order nuclear multipole moments of the ^{173}Yb nucleus, up to the magnetic 32-pole moment. The 1.58-year lifetime [LPH21] of the state allows us to employ high-precision measurement techniques that are normally reserved for atomic ground states – namely Ramsey spectroscopy – which can resolve the atomic hyperfine structure to the Hz level.

With Hertz-level precision, it may even be possible to determine the electric hexadecapole contribution to the hyperfine structure, which is likely to occur at a similar level of precision to the measurement. It is worth noting that all electric terms in the hyperfine structure come due to nuclear deformations rather than the valence nucleons, and are therefore not well-described by the nuclear shell model.

9.2 Theoretical Predictions of the Hyperfine Structure

If we wish to evaluate the hyperfine structure of the F state, it is worth exploring the theoretical work that has been done to predict the hyperfine structure. Three research groups have made predictions about the matter.

In 2011, Petrasiusnas et al. published a spectroscopy paper in which they detailed the spectra of a number of states in the ytterbium ion [PSW11]. In it, they reference a computation of the hyperfine coefficients of the $^2F_{7/2}^o$ state performed by Dr. W. M. Itano using the multiconfiguration Dirac Hartree-Fock (MCDHF) method using the GRASP atomic structure code, the same methods used in ref [Ita06].

Dzuba and Flambaum subsequently published a prediction of the hyperfine coefficients in 2016 [DF16]. This computation was carried out using a relativistic Hartree-Fock approach using the configuration interaction.

Reference	A	B	C/Ω	D/Π
[PSW11]	-304	-3680		
[DF16]	-240	-4762		
[XLC20]	-241	-5061	-6×10^{-4}	2.35×10^{-4}

Table 9.1: Predicted hyperfine structure coefficients of the $^2F_{7/2}^o$ in $^{173}\text{Yb}^+$ reported in the literature. All values are in MHz.

Finally, in preparation for this work, Drs. Di Xiao, Jiguang Li, and Andrei Derevianko performed a full MCDHF calculation of the hyperfine structure of the $^2F_{7/2}^o$ state, using an extended version of the GRASP2018 atomic structure code [XLC20]. The estimated uncertainty for the cited values was 10%.

Table 9.2 and Figure 9.2 show the predicted frequencies of these three different calculations.

9.3 Experimental Constraints

Given the extremely long lifetime of the $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$, the main limit on the accuracy and precision of the hyperfine structure measurement will be due to systematics and experimental imperfections. Here, we will evaluate the most likely sources of inaccuracy and imprecision.

9.3.1 Magnetic Field Sensitivity

The main broadening mechanism in this experiment is expected to be the residual sensitivity these states have to magnetic field noise. This sensitivity comes from the fact that the dependence of the zero-field clock qubits on magnetic field is quadratic for small B . An estimation of the magnetic sensitivity of the various microwave transition in the F state is

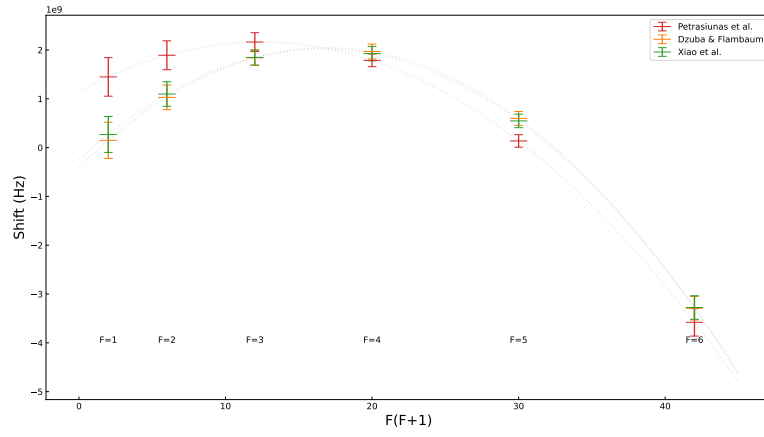


Figure 9.2: Predicted hyperfine spectra of the $^2F_{7/2}^o$ in $^{173}\text{Yb}^+$ reported in the literature. Reference [DF16] employed a Hartree-Fock method with the configuration interaction, while references [PSW11, XLC20] used an MCDHF method. Error bars represent 10% error on each of the hyperfine coefficients, as estimated in reference [XLC20]. 10% error is applied to the other two references, which did not provide error estimates.

derived in Section 2.4.2.

As the sensitivity of these states on external magnetic fields depends on the magnitude of the magnetic field, it is desirable to operate at as low a magnetic field as possible. Table [[Table - F State Clock Qubit Field Sensitivity]] shows the linewidth of the selected transitions assuming that we are operating at a magnetic field of 10 mG, with 5 mG of magnetic field noise. For all but two of the transitions, the broadening induced by magnetic field noise is expected to fall below 1 Hz.

Transition	Second-order sensitivity (kHz/G ²)	Linewidth at 10mG with 5mG noise
1 - 2	12.2	1.22
2 - 3	56.0	5.6
3 - 4	242.9	24.3
4 - 5	-4.8	0.48
5 - 6	-6.4	0.64

Table 9.2: The second-order magnetic field sensitivity of the five zero-field clock qubits in the $^{173}\text{Yb}^+ \ ^2F_{7/2}^o$ state, and the uncertainty in the qubit frequency that is induced by residual field noise at the planned magnetic field.

9.3.2 AC Zeeman Shift

In order to evaluate the AC Zeeman shift on the atomic energy levels, we may first start by noting the form of the AC Zeeman shift as [Hof14]

$$\Delta E_{\tilde{B}} = \frac{-\hbar\Omega_{\tilde{B}}^2}{4} \sum_j \frac{|\langle j|J_q|i\rangle|^2}{\omega_{ij} - \omega_{\tilde{B}}} \quad (9.2)$$

where q encodes the dependence of the Zeeman shift on the polarization of the magnetic field, and

$$\Omega_{\tilde{B}} = \frac{\mu_B g_F \langle \tilde{B} \rangle}{\hbar} \quad (9.3)$$

We wish to compute the effect of the AC Zeeman shift due to the secular motion and micromotion of the ion through static magnetic field gradients in the trap

To evaluate these gradients, we can investigate the shift in the frequency of the $F = 0, m_F = 0 \rightarrow F = 1, m_F = \pm 1$ transitions in $^{171}\text{Yb}^+$ as we move the ion a known distance using our trap electrodes. This direct measurement has yet to be performed, and for our error budget we will use a value of 50 G/m, as estimated using the geometry of our field coil.

We may then convert this into an RMS Zeeman shift if we know the amplitude of the motion in the trap. We can estimate the amplitude of the secular motion if we can estimate the temperature of the ion in the trap. We can take as fiducial a conservative value of 5 mK, which is approximately ten times higher than the Doppler limit. At this temperature, $\bar{n} \approx 500$, yielding an approximate motional amplitude of 1 micron along the z-axis, and 500 nm along the radial axes. If we instead assume a Doppler-limited temperature, the expected amplitudes are halved.

The amplitude of the micromotion is dependent on how far away the ion is from being well micromotion-compensated. As computed in section 5.2.1, the Mathieu q parameter in our trap is expected to be 0.12, which sets a rough lower bound on the micromotion amplitude at 50 nm. This corresponds $\langle \tilde{B} \rangle = 2.5 \mu\text{G}$. This yields $\Omega_{\tilde{B}} = 2\pi \times g_F \times 0.875 \text{Hz}$. In all cases, this results in an expected AC Zeeman shift of less than $1 \mu\text{Hz}$. For the AC Zeeman shift to exceed 1 Hz, The trap drive and the state splitting would have to be near-degenerate – within 1 kHz.

9.3.3 Quadratic Light Shifts

Although at the center of an ion trap the time-averaged electric field must be exactly zero, the same cannot be said for dynamic electric fields and static quadrupole electric fields.

The light shift due to the uncompensated electric field takes the form [DFB10]

$$\Delta E(\gamma, J, F, m_F, E) = -\frac{\langle E^2(t) \rangle}{4} \left(2\alpha_S(\gamma, J) + \alpha_T(\gamma, J, F) \frac{3m_F^2 - F(F+1)}{F(2F-1)} (3\cos^2 \beta - 1) \right) \quad (9.4)$$

where α_S and α_T are the scalar and tensor polarizability of the atom, respectively, and β is the angle between the electric field and the quantization axis. In order to evaluate the light shift due to the trap, we can write the time-averaged squared electric field seen by the ion as [BMB98]

$$\langle E_i^2(t) \rangle \approx \frac{m\Omega_{RF}^2 k_B T_i}{2e^2} \frac{a_i^2 + 2q_i^2}{2a_i + q_i^2} + 8 \left(\frac{q_i E_i}{2a_i + q_i^2} \right)^2 \quad (9.5)$$

In our trap, a_x and a_y are approximately 0, while q_x and q_y are approximately 0.1. In the limit that $a \ll q^2 \ll 1$, we may write

$$\langle E_i^2(t) \rangle \approx \frac{m\Omega_{RF}^2 k_B T_i}{e^2} + 8 \left(\frac{E_i}{q_i} \right)^2 \quad (9.6)$$

Given the small value of q , it is likely that the second term will dominate in the presence of even small stray electric fields.

We can get an order-of-magnitude estimate for the static polarizability of the $^2F_{7/2}^o$ state by inspecting the computed values for atomic ground states given in ref. [DFB10]. This gives a value of order of magnitude $10^{-10} \text{Hz}/(\text{V/m})^2$.

We can now evaluate the quadrupole light shift due to the trap. In the absence of uncompensated electric fields, the first term in 9.5 yields $\sum_i \langle E_i^2(t) \rangle \approx 2400 \text{V}^2/\text{m}^2$. Given our very rough estimate of the polarizability, this gives a rough shift of $0.1 \mu\text{Hz}$. We can therefore safely say that only the second term in equation 9.5 due to stray fields will matter. For this shift to be less than 1 Hz for a maximally high α of $10^{-9} \text{Hz}/(\text{V/m})^2$, we require that the stray fields are less than roughly 1000V/m . Given our trap dimensions, this corresponds to roughly 0.5 V placed on a single trap rod, a value that is significantly higher than the shim voltages that we currently apply.

9.3.4 Electric Quadrupole Shifts

An additional shift in the hyperfine structure of the F state will arise due to the coupling of the F state's electric quadrupole moment with the electric field gradient in the ion trap. In evaluating this effect, we will follow ref. [Ita01]. The Hamiltonian for this interaction is

$$H_Q = \nabla \mathbf{E}^{(2)} \cdot \mathbf{\Theta}^{(2)} \quad (9.7)$$

where $\mathbf{\Theta}$ is the electric quadrupole operator for the atom, and $\nabla \mathbf{E}^{(2)}$ is a symmetric traceless rank-two tensor describing the electric field gradient at the ion. The quadrupole moment of an atomic level is defined as stretched matrix element of the $q = 0$ component of the atomic electric quadrupole operator, $\Theta(\gamma, J) = \langle \gamma J J | \Theta_0^{(2)} | \gamma J J \rangle$. This moment has been measured in $^{171}\text{Yb}^+$ to be $\Theta(^2F_{7/2}^o) = -0.041(5)ea_0^2 = -1.84(22) \times 10^{-41} \text{C} \times \text{m}^2$ [HOL12].

In the natural axes of the ion trap (which is not necessarily aligned with the quantization axis), we may write the electric potential as

$$\phi(x', y', z') = V_0(x'^2 + y'^2 - 2z'^2) + \varepsilon(x'^2 - y'^2) \quad (9.8)$$

where the primed coordinates indicate that we are working in the frame of the ion trap, and not the frame in which the magnetic field is in the z -direction. The parameter ε encodes the asymmetry in the trapping potential. We can write the Hamiltonian 9.7 using the form of the gradient in the spherical basis given in ref [Ita01]. This yields

$$-2V_0\Theta_0^{(2)} + \sqrt{\frac{2}{3}}V_0\varepsilon(\Theta_2^{(2)} + \Theta_{-2}^{(2)}) \quad (9.9)$$

If we assume that the interaction is much smaller than the Zeeman interaction, we can consider this as a small perturbation in the $|F, m_F\rangle$ basis. Following ref [Ita01], we will write the matrix element of the Hamiltonian as

$$\begin{aligned} \langle \gamma J F m_F | H_Q | \gamma J F m_F \rangle = \\ - V_0 \langle F || \Theta^{(2)} || F \rangle \begin{pmatrix} F & 2 & F \\ -m_F & 0 & m_F \end{pmatrix} [(3 \cos^2 \beta - 1) - \epsilon \sin^2 \beta (\cos^2 \alpha - \sin^2 \alpha)] \end{aligned} \quad (9.10)$$

where α and β are the Euler angles that convert from the quantization axis frame to the ion trap frame. The reduced matrix element in the above expression is given by

$$\langle F || \Theta^{(2)} || F \rangle = (-1)^{I+J+F} (2F+1) \begin{Bmatrix} J & 2 & J \\ F & I & F \end{Bmatrix} \begin{pmatrix} J & 2 & J \\ -J & 0 & J \end{pmatrix}^{-1} \Theta(\gamma, J) \quad (9.11)$$

We can now evaluate the shifts due to electric field gradients in the trap. Let us assume a worst-case scenario in which the angular expression in equation 9.10 is equal to one. The sensitivities of each hyperfine interval are given in Table 9.3.

In the event that these shifts prove problematic due to uncancellable field gradients in our trap that are too large, it is possible to null out this effect by averaging over all π transitions in a hyperfine interval. It should also be possible to orient the magnetic field so that this shift is equal to zero, by carefully selecting the Euler angles α and β .

9.3.5 Doppler Effects

As the ion moves in the trap, it experiences Doppler shifts in the light it receives. In the lab frame, this results in a shift in the apparent transition frequency of

$$\frac{f - f_o}{f_o} = \frac{\Delta f_D}{f_o} = -\frac{v}{c} - \frac{v^2}{2c^2} + \dots \quad (9.12)$$

where the second term is the relativistic Doppler shift. Although the ion's oscillatory motion means that the first-order Doppler shift is zero and only results in symmetric sidebands, the relativistic term is of concern. If we compute the expectation value of the velocity in the

trap $\langle v^2 \rangle$, at the Doppler limit, we obtain a value of $\frac{3\hbar\gamma}{4m} = 0.05\text{m}^2/\text{s}^2$. This should result in a fractional shift in the observed frequency by a factor of

$$\frac{\Delta f_D}{f_0} = -\frac{v^2}{c^2} = -3 \times 10^{-19} \quad (9.13)$$

For a transition on the order of 1 GHz, this corresponds to a shift at the nanohertz level.

As the micromotion is considerably faster than the secular motion, we expect a larger Doppler shift from it. From ref [BMB98], we can write the relativistic Doppler shift due to the micromotion along a given axis i as

$$\left(\frac{\Delta f_D}{f_0} \right)_i \approx -\frac{1}{mc^2} \left[\frac{k_B T_i (a_i + q_i^2)}{2a_i + q_i^2} + \frac{4}{m} \left(\frac{eq_i E_i}{\Omega_{rf}(2a_i + q_i^2)} \right)^2 \right] \quad (9.14)$$

where E_i the stray field along axis i , and a_i and q_i are the Mathieu parameters of that axis.

Using equation 9.14, we compute the fractional shift as

$$\sum_i \left(\frac{\Delta f_D}{f_0} \right)_i \approx -1.4 \times 10^{-18} E_{xy}^2 \quad (9.15)$$

Where E_{xy} is the stray electric field in the xy plane. As there is no micromotion in the z direction, stray fields along that axis do not contribute. To keep this systematic below 1Hz, we require that the magnitude of the uncompensated electric field in the x-y plane is less than 10V/mm, which is a much larger field than we ever apply using our DC electrodes.

9.4 935 Spectroscopy

Spectroscopy of the 935 nm transition $^2D_{3/2} \rightarrow ^3[3/2]_{1/2}^o$ in $^{173}\text{Yb}^+$ was performed using a two-laser pump-probe spectroscopy technique. One laser – the pump – was placed at a stationary frequency with EOM sidebands added so that all lines except for one were addressed. This placement led to the ion being optically pumped into the hyperfine sublevel of the $^2D_{3/2}$ state that was not addressed by the pump. A second laser – the probe – was then scanned, and ion fluorescence was recorded. When the probe laser is resonant with

Interval	1-2	2-3	3-4	4-5	5-6
DC Zeeman	1220	5600	24200	-480	-640
AC Zeeman			$\approx 10^{-5}$		
Quadratic AC Stark			$\approx 10^{-4}$		
Electric Quadrupole	-147	68	-112	192	-296
Doppler			10		

Table 9.3: Expected systematic shifts of the hyperfine clock intervals in the $^2F_{7/2}^o$ manifold. All shifts are in mHz. Systematics are dominated by the nonlinear Zeeman effect, which contributes to systematic shifts at the Hz level. A 10 mG residual magnetic field is assumed.

the remaining transition not addressed by the pump laser, the ion is able to optically cycle and fluorescence is detected on the PMT. Figure 9.3 shows the result of performing this pump-probe spectroscopy on all six transitions between the $^2D_{3/2}$ and $^3[3/2]_{1/2}^o$ states.

![[Table - Pump laser placements]]

The hyperfine intervals measured in the spectroscopy allow us to extract the hyperfine A coefficient of the $^3[3/2]_{1/2}^o$ state and the hyperfine A and B coefficients of the $^2D_{3/2}$ state. As a notational shorthand, we will refer to the frequency of the transition $|^2D_{3/2}F = F\rangle \rightarrow |^3[3/2]_{1/2}F = F'\rangle$ as $T_F^{F'}$.

The $^3[3/2]_{1/2}^o$ A coefficient can be obtained directly in two different ways: $T_2^2 - T_2^3$, and $T_3^2 - T_3^3$. Averaging these two differences gives a hyperfine splitting of 1834.5(8)MHz, and an A coefficient of 611.5(3)MHz. The $^2D_{5/2}$ A and B coefficient can be obtained by first subtracting the $^1[3/2]_{3/2}$ hyperfine splitting from T_2^3 , T_3^3 , and T_4^3 , followed by a fit of all five measured frequencies to the formula obtained from equation 2.15

$$\frac{5}{4}(B - 5A) + \frac{1}{10}F(F+1)(5A - 3B) + \frac{B(F(F+1))^2}{80} \quad (9.16)$$

which is quadratic in $F(F+1)$. This procedure yields $A = -120(2)$ MHz, $B = 958(14)$ MHz.

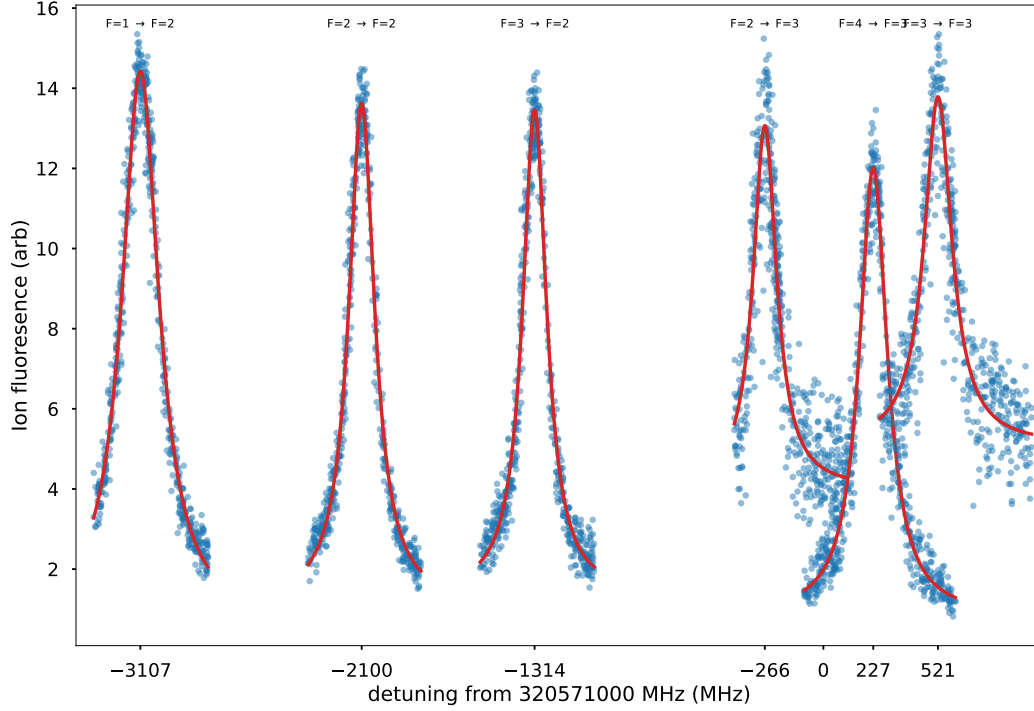


Figure 9.3: Hyperfine-resolved spectroscopy of the 935 nm $^2D_{3/2} \rightarrow ^3[3/2]_{1/2}^o$ transition, performed using a pump-probe technique in which the ion is pumped into a dark state using two EOM tones on a pump laser while a secondary laser is scanned across the spectrum. Signal-to-noise ratio on the $F = 2 \rightarrow F = 3$ and $F = 3 \rightarrow F = 3$ transitions are relatively poor due to their proximity to the strong $F = 4 \rightarrow F = 3$, which prevents a perfect dark state from being formed.

9.5 411 Spectroscopy

Spectroscopy of the 411 nm transition in $^{173}\text{Yb}^+$ were performed at a rather cursory level, with the primary aim of populating the $^2F_{7/2}^o$ state. As such, a full spectrum of the $^2S_{1/2} \rightarrow ^2D_{5/2}$ transitions has yet to be obtained, and a full exploration of the hyperfine structure of the $^2D_{5/2}$ state is incomplete.

The initial spectrum was obtained via direct interrogation using 411 nm light. Due to the absence of a repump in this scheme, successfully driven transition led to the ion being irretrievably driven into the $^2F_{7/2}^o$ state, at which point the ion had to be flushed from the trap and a new one loaded, a process that we dubbed *death spectroscopy*. After driving many transitions in this way, the average frequencies at which the ion was pumped into the $^2F_{7/2}^o$ state could be determined, resulting in Figure 9.4. Four or five transitions were ultimately driven, all with the lower state being $^2S_{1/2} F = 2$.

Ultimately after we located several 760 nm transitions that deshelve the ion from the $^2F_{7/2}^o$ state, we were able to perform quantum jump spectroscopy via an optical cycling process, in which population that is shelved is returned to the ground state. This results in a telegraphing signal, in which the ion rapidly alternates between bright and dark when the 411 nm laser is on resonance.

When performing this spectroscopy, the 411 nm laser was unlocked, and therefore the linewidth was rather large. This posed a problem when trying to distinguish the $^2S_{1/2} F = 2 \rightarrow ^2D_{5/2} F = 3$ and $^2S_{1/2} F = 2 \rightarrow ^2D_{5/2} F = 4$ transitions. While qualitatively we can state that two transitions were being driven, there is not sufficient separation to confidently assign the shelving events to two different transitions. Given this ambiguity, we can ignore this transition when computing the hyperfine structure constants, in which case we can state that $A_{2D_{5/2}} = 100 \pm 300\text{MHz}$, $B_{2D_{5/2}} = 1600 \pm 1300\text{MHz}$. Taking the transition we observed as $^2S_{1/2} F = 2 \rightarrow ^2D_{5/2} F = 3$, we can tentatively state $A_{2D_{5/2}} = 20(20)\text{MHz}$, $B_{2D_{5/2}} = 1200(80)\text{MHz}$.

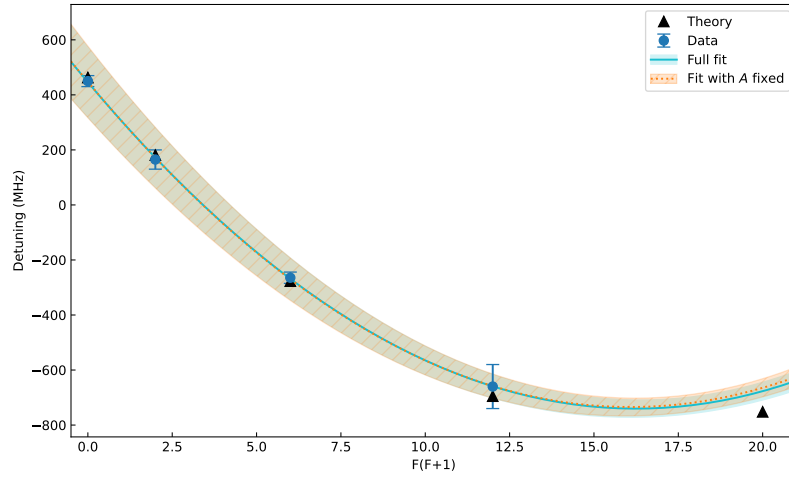


Figure 9.4: A summary of the 411 nm $^2S_{1/2} F = 2 \rightarrow ^2D_{5/2}$ 411 nm transitions in $^{173}\text{Yb}^+$. The $F = 2 \rightarrow F = 4$ transition was possibly observed but could not be clearly distinguished from the $F = 2 \rightarrow F = 3$ transition. Theory is the predicted structure by Itano [Ita06]. The fit in orange is a full quadratic fit of the hyperfine structure, while the fit in cyan fixes the hyperfine A coefficient of the $^2D_{5/2}$ state to be consistent with that in $^{171}\text{Yb}^+$

We can infer $A_{2D_{5/2}}$ from the $^2D_{5/2}$ state of $^{171}\text{Yb}^+$. Given that $A_{D_{5/2}}^{171} = -63.368(1)\text{MHz}$ [TEM21], we can use the nuclear magnetic moments of the two states infer that $A_{D_{5/2}}^{173} = 16.635\text{MHz}$. This is consistent with the values obtained during the four-point fit. If we treat this as fixed during the fit, we obtain $B_{D_{5/2}}^{173} = 1220(80)\text{MHz}$.

Further quantum jump spectroscopy of this transition is obviously required using a cavity-stabilized 411 nm source and full optical cycling. All the infrastructure for performing this measurement exists, but taking the data was postponed while the data in chapter 8 was being taken.

9.6 760 Spectroscopy

Performing spectroscopy of the 760 nm $^2F_{7/2}^o \rightarrow ^1[3/2]_{3/2}$ transition in $^{173}\text{Yb}^+$ is essential for being able to perform proper measurements of the $^2F_{7/2}^o$ hyperfine structure, as any readout of the ion's state in the $^2F_{7/2}^o$ manifold needs to be state-selectively outcoupled from specific sublevels of they hyperfine structure. Such readout processes include those used in heralded state preparation, in which the ion's state is prepared by out-coupling the states not being prepared to the ground state, and then projectively measuring the ion's state.

9.6.1 Binary Sweep Spectroscopy

Detecting whether the laser is resonant with the 760 nm transition was performed straightforwardly by measuring the ion's fluorescence with the 411 nm light turned on. When 760 nm light is resonant with a transition that out-couples every sublevel in the F state that is being populated, the ion is seen to be bright, rapidly flickering between the ground state and the $^2F_{7/2}^o$. If any state is unaddressed by the 760 nm light, the ion is optically pumped into that state and does not fluoresce.

Using one 760 nm laser, this fluorescence technique only works when one of the hyperfine states in the $^2F_{7/2}^o$ is being populated. This is true when populating the $^2F_{7/2}^o$ state via the

$^2D_{5/2}$ $F = 0$ state, in which case only the $F = 1$ sublevel is populated.

In the case that the $^2D_{5/2}F = 1$ state is being used to populate the F state, the $F = 1$ and $F = 2$ states are both populated by the decay. In this case, two laser tones are needed to avoid pumping into a dark state, but the spectroscopy can still be performed. In order to find the initial transitions, a binary search of the laser frequencies was performed. The laser was initially swept over a roughly 10 GHz range, and the sweep range was progressively reduced by half. Each half of the previous range was tested until the sweep range became less than the resolution of the wavemeter lock.

Populating the $^2F_{7/2}^o$ state via any of the other sublevels in the $^2D_{5/2}$ manifold leads to three states in the F state being populated by the decay. In order to prevent optical pumping into a dark state, three 760 nm tones must be supplied. Alternatively, one of the 760 nm lasers can be modulated in order to drive two transitions while the second 760 nm laser is used as a probe. Due to the slow timescales involved in this spectroscopy, this modulation can be achieved either via an EOM or via simply sweeping the laser at a low frequency (perhaps 100 Hz) such that two of the three transitions which out-couple the $^2F_{7/2}^o$ states are driven while the third is driven by the probe laser.

In the preliminary work presented in this thesis, only two 760 nm lasers were available to perform the spectroscopy, and each was stabilized to roughly 50 MHz as described in Section 5.5.4. As such, spectroscopy of the 760 nm transitions was performed only on the transitions $^2F_{7/2}^o$ $F = 1, 2 \rightarrow ^1[3/2]_{3/2}^o$. However, this is sufficient to extract the hyperfine structure of the $^1[3/2]_{3/2}^o$ state and to obtain the approximate splitting between the $F = 1$ and $F = 2$ sublevels of the $^2F_{7/2}^o$ manifold.

Figure 9.5 shows the spectroscopy scheme in more detail, and Table 9.6.1 details the frequencies of the 760 nm transitions as measured on our wavemeter and the associated hyperfine splittings.

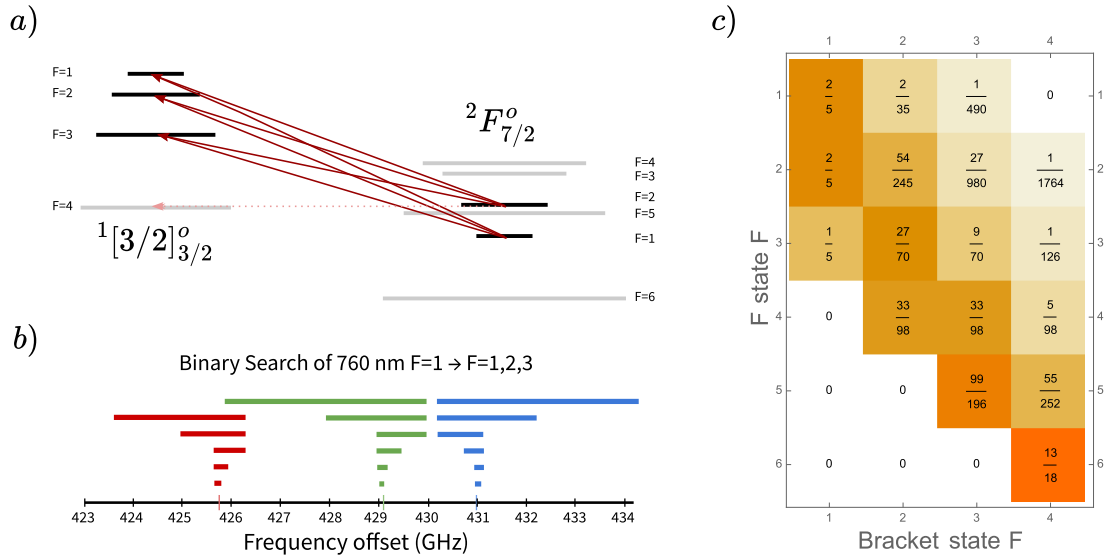


Figure 9.5: Spectroscopy of the 760 nm $^2F_{7/2}^o \rightarrow ^1[3/2]_{3/2}^o$ transition. a) Diagrammatically represents the transitions we were able to probe using only two 760 nm lasers, including one weak transition what we were unable to observe. b) shows the sweep ranges used and the binary search process used to converge on the transition frequencies. c) shows the relative line strengths of the transitions between sublevels.

Transition (${}^2F_{7/2}^o$)	Wavemeter Frequency (THz)	Relative Splitting from $F \rightarrow 1$ (GHz)
$1 \rightarrow 1$	394.431090	0
$1 \rightarrow 2$	394.429100	1.99
$1 \rightarrow 3$	394.425684	5.40
$2 \rightarrow 1$	394.430040	0
$2 \rightarrow 2$	394.428056	1.98
$2 \rightarrow 3$	394.424640	5.40

Table 9.4: Frequencies of the observed 760 nm transitions out of the ${}^2F_{7/2}^o$ state. All wavemeter frequencies and splittings should be taken as having ± 50 MHz precision, and low absolute accuracy.

9.6.2 Line Assignment

While hyperfine splittings of the ${}^1[3/2]_{3/2}^o$ state can be determined from the measured 760 nm transitions, the assignment of the levels is less straightforward. We can estimate the hyperfine A coefficient by rescaling the value in ${}^{171}\text{Yb}^+$ [Rom21]. This leads to an estimated A coefficient $A_{[3/2]_{3/2}}^{173} = -1163$ MHz. Enumerating every possible level assignment, only one yields a fitted hyperfine A coefficient that is consistent with the rescaled value. We can therefore state that $A_{[3/2]_{3/2}}^{173} = -1230(30)$ MHz and $B_{[3/2]_{3/2}}^{173} = -580(120)$ MHz.

9.6.3 Hyperfine Splitting in the ${}^2F_{7/2}^o$

From the measured lines, we can also extract the splitting between the $F = 1$ and $F = 2$ sublevels of the F state. Subtracting the relevant pairs of transitions, we obtain a splitting of 1050(50) MHz. If we take the hyperfine A coefficient of the F state to be -237 MHz by rescaling from the hyperfine structure in ${}^{171}\text{Yb}^+$, we can use this to approximately infer the hyperfine B coefficient of the F state in ${}^{173}\text{Yb}^+$. This gives us $B = -5300 \pm 300$ MHz.

9.7 The $^{173}\text{Yb}^+$ Ground State Qubit

While it is possible to fully shelf both hyperfine sublevels of the $^{173}\text{Yb}^+$ ground state simply by Doppler cooling while applying 411 nm light, it would be beneficial to many experiments to be able to prepare a qubit in the $^2S_{1/2}$ manifold. Furthermore, any experiments that involve using $^{173}\text{Yb}^+$ for computation would benefit greatly from having a defined ground-state and metastable-state qubit.

9.7.1 State preparation

State preparation of a ground state qubit can happen in one of two ways: Narrowband Optical Pumping (NBOP) or Microwave-Assisted Optical Pumping (MAOP). In the following discussion, I will assume that the goal is to prepare the pure clock state $|^2S_{1/2}F=2, m_F=0\rangle$. Most techniques can trivially be switched to prepare the $|^2S_{1/2}F=3, m_F=0\rangle$ state or any other Zeeman sublevel. Figure 9.6 schematically depicts both optical pumping schemes.

9.7.1.1 Narrowband Optical Pumping

Narrowband optical pumping for state preparation would essentially act as an extension of the quadrupole optical pumping discussed in section 6.4.1.1. It was further demonstrated in reference [ARS22], along with MAOP. In this scheme, the narrow linewidth of the 411 nm transition would be paired with the 976 nm $^2D_{5/2} \rightarrow ^1[3/2]_{3/2}^o$ transition, creating an optical cycling transition. The narrow linewidth of the 411 step allows for selectivity of Zeeman sublevels, allowing for optical pumping into a single Zeeman sublevel, either by rapidly alternating between different 411 transitions or with well-chosen EOM sidebands.

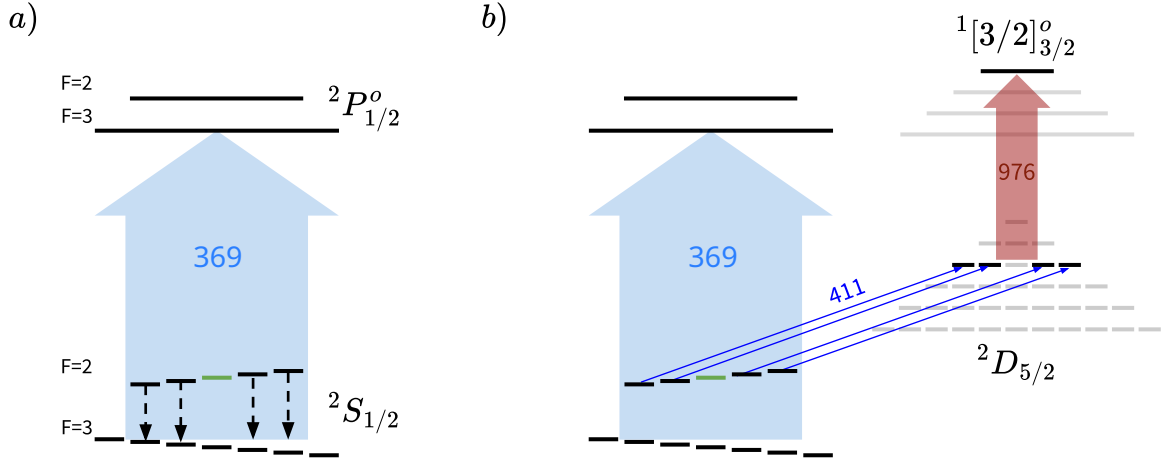


Figure 9.6: Schematic of Magnetically Assisted Optical Pumping (MAOP) and Narrowband Optical Pumping (NBOP) used to prepare the $F = 2, m_F = 0$ state in the ground state of $^{173}\text{Yb}^+$. a) depicts MAOP, in which microwaves are used to pump population out of the undesired Zeeman sublevels. b) depicts NBOP, in which the narrowband 411 nm transition is used to the same end, with a 976 nm transition moving the atom back into the ground state via the short-lived $^1[3/2]_{3/2}^o$ state. In both cases, 369 nm light cycles on the $^2S_{1/2} F = 3 \leftrightarrow ^2P_{1/2}^o F = 3$ transition.

9.7.1.2 Microwave-Assisted Optical Pumping

Microwave-assisted optical pumping has been demonstrated as a powerful technique for initializing qubits in systems with $I > 1/2$ [TPL21, ARS22]. In this technique, 369 nm addresses the transition $^2S_{1/2}F = 3 \rightarrow ^2P_{1/2}^oF = 3$. Four microwave tones are used to transfer population from the undesired Zeeman sublevels to the $F = 3$ manifold, where they are optically pumped. These microwave pulses can be cycled pi-pulses, or can be applied in a continuous fashion using an arbitrary waveform generator or by otherwise superimposing the tones required.

9.7.1.3 Polarization-based Optical Pumping

The final common technique for optically pumping in systems with $I > 1/2$ is to use a purely σ_+ polarized beam to optically pump the ion into the stretched state $^2S_{1/2}F = 3, m_F = 3$ [HAB14]. From here, microwave pulses can be used to transfer the qubit population back into the clock qubit. While technically straightforward, the fidelity of such a scheme is limited by the polarization purity that can be achieved at the ion. Additionally, it requires the cooling light to be oriented along the quantization axis, requiring either a change in the trap geometry or a separate beam path dedicated to optical pumping.

9.7.2 State detection

Without the ability to perform the closed-cycle state detection found in spin-1/2 isotopes, shelving to the $^2F_{7/2}^o$ is required to read out the state of the ground state qubit. In an *omg* scheme, shelving would have to occur to a F sublevel that does not interact with the quantum information subspace of the metastable qubit.

9.8 Microwave Spectroscopy of the F State

Microwave spectroscopy of the $^2F_{7/2}^o$ state requires a number of components:

- A way to deterministically prepare a single Zeeman sublevel in the $^2F_{7/2}^o$ manifold
- A way to deliver broadband microwaves to the trap
- A state-selective way to out-couple population to the ground state for state detection

9.8.1 Initial Pulse Sequence

Without knowing the hyperfine structure of the F state, it will not be possible to prepare a single pure Zeeman sublevel needed for high-fidelity Ramsey spectroscopy. However, by optically pumping into the $^2F_{7/2}^o$ via the $^2D_{5/2}$ $F = 0$ state, only the three Zeeman sublevels of the $^2F_{7/2}^o$ $F = 1$ are populated. This initial population is sufficient to perform a straightforward microwave line scan with 1/3 contrast. State detection would be performed by applying a 760 nm tone that only depopulates the $F = 2$ state, followed by checking for fluorescence via ground state Doppler cooling.

9.8.2 Spectroscopy of Subsequent Lines

Following the determination of the $F = 0 \rightarrow F = 1$ microwave transition, state preparation in the $^2F_{7/2}^o$ state can become more sophisticated.

9.8.2.1 MAOP and heralding in the F State

State preparation of the $F = 1, m_F = 0$ state can proceed via heralding once one microwave transition has been found. This is done by driving the $F = 1, m_F = \pm 1 \rightarrow F = 2, m_F = \pm 1$ transitions, out-coupling the population in the $F = 2$ manifold, and checking for fluorescence. This fluorescence check constitutes a projective measurement, and if no fluorescence

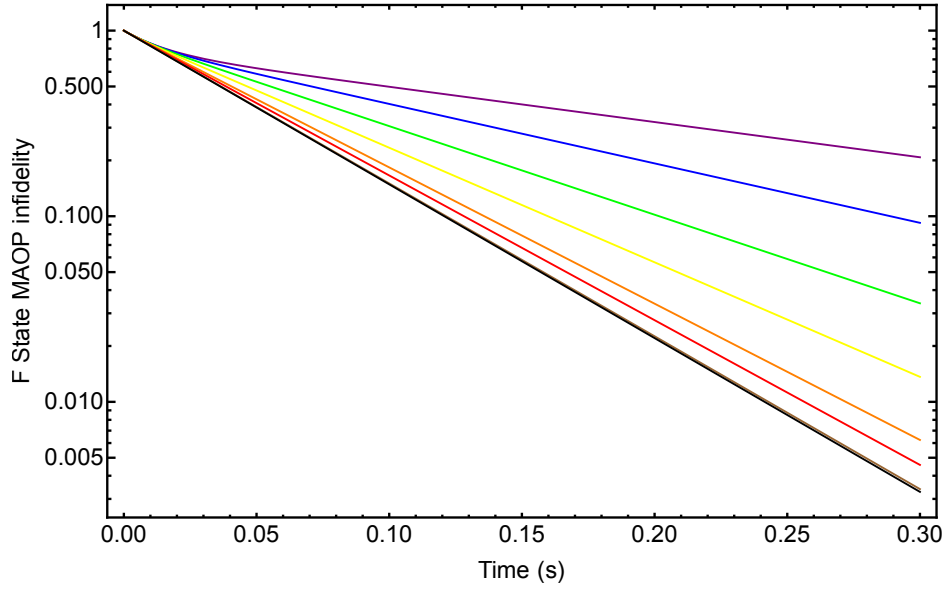


Figure 9.7: Infidelity of MAOP state prep in the $^2F_{7/2}^o$ state as a function of optical pumping duration. Assuming a reasonable microwave Rabi rate of 1 kHz can be achieved, the pumping rate is essentially set by the lifetime of the $^2D_{5/2}$ state.

is detected, the ion has been projected into the $F = 1, m_F = 0$ state. This procedure will, however, have a 2/3 failure rate, requiring a great deal of post-processing or conditional logic in the experiment.

This error rate can be improved by first performing a microwave-assisted optical pumping step in the F state. In this case, the microwaves driving $F = 1, m_F = \pm 1 \rightarrow F = 2, m_F = \pm 1$ are simply turned on while the 411 nm narrowband optical pumping is still ongoing. If the Rabi frequency of the microwaves is faster than $2\pi/7.2\text{ms}$, this will cause population to transfer into the $F = 1, m_F = 0$ without any increase in the duration of the pulse sequence, as the shelving rate will continue to dominate.

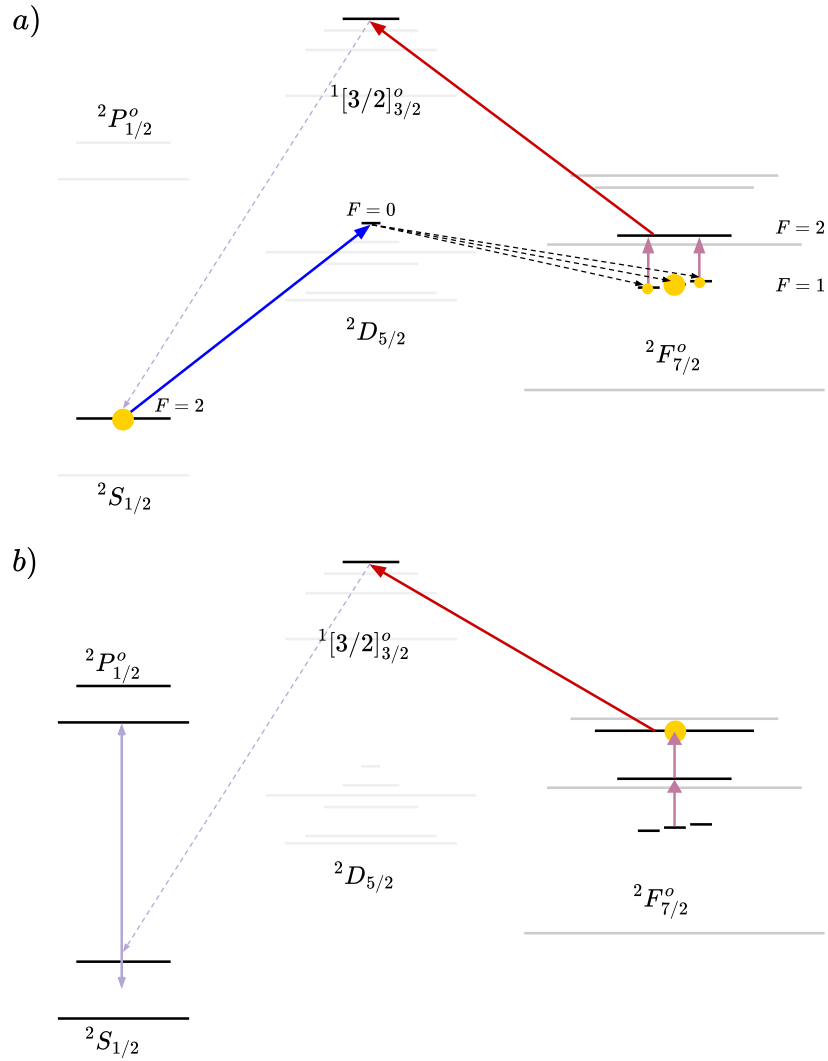


Figure 9.8: The scheme for hyperfine spectroscopy in the $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$. a) depicts the process of preparing the $F=1, m_F=0$ state in the $^2F_{7/2}^o$. 411 nm light resonant with the $^2S_{1/2} F=2 \rightarrow ^2D_{5/2} F=0$ transition, which decays indiscriminately into the $^2F_{7/2}^o F=1$ state. By applying resonant microwaves (purple) to clear out the $m_F = \pm 1$ states while 760 nm light is resonant with the $F=2 \rightarrow ^1[3/2]_{3/2}^o$ transition, we optically pump into the $m_F=0$ state. b) depicts spectroscopy of the $F=2 \rightarrow F=3$ transition. Following preparation of $F=0, m_F=0$, pi pulses are applied until the probe transition is reached, and then spectroscopy is performed.

9.8.2.2 Hyperfine ladder

In order to measure the splitting between state F and state $F + 1$, we always begin the pulse sequence by initializing the ion to the $F = 1, m_F = 0$ state. We then apply microwave π pulses up each known transition in order to populate the $F = F, m_F = 0$ state needed to start the spectroscopy. Then, the same type of measurement is performed as with the $F = 1 \rightarrow F = 2$ transition. Microwave interrogation is attempted, and population in the F manifold is out-coupled to the ground state and is measured. When a microwave transition has been successfully found, the ion will not fluoresce and register as dark. Following successful identification of the microwave transition via straightforward microwave interrogation, a Ramsey sequence is performed in order to determine the transition frequency to the Hz level. Figure [[Fig - Hyperfine Spectroscopy Scheme]] shows a schematic of the whole procedure described in this section.

CHAPTER 10

The future of $^{173}\text{Yb}^+$

Following spectroscopy of the metastable state of $^{173}\text{Yb}^+$, we will have unlocked a trapped ion system with a hyperfine ground state qubit whose cousins have exhibited exceptional SPAM fidelity [ARS22], and a metastable excited state whose long lifetime only increases the number of potential experiments and quantum information schemes that can be explored. In this chapter, we will discuss several potential avenues of future research for the $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$.

10.1 Quantum Computation

10.1.1 The $^{173}\text{Yb}^+$ Qudit

An immediate feature of the F state is the large number of states available, even with a relatively modest level of control. Such a structure allows us to encode quantum information in more than a simple two-level system and use a higher-dimensional Hilbert space. Such *qudits* are a potentially attractive option for quantum computing, especially in the regime where hardware places strong limits on the number of ions that can be trapped together.

10.1.1.1 Why qudits?

Qudits allow you to use fewer ions When trapped ions are stored in an ion trap, the physical parameters of the trap generally limit the number of ions that can be stored in a simple chain of the type desirable for quantum computation, with state-of-the-art ion

chains containing 61 ions in quantum simulation experiments [LWM23], and roughly 30 ions in quantum computation experiments [CNE24]. As ion chains become large, the shared motional modes that allow one to perform gates become harder and harder to manage, as the spectral density of motional modes increases with each ion added to the chain. This poses a technical limit on the number of ions that can be trapped within a single chain before the motional modes become difficult to address. While schemes have been proposed that mitigate some of these crowding effects [LWL19], these schemes add control complexity.

While there are ways to sidestep this using shuttling-based architectures and microtraps [MBA23], these schemes may be undesirable in quantum simulation experiments where constant all-to-all connectivity is an important feature. In these situations, qudits pose a clear advantage, as the Hilbert space spanned by n d -qudits scales as d^n , rather than 2^n for qubits. This represents an increase in the size of the Hilbert space by a factor of $(d/2)^n$.

Qudits may simplify certain quantum algorithms Qudit-based forms of certain quantum algorithms have also been investigated/ For instance, a qudit implementation Grover’s algorithm for quantum search has been proposed that utilizes the higher-dimensional properties of the qudits to reduce the number of ions that need to be controlled [ITV12]. Algorithms for the quantum Fourier transform and quantum phase estimation have also been found [CPZ11]. For a good review of the state of qudit quantum computing, see [WHS20].

10.1.1.2 Six-state clock qudit

Perhaps most straightforwardly, we can utilize the six $m_F = 0$ zero-field clock states in the $^2F_{7/2}^o$ state to create a six-level qudit. Full control of the system can be achieved by using $m_F \neq 0$ state as ancillae for state transfer, allowing for control of the qudit without having to exit the metastable manifold. State preparation of the qudit can occur dissipatively in the same way as discussed in section 9.8.2.1, or can be performed directly either by direct transfer at 467 nm, or the two-step 411nm + 3.4 μ m process.

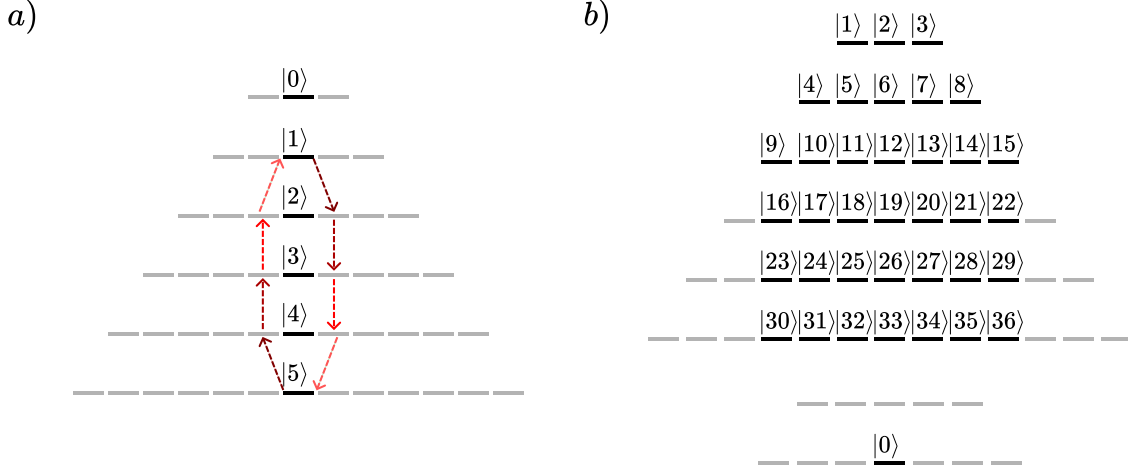


Figure 10.1: Qudit schemes in the $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$. a) depicts the six-level clock qudit, and shows an example of one of the $\text{SU}(6)$ primitives (a rotation between $|1\rangle$ and $|5\rangle$) using the Zeeman states as ancillae. b) shows a potential for a 37-level optical qudit, composed of all the states directly accessible via the $^2S_{1/2} F = 3, m_F = 0$ state. With the addition of single-qubit gates in the ground state, the full 48-level qudit is potentially accessible.

10.1.1.3 Many-state qudit

In addition to the six-state clock qudit scheme, it is also possible to utilize the Zeeman structure of the $^2F_{7/2}^o$ state to create a much higher-multiplicity qudit. This follows the work of the Low et al. at the University of Waterloo, in which a 13-state qudit in the $^2D_{5/2}$ state of $^{137}\text{Ba}^+$ was controlled, with a potential increase to a 25-level scheme [LWS23]. Employing a similar scheme and using either direct or two-step transfer between the ground and metastable states, the 48 Zeeman sublevels of the $^2F_{7/2}^o$ state can all be used as qubit levels, with the main difficulty arising from the difficulty of frequency calibration and dephasing due to the magnetic sensitivity of the states.

Quantum control of such a many-state system may prove difficult, and the implementation in [LWS23] requires the use of a single ground-state level that acts as a bridge connecting the different states via an optical transition.

10.1.2 FOFI qubits in the F state

In the presence of magnetic fields, and given sufficient higher-order Zeeman effect terms, there exist values for the magnetic field for which the linear frequency dependence of the magnetic field equals zero. These are referred to as First-Order Field-Insensitive (FOFI) qubits, and a well-known example is implemented between the (zero-field) $F = 4, m_F = 0$ and $F = 3, m_F = 1$ states of the $^{43}\text{Ca}^+$ ground state at 146 Gauss [Tho13]. These states are desirable, since coherent population trapping [BB02] makes it impractical to operate an ion trap at zero magnetic field.

The ground and metastable states of $^{173}\text{Yb}^+$ are likely to exhibit such qubits as well, potentially even with lower second order magnetic field dependence than the zero-field clock qubits. To perform the analysis, we look at the derivative of the frequency between pairs of states with respect to magnetic field. If this crosses zero, we have found a FOFI qubit. A full table of these qubits are displayed in Table G.2.

It is worth noting however, that while these qubits are interesting to think about, they pose a number of problems:

- They are difficult to predict in systems as complex as this one, as the exact magnetic field dependence of the energy levels exhibits chaos, especially at higher magnetic fields [PS24].
- As the number of states at each m_F is preserved, qubits defined with $m_F \neq 0$ can see asymmetric couplings between σ_+ and σ_- polarization.
- FOFI qubits are ill-suited to performing certain types of geometric phase gates [BCM19]
- The large number of other states that *aren't* magnetic field insensitive makes state preparation difficult by any means other than by direct optical transfer the ground state.

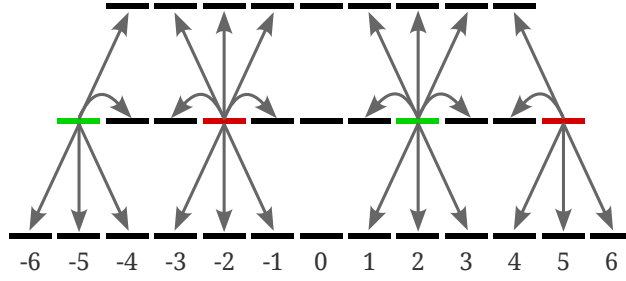


Figure 10.2: An implementation of the simplest $\mathcal{A}$ code in the $F = 5$ manifold in the ${}^2F_{7/2}^o$ state. The codewords (red and green) are symmetrically spaced, making them more robust to the Zeeman effect, and spaced far enough apart that rank-1 errors will not mix them. The amplitudes of each sublevel are tuned so that no information can be obtained from the polarization of a detected photon.

10.1.3 Robust qubit encodings

The high multiplicity of the ${}^2F_{7/2}^o$ state can also be used as a resource to improve the robustness of quantum information to certain typed of errors. As a general concept, this works by defining a number of *codewords* – superpositions of quantum states – in which errors can be corrected without directly measuring their state.

10.1.3.1 $\mathcal{A}$ codes

A specific proposal for the ${}^2F_{7/2}^o$ state is the $\mathcal{A}$ code [JHC23]. While this scheme was originally proposed as a way to leverage the high multiplicity of molecular rotational states for error correction, the large number of states in the ${}^2F_{7/2}^o$ allows us to implement the simplest example of the code in an atomic system.

Ultimately, the $\mathcal{A}$ code seeks to insulate the quantum information from the specific noise channels that are often the most problematic in trapped ion experiments: Rank-1 error such as uncontrolled Zeeman shifts, unwanted dipole excitations, and unwanted dipole decays. What is important about these processes is that they have a well-defined state space into

which they will mix the atom: those with $|\Delta m| \leq 1$. If there is sufficient Zeeman structure, we may therefore define codewords in which the individual codewords consist only of states with $|\Delta m| \geq 2$, so that any dipole-like error process cannot mix the states. This is essentially the Knill-Laflamme condition $\langle \psi_i | \hat{E}_a^\dagger \hat{E}_b | \psi_j \rangle = \delta_{ij} c_{ab}$, which essentially states that an error channel with Kraus operators $\{\hat{E}\}$ must populate a unique set of states for each codeword $|\psi\rangle$, equivalent to saying that errors cannot mix codewords [KL97]. We may suggest, for instance, that we define the following two codewords in the ${}^2F_{7/2}^o$ $F = 5$ state which E1 processes connect into disjoint subspaces:

$$|\psi_1\rangle = |m_F = -5\rangle + |m_F = 2\rangle \quad (10.1)$$

$$|\psi_2\rangle = |m_F = 5\rangle + |m_F = -2\rangle \quad (10.2)$$

In reality, of course, atomic physics makes this code impractical, as the two states will preferentially emit and absorb different *polarizations* of light: $|\psi_1\rangle$ will preferentially emit and absorb σ_- light, and $|\psi_2\rangle$ will preferentially absorb σ_+ light. This asymmetry between the two states means that we cannot satisfy the Knill-Laflamme condition, as the coefficient associated with a given error channel c_{ab} is not independent of the codewords. In order to counteract this, we must define our codewords as superpositions that preserve the quantum information with regard to polarization as well. Such a codeword is:

$$|\psi_1\rangle = \sqrt{\frac{5}{7}} |m_F = 2\rangle + \sqrt{\frac{2}{7}} |m_F = -5\rangle \quad (10.3)$$

$$|\psi_1\rangle = \sqrt{\frac{5}{7}} |m_F = -2\rangle + \sqrt{\frac{2}{7}} |m_F = 5\rangle \quad (10.4)$$

Such an encoding should be sufficient to correct rank-1 error processes, and sufficient to detect rank-2 error processes.

This fairly loose demonstration of the concept of $\mathbb{A}$ codes is far from rigorous, and a much more complete theoretical treatment is given in the original paper [JHC23].

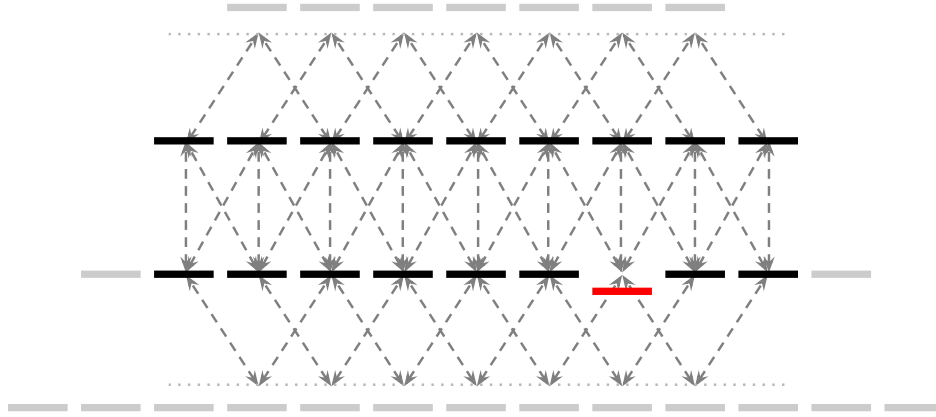


Figure 10.3: An implementation of a lattice model implemented in the $^{173}\text{Yb}^+$ metastable state. Microwave tones (dashed) are used to connect the Zeeman sublevels, generating a highly connected graph. By shifting the energy of one of the Zeeman sublevels (red), we can cause the ion to evolve into the selected sublevel.

10.2 Quantum Simulation

10.2.1 Quantum Random Walks

The large number of Zeeman states in the metastable manifold that are connected by microwave transitions allows us, with sufficient microwave control, to implement a lattice-like model inside the F state, in which each state is connected to up to six neighboring states with some amplitude. In such a model, quantum random walk-based search schemes have been proposed [Won22].

Quantum search schemes require the existence of an *oracle* – an operator that imparts a phase shift onto the state being searched for. In the metastable manifold, we could create such a phase shift by using a laser detuned from a narrow transition to introduce a light shift on one of the Zeeman sublevels after the initialization of a uniform superposition of all the Zeeman sublevels. Following this initialization step, microwaves can be turned on continuously, allowing the ion to evolve into the selected state in faster-than-classical time.

10.3 Precision Measurement

10.3.1 Quenching of F state Lifetimes

The $^2F_{7/2}^o$ also poses potential advantages for atomic clocks. Due to the large nuclear spin of ^{173}Yb , the hyperfine interaction causes admixture between the $^2F_{7/2}^o$ and $^2P_{3/2}^o$ states. This admixture has been predicted to quench the lifetime of the $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$ from its measured $^{171}\text{Yb}^+$ value of 1.58(3) years [LPH21] to as little as a few days [DF16]. This small hyperfine-induced transition dipole moment significantly broadens the transition between the S state and the F state, making it easier to drive without broadening it so much that it ceases to be useful as a clock. Broadening the transition also reduces the power required to drive the transition, helping to mitigate light shifts associated with the probe laser.

10.3.2 Sensitivity to fundamental physics

The presence of a large multipole hyperfine structure that makes the F state such a powerful microscope on the multipole structure of the nucleus also makes it a good candidate for investigating effects that would *change* the nuclear structure over time. For instance, certain theories of axion dark matter predict oscillations the nuclear radius [BBF23]. When considering only dipole hyperfine structure, this effect is small. However, the nuclear quadrupole moment is significantly more sensitive to these effects, and by searching for variation of the ratio between the dipole and quadrupole terms in the hyperfine structure, these theories can be tested more effectively. The $^2F_{7/2}^o$ state in $^{173}\text{Yb}^+$ has been shown to have a high sensitivity to such effects [FM23].

Ytterbium ions have also been used as a probe of Lorentz symmetry violations that would arise from certain theories that aim to unify gravity with the other fundamental forces [DFS16, SHL19]. The sensitivity of the F state to this effect comes from the high degree of anisotropy in the stretched Zeeman states of the F state, and Lorentz symmetry

violations would correspond to orientation-dependent shifts of this highly anisotropic electron distribution. By probing a transition within the F state and monitoring the clock as the Earth orbits the sun, for instance, a small shift in the clock frequency could indicate a Lorentz symmetry violation.

To this effect, several experiments are now pursuing $^{173}\text{Yb}^+$ E3 optical clocks in [YGY24, ASL24].

APPENDIX A

Useful Angular Momentum Formulas

A.0.1 Recoupling

A.0.1.1 Recoupling two angular momenta

This is the one that you're taught in Sakurai: how do you convert from the $|j_1, m_1; j_2, m_2\rangle$ basis to the $|j_1, j_2, J, m\rangle$ basis?

$$|j_1, j_2, J, m\rangle = \sum_{m_1, m_2} \langle j_1, m_1; j_2, m_2 | J, m \rangle |j_1, m_1; j_2, m_2\rangle \quad (\text{A.1})$$

$$= \sum_{m_1, m_2} (-)^{j_1-j_2+m} \sqrt{2j+1} \begin{pmatrix} j_1 & j_2 & J \\ m_1 & m_2 & -m \end{pmatrix} |j_1, m_1; j_2, m_2\rangle \quad (\text{A.2})$$

A.0.1.2 Recoupling three angular momenta

Recoupling three angular momenta can be done easily if they have the right form:

$$\langle j_{12}, j_3, j | j_1, j_{23}, j \rangle = (-)^{j_1+j_2+j_3+j} \sqrt{(2j_{12}+1)(2j_{23}+1)} \begin{Bmatrix} j_1 & j_2 & j_{12} \\ j_3 & j & j_{23} \end{Bmatrix} \quad (\text{A.3})$$

A.0.1.3 Recoupling four angular momenta

Same goes for four:

$$\langle j_{12}, j_{34}, j | j_{13}, j_{24}, j \rangle = \sqrt{(2j_{13} + 1)(2j_{24} + 1)(2j_{14} + 1)(2j_{23} + 1)} \begin{Bmatrix} j_1 & j_2 & j_{12} \\ j_3 & j_4 & j_{34} \\ j_{13} & j_{24} & j \end{Bmatrix} \quad (\text{A.4})$$

Example: LS to JJ For instance, converting from LS to JJ coupling the coefficients are expressed:

$$\langle (l_c s_c) j_c, (l_o s_o) j_o, J | (l_s l_o) L, (s_c s_o) S, J \rangle = \sqrt{(2j_c + 1)(2j_o + 1)(2L + 1)(2S + 1)} \begin{Bmatrix} l_c & l_o & L \\ s_c & s_o & S \\ j_c & j_o & J \end{Bmatrix} \quad (\text{A.5})$$

A.0.2 More complicated recouplings

A.0.2.1 JK to LS

Decomposing a state in the JK-coupled basis into LS/Russell-Saunders coupled states is more complex than simply a single $6j$ symbol. The formula is:

$$|J_C, K, J, m_J\rangle = \sum_{L, S} (-)^{-l_c - l_o - 2s_c - s_o - K - L - J} \sqrt{(2J_c + 1)(2L + 1)} \sqrt{(2K + 1)(2S + 1)} \\ \times \begin{Bmatrix} s_c & l_c & J_c \\ l_o & K & L \end{Bmatrix} \begin{Bmatrix} L & s_c & K \\ s_o & J & S \end{Bmatrix} |L, S, J, m_J\rangle \quad (\text{A.6})$$

A.0.3 Reduction Theorems

These theorems are used in almost every atomic matrix element calculation to simplify the problem. In Wes' words, "Use the Wigner-Eckart theorem whenever you can."

A.0.3.1 The Wigner-Eckart Theorem

The Wigner-Eckart theorem tells us that for the q -th element a spherical tensor operator or rank k , the matrix element can be reduced to a $3j$ symbol that encodes all the rotational dependence, and a rotationally invariant *reduced matrix element*, denoted with double bars $\langle j || T^{(k)} || j' \rangle$:

$$\langle j, m | T_q^{(k)} | j', m' \rangle = (-)^{j-m} \begin{pmatrix} j & k & j' \\ -m & q & m' \end{pmatrix} \langle j || T^{(k)} || j' \rangle \quad (\text{A.7})$$

A.0.3.2 The repeated reduction formula

Also called the spectator theorem, this allows you to "factor out" parts of a reduced matrix element that do not participate in the interaction. If $J = j_1 + j_2$, and T only acts on the j_1 subspace:

$$\langle j_1, j_2, J || T^{(k), j_1} || j'_1, j'_2, J' \rangle = \delta_{j_2 j'_2} (-)^{J+j'_1+j'_2+k} \sqrt{(2J+1)(2J'+1)} \begin{Bmatrix} j_1 & j'_1 & k \\ J' & J & j_2 \end{Bmatrix} \langle j_1 || T^{(k)} || j'_1 \rangle \quad (\text{A.8})$$

A.0.4 Tensor Operator Math

A.0.4.1 Dot products of tensor operators

$$A^{(k)} \cdot B^{(k)} = \sum_q (-)^q A_q^{(k)} B_{-q}^{(k)} \quad (\text{A.9})$$

$$A^{(k)} \cdot B^{(k)} = (-)^k \sqrt{2k+1} [A^{(k)} \otimes B^{(k)}]_0^{(0)} \quad (\text{A.10})$$

$$\begin{aligned} \langle \gamma, j_1, j_2, J, m | \mathbf{T}^{(k,1)} \cdot \mathbf{U}^{(k,2)} | \gamma', j'_1, j'_2, J', m' \rangle = \\ \delta_{J,J'} \delta_{m,m'} (-)^{j'_1+j_2+J} \begin{Bmatrix} j_1 & j_2 & J \\ j'_2 & j'_1 & k \end{Bmatrix} \sum_{\gamma''} \langle \gamma, j_1 || T^{(k)} || \gamma'', j'_1 \rangle \langle \gamma'', j_2 || U^{(k)} || \gamma', j'_2 \rangle \end{aligned} \quad (\text{A.11})$$

A.0.4.2 Multiplication (contraction) of tensor operators

$$T_q^{(k)} = \sum_{q_1, q_2} \langle k_1, q_1; k_2, q_2 | k, q \rangle T_{q_1}^{(k_1)} T_{q_2}^{(k_2)} \quad (\text{A.12})$$

$$\equiv [T^{(k_1)} \otimes T^{(k_2)}]_q^{(k)} \quad (\text{A.13})$$

$$= \sum_{q_1, q_2} (-)^{k_1-k_2+q} \sqrt{2k+1} \begin{pmatrix} k_1 & k_2 & k \\ q_1 & q_2 & -q \end{pmatrix} T_{q_1}^{(k_1)} T_{q_2}^{(k_2)} \quad (\text{A.14})$$

A.0.5 Explicit values for reduced matrix elements

$$\langle j || J^{(0)} || j \rangle = \sqrt{2j+1} \quad (\text{A.15})$$

$$\langle j || J^{(1)} || j \rangle = \sqrt{j(j+1)(2j+1)} \quad (\text{A.16})$$

$$\langle j || J^{(2)} || j \rangle = \sqrt{\frac{j(j+1)(2j-1)(2j+1)(2j+3)}{6}} \quad (\text{A.17})$$

For more of these, see Application 13 in chapter 5 of Zare [Zar91]

APPENDIX B

Equivalence of Zeeman Matrix Elements

In deriving the matrix element of the Zeeman effect, one can arrive at two different expressions, depending on whether one begins with the Wigner-Eckart theorem, or with a change of basis. Here, we will use diagrammatic methods to demonstrate their equality. A good introduction to these methods can be found in [Zar91], with a very in-depth treatment given in [KMV88]. In the derivation below, we use the relatively common shorthand $[F] \equiv (2F + 1)$.

We begin with the expression 2.29 obtained by recoupling from the F, m_F basis to the m_I, m_J basis:

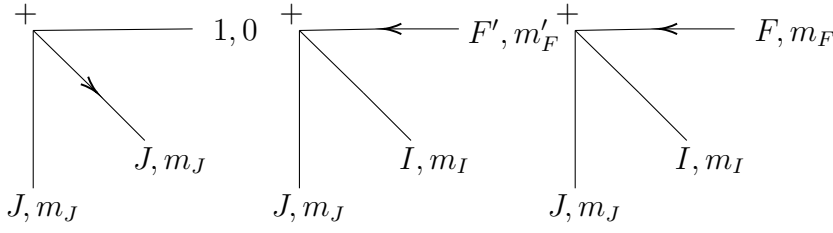
$$[F']^{1/2}[F]^{1/2} \sum_{m_I, m_J} (-)^{m'_F + m_F + 2J - 2I} m_J \begin{pmatrix} J & I & F' \\ m_J & m_I & -m'_F \end{pmatrix} \begin{pmatrix} J & I & F \\ m_J & m_I & -m_F \end{pmatrix} \quad (\text{B.1})$$

We must first recognize that m_J represents the z-component of a rank-1 spherical tensor: $m_J = \langle J | J_0^{(1)} | J \rangle$. Applying the Wigner-Eckart theorem and noting that $\langle J || J || J \rangle = \sqrt{J(J+1)(2J+1)}$, this allows us to write $m_J = \sqrt{J(J+1)(2J+1)} \begin{pmatrix} J & J & 1 \\ m_J & -m_J & 0 \end{pmatrix}$, giving us

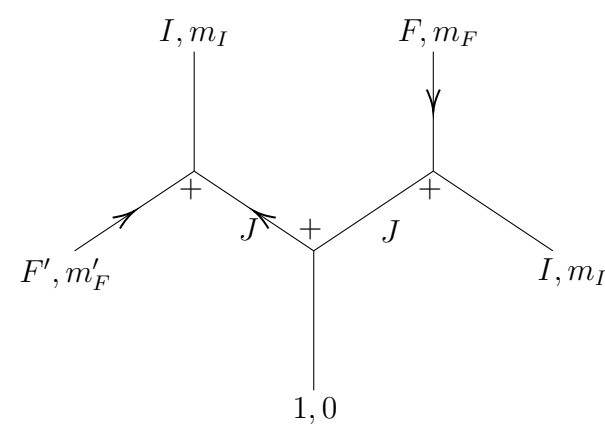
$$= [F']^{1/2}[F]^{1/2} \sqrt{J(J+1)(2J+1)} \sum_{m_I, m_J} (-)^{m'_F + m_F + 2J - 2I + m_J} \begin{pmatrix} J & J & 1 \\ m_J & -m_J & 0 \end{pmatrix} \begin{pmatrix} J & I & F' \\ m_J & m_I & -m'_F \end{pmatrix} \begin{pmatrix} J & I & F \\ m_J & m_I & -m_F \end{pmatrix} \quad (\text{B.2})$$

We can now start to use diagrammatic methods. We convert each of our $3j$ symbols into vertices, and start combining edges where the projection quantum numbers m are being

summed over:

$$= [F']^{1/2} [F]^{1/2} \sqrt{J(J+1)(2J+1)} \sum_{m_I, m_J} (-)^{-F-F'-2I}$$


(B.3)

$$= [F']^{1/2} [F]^{1/2} \sqrt{J(J+1)(2J+1)} \sum_{m_I} (-)^{-F-F'-2I}$$


(B.4)

$$= [F']^{1/2} [F]^{1/2} \sqrt{J(J+1)(2J+1)} (-)^{-F-F'-2I}$$

(B.5)

We now use identity 4.55 from [Zar91] to remove the three remaining geometrically-dependent legs and turn them into a new $3j$ symbol and a geometrically invariant part. This is essentially a form of the Wigner-Eckart theorem.

$$= [F']^{1/2} [F]^{1/2} \sqrt{J(J+1)(2J+1)} (-)^{J-I-F'}$$

(B.6)

$$= [F']^{1/2} [F]^{1/2} \sqrt{J(J+1)(2J+1)} (-)^{3J+I-2F+m_F-1}$$

$$(B.7)$$

Finally, we convert back into $3j$ and $6j$ symbols to recover equation 2.30.

$$= [F']^{1/2} [F]^{1/2} \sqrt{J(J+1)(2J+1)} (-)^{J-I+2F-m_F+1} \begin{Bmatrix} F' & F & 1 \\ J & J & I \end{Bmatrix} \begin{pmatrix} F & 1 & F' \\ -m_F & 0 & m_F' \end{pmatrix} \quad (B.8)$$

APPENDIX C

Ytterbium Hyperfine Literature

Table C.1: Hyperfine coefficients reported in the literature for $^{171}\text{Yb}^+$

State	A (MHz)	Type	Source
$^2S_{1/2}$	12642.8121242(14)	Experiment	[BSW83]
	12645(2)	Experiment	[MGH94]
	12642.812118466(2)	Experiment	[FSL97]
	12642.812118471(9)	Experiment	[TSB95]
	12642.8121184690(8)	Experiment	[HXZ24]
$^2P_{1/2}^o$	2104.9(13)	Experiment	[MGH94]
$^3[3/2]_{1/2}^o$	-2500	Experiment	[ET96]
	-2210	Experiment	[Rom21]
$^2P_{3/2}^o$	877(20)	Experiment	[BM92]
	875.4(10)	Experiment	[FFE17]
$^2D_{3/2}$	430	Experiment	[ET96]
	400.48	Theory	[Ita06]
$^1[3/2]_{3/2}^o$	4430.5(30)	Experiment	[Rom21]
$^1[5/2]_{5/2}^o$	-107(6)	Experiment	[FFE17]
	199	Theory	[PSW11]
$^2D_{5/2}$	-63.6	Experiment	[RTG99]
	-12.58	Theory	[Ita06]
	-63.368(1)	Experiment	[TEM21]
$^2F_{7/2}^o$	905.0(5)	Experiment	[TRM99]
	1105	Theory	[PSW11]
	905.134(5)	Experiment	[Rom21]

Table C.2: Hyperfine coefficients reported in the literature for $^{173}\text{Yb}^+$

State	A (MHz)	B (MHz)	C (MHz)	Type	Source
$^2S_{1/2}$	-3508	—	—	Experiment	[KN56]
	-3497.24007985(9)	—	—	Experiment	[MBG87]
	-3497.5(6)	—	—	Experiment	[MGH94]
	-3529.66	—	—	Theory	[MA11]
$^2P_{1/2}^o$	-600	—	—	Experiment	[KN56]
	-581.2(4)	—	—	Experiment	[MGH94]
	-612.36	—	—	Theory	[MA11]
	-582(1)	—	—	Experiment	This Work
$^3[3/2]_{1/2}^o$	608	—	—	Theory	[Rom21]
	611.5(3)	—	—	Experiment	This Work
$^2P_{3/2}^o$	-245(10)	-1460(50)	—	Experiment	[BM92]
	-88.973	-1839.779	—	Theory	[MA11]
$^2D_{3/2}$	-110.31	951.4	—	Theory	[Ita06]
	-104.479	902.301	—	Theory	[MA11]
	-117.87(3)	963.3(21)	—	Experiment	This Work
$^1[3/2]_{3/2}^o$	-1230(30)	-580(120)	—	Experiment	This Work
$^1[5/2]_{5/2}^o$	-55	-1720	0	Theory	[PSW11]
$^2D_{5/2}$	3.47	1190.4	0	Theory	[Ita06]
	22.078	1165.046	0	Theory	[MA11]
	20(20)	1210(90)	0	Experiment	This Work
$^2F_{7/2}^o$	-304	-3680	0	Theory	[PSW11]
	-240	-4762	0	Theory	[DF16]
	-241	-5330	0.00	Theory	[XLC20]
	-252	-5622	0.00	Theory	[XLC20]
	-239	-5330	0.02	Theory	[XLC20]

APPENDIX D

Hyperfine Decay Branching Ratios & Lifetimes

If we wish to determine the relative amplitude of various hyperfine components of a decay, the repeated reduction formula (A.8) is our friend. Take, for instance, the possible decays of the $^2F_{7/2}^o$ state in $^{171}\text{Yb}^+$. The $F = 3$ level can decay via electric octopole decay to both the $F = 0$ and $F = 1$ state in the $^2S_{1/2}$, but the $F = 4$ state is only able to decay to the $F = 1$ state. We know from Fermi's golden rule that the decay rate is proportional to the square of the reduced matrix element of the transition:

$$\Gamma_{F,F'} \propto \frac{\left| \langle ^2S_{1/2} F' || T^{(3)} || ^2F_{7/2}^o F \rangle \right|^2}{2F + 1} \quad (\text{D.1})$$

We can apply the repeated reduction formula to extract the dependence on F into a geometric prefactor applied to the total lifetime of the state:

$$\Gamma_{F,F'} \propto \left| \begin{Bmatrix} 1/2 & 7/2 & 3 \\ F & F' & 1/2 \end{Bmatrix} \langle ^2S_{1/2} || T^{(3)} || ^2F_{7/2}^o \rangle \right|^2 \quad (\text{D.2})$$

$$= (2F' + 1)(2J + 1) \begin{Bmatrix} 1/2 & 7/2 & 3 \\ F & F' & 1/2 \end{Bmatrix}^2 \Gamma_{^2F_{7/2}} \quad (\text{D.3})$$

Plugging in the different values of F and F' , we obtain the following prefactors modifying each decay:

$$\Gamma_{3,0} = \frac{4}{7} \Gamma_{^2F_{7/2}} \quad (\text{D.4})$$

$$\Gamma_{3,1} = \frac{3}{7} \Gamma_{^2F_{7/2}} \quad (\text{D.5})$$

$$\Gamma_{4,1} = \Gamma_{^2F_{7/2}} \quad (\text{D.6})$$

Of note is that when we total the decay rate of the $F = 3$ and $F = 4$, we see that the total decay rate of the F state is indeed preserved. Figure D.1 shows a schematic of all the relative line

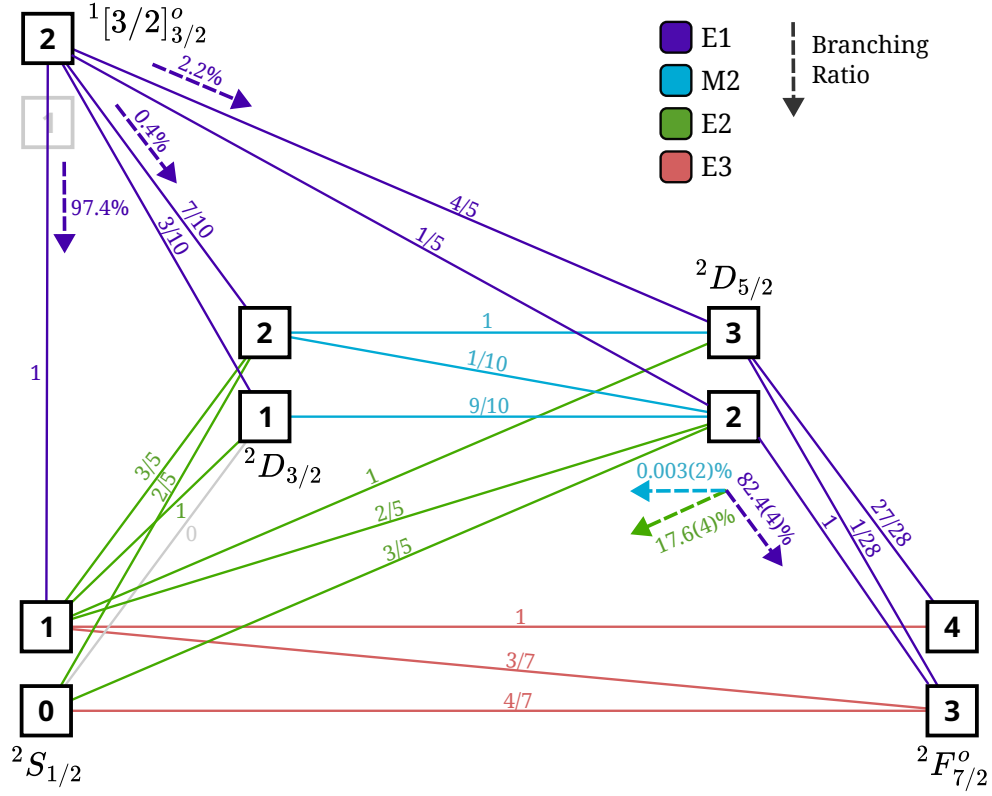


Figure D.1: A diagram of the branching ratios of states involved in shelving. Branching ratios from the $2D_{5/2}$ are from [TEM21] and this work, while those of the $1[3/2]_{3/2}^o$ are inferred from the DREAM database [QP20].

strengths involved in the shelving process.

APPENDIX E

Magnetometry Procedure

In order to correctly determine the magnetic field on the ion, we performed detailed magnetometry of the trap center. While the ion itself is a sensitive probe of the magnetic field magnitude, its direction must be inferred from the magnitude's dependence on the various coil currents. Let us begin by writing out the form of the field magnitude as a function of the current applied to a magnetic field coil:

$$|B(I_n)| = \sqrt{(\eta_n I_n + B_{BG,\parallel})^2 + B_{BG,\perp}^2} \quad (\text{E.1})$$

This function describes a hyperbola with slope η_n , minimum value $B_{BG,\perp}$, and horizontal offset $B_{BG,\parallel}$. By scanning the current across a wide range, the coil constant η can be determined, as well as the projection of the coil's magnetic field onto the background magnetic field.

Performing such scans for each of the three coils allows one to determine the background magnetic field along the three axes of the coils. If the coils are perfectly orthogonal, we can stop here. However, in the case that the coils are not orthogonal, we can still determine the geometry of the magnetic field by quantifying the angles between the coils. To do this, we perform six further scans, two for each coil, in which a constant current is applied on one of the other coils during the scan. In this case, the form of the hyperbola becomes

$$|B(I_n)| = \sqrt{(\eta_n I_n + B_{BG,\parallel} + \eta_m I_m \cos \theta_{nm})^2 + (B_{BG,\perp} + \eta_m I_m \sin \theta_{nm})^2} \quad (\text{E.2})$$

By comparing the difference in offsets between the unperturbed hyperbola and the perturbed hyperbola, we can extract the quantity $\cos \theta_{nm}$ for each pair of coils.

From here, extracting the magnetic field due to the coils is a matter of somewhat convoluted geometry. We must first construct an orthogonal system of axes on which to define the axes of our

coils. One coil can be chosen to define the z-axis, and a second coil can be chosen to define the x-z plane. The third axis is then fixed by its orthogonality to the first two. Skipping the tedious algebra, we can write the expressions for the three coil axes in terms of the angles θ_{nm} :

$$\hat{u}_1 = \begin{pmatrix} \sin \theta_{13} \\ 0 \\ \cos \theta_{13} \end{pmatrix}, \hat{u}_2 = \begin{pmatrix} \cos \varphi \sin \theta_{23} \\ \sin \varphi \sin \theta_{23} \\ \cos \theta_{23} \end{pmatrix}, \hat{u}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (\text{E.3})$$

$$\cos \varphi = \frac{\cos \theta_{12} - \cos \theta_{13} \cos \theta_{23}}{\sin \theta_{13} \sin \theta_{23}} \quad (\text{E.4})$$

The background field may be computed as

$$\mathbf{B}_{BG} = \left(\frac{B_{1\parallel} - \hat{u}_{1z} B_{3\parallel}}{\hat{u}_{1x}} \right) \hat{x} + \left(\frac{B_{2\parallel} - \hat{u}_{2x} B_{1\parallel} - \hat{u}_{2z} B_{3\parallel}}{\hat{u}_{2y}} \right) \hat{y} + (\hat{u}_3 \cdot B_{3\parallel}) \hat{z} \quad (\text{E.5})$$

Where $B_{i\parallel}$ is defined as the magnetic field along the axis of coil i , as determined by E.2. We can now write a formula for the magnetic field as a function of the coil currents:

$$\mathbf{B}(\mathbf{I}) = \hat{\mathbf{U}}\mathbf{I} + \mathbf{B}_{BG} \quad (\text{E.6})$$

Where $\hat{\mathbf{U}}$ is the transformation matrix whose columns are $\hat{u}_i$, and $\mathbf{I}$ is the vector of coil currents. We can invert the above formula to compute the coil currents needed to produce a total field. We tested this procedure, and found that after proper calibration, we can estimate the magnetic field up to several milligauss.

APPENDIX F

Zeeman Dependence of the F State in $^{173}\text{Yb}^+$

Table F.1: The magnetic field dependence of the F state in
at low field to fourth order.

F	m_F	1st-Order (kHz/G)	2nd-Order (Hz/G ²)	3rd-Order (Hz/G ³)	4th-Order (Hz/G ⁴)
6	-6	5.59961×10^6	0	0	0
6	-5	4.66634×10^6	-162.694	0.0113	0
5	-5	4.93305×10^6	162.694	-0.0113	0
6	-4	3.73308×10^6	-295.807	0.0165	0
5	-4	3.94644×10^6	-675.851	0.2466	0.0004
4	-4	4.31983×10^6	971.658	-0.2631	-0.0004
6	-3	2.79981×10^6	-399.339	0.0167	0
5	-3	2.95983×10^6	-1328.05	0.3337	-0.0004
4	-3	3.79993×10^6	-27654.8	-206.275	11.1994
3	-3	3.23989×10^6	29382.2	205.925	-11.1990
6	-2	1.86654×10^6	-473.291	0.0132	0
5	-2	1.97322×10^6	-1793.91	0.2932	-0.0016
4	-2	3.46677×10^6	-4335.93	-5.3956	-0.0084
3	-2	2.5342×10^6	-43544.4	-138.176	22.6236
2	-2	2.15901×10^6	50147.6	143.265	-22.6136
6	-1	933269	-517.662	0.0072	0
5	-1	3.60032×10^6	-5214.78	-11.7664	-0.01327
4	-1	986610.	-2073.42	0.1679	-0.0026
3	-1	1.73338×10^6	-1722.72	7.4583	-0.0044
2	-1	1.26871×10^6	-53408.4	76.4575	26.0203
1	-1	1.07789×10^6	62937.	-72.3246	-25.9999
6	0	0	-532.453	0	0

Table F.1: (continued) Magnetic field dependence of the F
state

F	m_F	1st-Order (Hz/G)	2nd-Order (Hz/G ²)	3rd-Order (Hz/G ³)	4th-Order (Hz/G ⁴)
5	0	0	-6953.12	0.0015	0.0271
4	0	0	-2166.6	0.0003	-0.0030
3	0	0	-851.496	-0.0064	-0.0466
2	0	0	-56819.4	290.463	23.0896
1	0	0	67323.1	-290.458	-23.0670
6	1	-933269	-517.662	-0.0072	0
5	1	-3.60032×10^6	-5214.87	11.763	-0.01530
4	1	-986610	-2073.43	-0.1675	-0.0027
3	1	-1.73338×10^6	-1722.51	-7.4613	0.0006
2	1	-1.26426×10^6	-53617.9	409.107	16.7767
1	1	-1.08235×10^6	63146.3	-413.234	-16.7593
6	2	-1.86654×10^6	-473.291	-0.0132	0
5	2	-1.97322×10^6	-1793.91	-0.2930	-0.0017
4	2	-3.46677×10^6	-4335.88	5.3926	-0.0073
3	2	-2.53181×10^6	-43898.4	394.853	9.3976
2	2	-2.1614×10^6	50501.5	-399.939	-9.3886
6	3	-2.79981×10^6	-399.339	-0.0167	0
5	3	-2.95983×10^6	-1328.05	-0.3337	-0.0004
4	3	-3.79947×10^6	-27934.6	254.278	2.8378
3	3	-3.24035×10^6	29661.9	-253.928	-2.8374
6	4	-3.73308×10^6	-295.807	-0.0165	0
5	4	-3.94644×10^6	-675.85	-0.2466	0.0005
4	4	-4.31983×10^6	971.657	0.2631	-0.0005
6	5	-4.66634×10^6	-162.694	-0.0113	0
5	5	-4.93305×10^6	162.694	0.0113	0
6	6	-5.59961×10^6	0	0	0

APPENDIX G

FOFI Qubits

This appendix contains data on all the First-Order Field-Insensitive qubits I computed with my Zeeman structure code.

G.1 $^{173}\text{Yb}^+$

G.1.1 Ground State

Table G.1: All first-order field-insensitive qubits in the ground state of $^{173}\text{Yb}^+$

B (G)	Qubit ($ F, m_F\rangle \leftrightarrow F', m'_F\rangle$)	splitting (MHz)	2nd order (kHz/G ²)
642.137	$ 2, -2\rangle \leftrightarrow 3, -1\rangle$	-10,340.0	-0.74
642.673	$ 2, 1\rangle \leftrightarrow 3, 0\rangle$	-10,339.7	-0.74
1248.04	$ 2, -1\rangle \leftrightarrow 3, -1\rangle$	-9,890.9	-0.79
1944.73	$ 2, -3\rangle \leftrightarrow 3, -2\rangle$	-9,026.3	-0.85
1945.2	$ 2, 0\rangle \leftrightarrow 3, -1\rangle$	-9,025.5	-0.85
2495.85	$ 2, 0\rangle \leftrightarrow 3, -1\rangle$	-7,819.0	-1.00

G.1.2 F State

Table G.2: All first-order field-insensitive qubits below 2000 Gauss in the F state of $^{173}\text{Yb}^+$. Qubits are labeled by their zero-field quantum numbers, but it should be remembered that these quantum numbers no longer apply in the intermediate-field regime. Due to the chaotic nature of the Zeeman effect [PS24], the nature of these qubits is also highly dependent on the exact hyperfine structure of the F state, which has not yet been measured with precision.

B (G)	Qubit $F, m_F\rangle \leftrightarrow F', m'_F\rangle$	splitting (MHz)	2nd order (kHz/G²)
0.0	$ 3, 0\rangle \leftrightarrow 4, 0\rangle$	79	242.9
0.0	$ 5, 0\rangle \leftrightarrow 6, 0\rangle$	-5,970	-6.4
0.0	$ 4, 0\rangle \leftrightarrow 5, 0\rangle$	-278	-4.8
0.0	$ 2, 0\rangle \leftrightarrow 3, 0\rangle$	750	56.0
0.0	$ 1, 0\rangle \leftrightarrow 2, 0\rangle$	828	12.2
0.7	$ 4, 0\rangle \leftrightarrow 5, 1\rangle$	-1,372	-127.1
0.8	$ 3, -1\rangle \leftrightarrow 4, -1\rangle$	79	229.2
1.8	$ 4, 2\rangle \leftrightarrow 5, 2\rangle$	-1,379	-99.6
2.0	$ 3, -2\rangle \leftrightarrow 4, -2\rangle$	79	188.3
4.6	$ 3, 1\rangle \leftrightarrow 4, 2\rangle$	77	179.4
4.7	$ 3, -3\rangle \leftrightarrow 4, -3\rangle$	78	119.8
4.8	$ 3, 0\rangle \leftrightarrow 4, 1\rangle$	77	208.8
4.8	$ 4, 3\rangle \leftrightarrow 5, 3\rangle$	-1,379	-53.8
4.9	$ 2, 1\rangle \leftrightarrow 3, 1\rangle$	751	-86.9
5.2	$ 3, 2\rangle \leftrightarrow 4, 3\rangle$	78	122.8
5.5	$ 3, -1\rangle \leftrightarrow 4, 0\rangle$	76	211.1
6.8	$ 2, -2\rangle \leftrightarrow 3, -3\rangle$	751	-48.6
7.1	$ 3, -2\rangle \leftrightarrow 4, -1\rangle$	74	185.9
7.3	$ 4, -3\rangle \leftrightarrow 5, -2\rangle$	-1,376	-109.4
7.8	$ 4, -2\rangle \leftrightarrow 5, -1\rangle$	-1,376	-108.1

Table G.2: (continued) All first-order field-insensitive qubits ($^{173}\text{Yb}^+$, F state).

B (G)	Qubit $F, m_F\rangle \leftrightarrow F', m'_F\rangle$	splitting (MHz)	2nd order (kHz/G²)
8.0	$ 4, -4\rangle \leftrightarrow 5, -3\rangle$	-1,376	-92.6
9.6	$ 4, -1\rangle \leftrightarrow 5, 0\rangle$	-1,374	-89.3
10.2	$ 2, -1\rangle \leftrightarrow 3, -2\rangle$	753	-65.5
10.8	$ 3, 3\rangle \leftrightarrow 4, 4\rangle$	77	37.9
10.9	$ 3, -3\rangle \leftrightarrow 4, -2\rangle$	71	132.5
11.5	$ 4, -5\rangle \leftrightarrow 5, -4\rangle$	-1,375	-56.9
14.5	$ 4, 0\rangle \leftrightarrow 5, 1\rangle$	-1,372	-54.6
16.3	$ 2, 0\rangle \leftrightarrow 3, -1\rangle$	758	-39.6
17.2	$ 5, -1\rangle \leftrightarrow 6, -1\rangle$	-3,824	3.1
20.9	$ 2, 2\rangle \leftrightarrow 3, 2\rangle$	757	-12.9
39.4	$ 4, 1\rangle \leftrightarrow 5, 2\rangle$	-1,361	-13.2
40.8	$ 5, -2\rangle \leftrightarrow 6, -2\rangle$	-3,826	2.6
41.4	$ 2, 2\rangle \leftrightarrow 3, 2\rangle$	756	7.5
49.1	$ 1, -1\rangle \leftrightarrow 2, -2\rangle$	825	3.7
60.7	$ 2, 0\rangle \leftrightarrow 3, -1\rangle$	751	15.0
89.9	$ 5, -3\rangle \leftrightarrow 6, -3\rangle$	-3,831	1.7
92.1	$ 2, 1\rangle \leftrightarrow 3, 1\rangle$	705	17.7
109.4	$ 4, 4\rangle \leftrightarrow 5, 4\rangle$	-1,359	-3.5
111.9	$ 2, -2\rangle \leftrightarrow 3, -3\rangle$	703	13.6
112.9	$ 2, -1\rangle \leftrightarrow 3, -2\rangle$	710	12.4
122.8	$ 2, 0\rangle \leftrightarrow 3, 0\rangle$	637	20.5
129.2	$ 1, -1\rangle \leftrightarrow 2, -1\rangle$	686	31.2
139.6	$ 5, -1\rangle \leftrightarrow 6, -1\rangle$	-3,803	-42.5
141.7	$ 2, -2\rangle \leftrightarrow 3, -2\rangle$	555	23.6
162.0	$ 2, -1\rangle \leftrightarrow 3, -1\rangle$	554	18.0
167.1	$ 2, -2\rangle \leftrightarrow 3, -1\rangle$	389	27.6
197.0	$ 5, -1\rangle \leftrightarrow 6, -1\rangle$	-3,816	8.8
205.2	$ 5, 4\rangle \leftrightarrow 6, 3\rangle$	-3,925	9.2
211.3	$ 5, 3\rangle \leftrightarrow 6, 2\rangle$	-3,928	6.3

Table G.2: (continued) All first-order field-insensitive qubits ($^{173}\text{Yb}^+$, F state).

B (G)	Qubit $F, m_F\rangle \leftrightarrow F', m'_F\rangle$	splitting (MHz)	2nd order (kHz/G²)
243.4	$ 1, 0\rangle \leftrightarrow 2, 1\rangle$	622	9.3
247.6	$ 5, 2\rangle \leftrightarrow 6, 1\rangle$	-3,944	3.6
250.3	$ 1, -1\rangle \leftrightarrow 2, 0\rangle$	372	11.6
268.8	$ 4, 2\rangle \leftrightarrow 5, 3\rangle$	-1,191	-5.4
271.5	$ 2, 0\rangle \leftrightarrow 3, 1\rangle$	404	3.0
307.6	$ 5, 5\rangle \leftrightarrow 6, 4\rangle$	-3,950	2.9
350.0	$ 3, -1\rangle \leftrightarrow 4, 0\rangle$	547	-3.0
380.4	$ 2, -1\rangle \leftrightarrow 3, 0\rangle$	253	1.7
391.2	$ 5, 0\rangle \leftrightarrow 6, -1\rangle$	-4,029	2.2
398.4	$ 5, 5\rangle \leftrightarrow 6, 5\rangle$	-3,770	-0.7
667.3	$ 2, 1\rangle \leftrightarrow 3, 2\rangle$	483	0.4
702.1	$ 4, -6\rangle \leftrightarrow 5, -5\rangle$	-1,222	-0.3
720.2	$ 5, 2\rangle \leftrightarrow 6, 1\rangle$	-3,847	-1.8
731.5	$ 5, -1\rangle \leftrightarrow 6, -1\rangle$	-3,688	-1.4
863.4	$ 2, 2\rangle \leftrightarrow 3, 3\rangle$	324	0.6
919.8	$ 5, 0\rangle \leftrightarrow 6, -1\rangle$	-3,944	-1.4
982.0	$ 5, 1\rangle \leftrightarrow 6, 1\rangle$	-2,410	-40.8
1007.7	$ 1, 1\rangle \leftrightarrow 2, 2\rangle$	2,379	-28.9
1059.6	$ 5, 0\rangle \leftrightarrow 6, 1\rangle$	-848	-6.3
1111.6	$ 5, 0\rangle \leftrightarrow 6, 0\rangle$	-2,977	-3.5
1177.5	$ 5, 3\rangle \leftrightarrow 6, 2\rangle$	-3,181	-3.2
1253.7	$ 5, 3\rangle \leftrightarrow 6, 4\rangle$	-3,071	-0.9
1293.6	$ 5, -3\rangle \leftrightarrow 6, -3\rangle$	-3,478	-0.9
1353.0	$ 5, 4\rangle \leftrightarrow 6, 4\rangle$	-3,350	-0.9
1438.7	$ 5, -1\rangle \leftrightarrow 6, 0\rangle$	-866	-5.6
1498.4	$ 5, -3\rangle \leftrightarrow 6, -2\rangle$	-2,577	-1.3
1513.0	$ 1, 1\rangle \leftrightarrow 2, 2\rangle$	2,166	1.9
1515.9	$ 5, 5\rangle \leftrightarrow 6, 4\rangle$	-3,414	-1.3
1537.9	$ 5, 4\rangle \leftrightarrow 6, 3\rangle$	-2,036	-3.8

Table G.2: (continued) All first-order field-insensitive qubits ($^{173}\text{Yb}^+$, F state).

B (G)	Qubit $ F, m_F\rangle \leftrightarrow F', m'_F\rangle$	splitting (MHz)	2nd order (kHz/G ²)
1601.7	$ 5, 1\rangle \leftrightarrow 6, 2\rangle$	-1,239	-3.8
1613.5	$ 5, -2\rangle \leftrightarrow 6, -1\rangle$	-1,585	-2.3
1645.7	$ 5, -2\rangle \leftrightarrow 6, -2\rangle$	-2,627	-1.6
1689.7	$ 5, 2\rangle \leftrightarrow 6, 3\rangle$	-2,239	-1.5
1729.7	$ 5, 3\rangle \leftrightarrow 6, 3\rangle$	-2,270	-1.9
1937.0	$ 2, 2\rangle \leftrightarrow 3, 3\rangle$	375	-0.1

G.2 Other Species' Ground States

G.2.1 $^9\text{Be}^+$

B (G)	Qubit ($ F, m_F\rangle \leftrightarrow F', m'_F\rangle$)	splitting (MHz)	2nd order (kHz/G ²)
119.472	$ 1, 0\rangle \leftrightarrow 2, 1\rangle$	1,207.5	6.10
119.766	$ 1, 1\rangle \leftrightarrow 2, 0\rangle$	1,207.3	6.10
223.162	$ 1, 1\rangle \leftrightarrow 2, 1\rangle$	1,082.4	7.25

G.2.2 $^{25}\text{Mg}^+$

B (G)	Qubit ($ F, m_F\rangle \leftrightarrow F', m'_F\rangle$)	splitting (MHz)	2nd order (kHz/G ²)
109.438	$ 2, -2\rangle \leftrightarrow 3, -1\rangle$	-1,763.1	-4.33
109.739	$ 2, 1\rangle \leftrightarrow 3, 0\rangle$	-1,763.0	-4.33
212.894	$ 2, -1\rangle \leftrightarrow 3, -1\rangle$	-1,686.5	-4.66
331.611	$ 2, -3\rangle \leftrightarrow 3, -2\rangle$	-1,539.1	-4.98
331.873	$ 2, 0\rangle \leftrightarrow 3, -1\rangle$	-1,538.7	-4.98
425.656	$ 2, -2\rangle \leftrightarrow 3, -2\rangle$	-1,333.1	-5.89

G.2.3 $^{43}\text{Ca}^+$

B (G)	Qubit ($ F, m_F\rangle \leftrightarrow F', m'_F\rangle$)	splitting (MHz)	2nd order (kHz/G ²)
145.931	$ 3, -2\rangle \leftrightarrow 4, -1\rangle$	-3,200.0	-2.41
146.763	$ 3, 1\rangle \leftrightarrow 4, 0\rangle$	-3,199.7	-2.41
288.028	$ 3, -1\rangle \leftrightarrow 4, -1\rangle$	-3,123.0	-2.51
439.627	$ 3, -3\rangle \leftrightarrow 4, -2\rangle$	-2,985.8	-2.59
440.403	$ 3, -2\rangle \leftrightarrow 4, -1\rangle$	-2,984.9	-2.59
575.901	$ 3, -2\rangle \leftrightarrow 4, -2\rangle$	-2,793.0	-2.81
738.667	$ 3, -4\rangle \leftrightarrow 4, -3\rangle$	-2,496.3	-3.12
739.310	$ 3, -1\rangle \leftrightarrow 4, -2\rangle$	-2,494.8	-3.12
863.465	$ 3, -3\rangle \leftrightarrow 4, -3\rangle$	-2,132.7	-3.68

G.2.4 $^{87}\text{Sr}^+$

B (G)	Qubit ($ F, m_F\rangle \leftrightarrow F', m'_F\rangle$)	splitting (MHz)	2nd order (kHz/G ²)
179.944	$ 4, -2\rangle \leftrightarrow 5, -1\rangle$	-4,977.2	-1.56
181.011	$ 4, 1\rangle \leftrightarrow 5, 0\rangle$	-4,976.9	-1.56
357.295	$ 4, -1\rangle \leftrightarrow 5, -1\rangle$	-4,901.2	-1.60
541.360	$ 4, -3\rangle \leftrightarrow 5, -2\rangle$	-4,769.5	-1.63
542.382	$ 4, 0\rangle \leftrightarrow 5, -1\rangle$	-4,768.6	-1.63
714.489	$ 4, -3\rangle \leftrightarrow 5, -2\rangle$	-4,584.3	-1.71
904.669	$ 4, -4\rangle \leftrightarrow 5, -3\rangle$	-4,322.5	-1.80
905.594	$ 4, -1\rangle \leftrightarrow 5, -2\rangle$	-4,321.0	-1.80
1071.48	$ 4, -3\rangle \leftrightarrow 5, -3\rangle$	-4,001.0	-1.96
1274.97	$ 4, -5\rangle \leftrightarrow 5, -4\rangle$	-3,537.2	-2.22
1275.72	$ 4, -2\rangle \leftrightarrow 5, -3\rangle$	-3,535.1	-2.22
1428.16	$ 4, -4\rangle \leftrightarrow 5, -4\rangle$	-3,000.3	-2.62

G.2.5 $^{135}\text{Ba}^+$

B (G)	Qubit ($ F, m_F\rangle \leftrightarrow F', m'_F\rangle$)	splitting (MHz)	2nd order (kHz/G ²)
686.122	$ 1, 1\rangle \leftrightarrow 2, 0\rangle$	6,939.0	1.06
686.924	$ 1, 0\rangle \leftrightarrow 2, 1\rangle$	6,938.4	1.06
1281.21	$ 1, 1\rangle \leftrightarrow 2, 1\rangle$	6,221.4	1.26

G.2.6 $^{137}\text{Ba}^+$

B (G)	Qubit ($ F, m_F\rangle \leftrightarrow F', m'_F\rangle$)	splitting (MHz)	2nd order (kHz/G ²)
767.640	$ 1, 1\rangle \leftrightarrow 2, 0\rangle$	7,764.4	0.95
768.644	$ 1, 0\rangle \leftrightarrow 2, 1\rangle$	7,763.7	0.95
1433.55	$ 1, 1\rangle \leftrightarrow 2, 1\rangle$	6,961.5	1.13

REFERENCES

- [AAA25] H. Aghaee Rad, T. Ainsworth, R. N. Alexander, B. Altieri, M. F. Askarani, R. Baby, L. Banchi, B. Q. Baragiola, J. E. Bourassa, R. S. Chadwick, I. Charania, H. Chen, M. J. Collins, P. Contu, N. D’Arcy, G. Dauphinais, R. De Prins, D. Deschenes, I. Di Luch, S. Duque, P. Edke, S. E. Fayer, S. Ferracin, H. Ferretti, J. Gefaell, S. Glancy, C. González-Arciniegas, T. Grainge, Z. Han, J. Hastrup, L. G. Helt, T. Hillmann, J. Hundal, S. Izumi, T. Jaeken, M. Jonas, S. Kocsis, I. Krasnokutska, M. V. Larsen, P. Laskowski, F. Laudенbach, J. Lavoie, M. Li, E. Lomonte, C. E. Lopetegui, B. Luey, A. P. Lund, C. Ma, L. S. Madsen, D. H. Mahler, L. Mantilla Calderón, M. Menotti, F. M. Miatto, B. Morrison, P. J. Nadkarni, T. Nakamura, L. Neuhaus, Z. Niu, R. Noro, K. Papirov, A. Pesah, D. S. Phillips, W. N. Plick, T. Rogalsky, F. Rortais, J. Sabines-Chesterking, S. Safavi-Bayat, E. Sazhaev, M. Seymour, K. Rezaei Shad, M. Silverman, S. A. Srinivasan, M. Stephan, Q. Y. Tang, J. F. Tasker, Y. S. Teo, R. B. Then, J. E. Tremblay, I. Tzitrin, V. D. Vaidya, M. Vasmer, Z. Vernon, L. F. S. S. M. Villalobos, B. W. Walshe, R. Weil, X. Xin, X. Yan, Y. Yao, M. Zamani Abnili, and Y. Zhang. “Scaling and networking a modular photonic quantum computer.” *Nature*, pp. 1–8, January 2025. Publisher: Nature Publishing Group.
- [AAB19] Frank Arute, Kunal Arya, Ryan Babbush, Dave Bacon, Joseph C. Bardin, Rami Barends, Rupak Biswas, Sergio Boixo, Fernando G.S.L. Brandao, David A. Buell, Brian Burkett, Yu Chen, Zijun Chen, Ben Chiaro, Roberto Collins, William Courtney, Andrew Dunsworth, Edward Farhi, Brooks Foxen, Austin Fowler, Craig Gidney, Marissa Giustina, Rob Graff, Keith Guerin, Steve Habegger, Matthew P. Harrigan, Michael J. Hartmann, Alan Ho, Markus Hoffmann, Trent Huang, Travis S. Humble, Sergei V. Isakov, Evan Jeffrey, Zhang Jiang, Dvir Kafri, Kostyantyn Kechedzhi, Julian Kelly, Paul V. Klimov, Sergey Knysh, Alexander Korotkov, Fedor Kostritsa, David Landhuis, Mike Lindmark, Erik Lucero, Dmitry Lyakh, Salvatore Mandrà, Jarrod R. McClean, Matthew McEwen, Anthony Megrant, Xiao Mi, Kristel Michielsen, Masoud Mohseni, Josh Mutus, Ofer Naaman, Matthew Neeley, Charles Neill, Murphy Yuezhen Niu, Eric Ostby, Andre Petukhov, John C. Platt, Chris Quintana, Eleanor G. Rieffel, Pedram Roushan, Nicholas C. Rubin, Daniel Sank, Kevin J. Satzinger, Vadim Smelyanskiy, Kevin J. Sung, Matthew D. Trevithick, Amit Vainsencher, Benjamin Villalonga, Theodore White, Z. Jamie Yao, Ping Yeh, Adam Zalcman, Hartmut Neven, and John M. Martinis. “Quantum supremacy using a programmable superconducting processor.” *Nature*, **574**(7779):505–510, October 2019.
- [ACC21] D. T. C. Allcock, W. C. Campbell, J. Chiaverini, I. L. Chuang, E. R. Hudson, I. D. Moore, A. Ransford, C. Roman, J. M. Sage, and D. J. Wineland. “*omg* blueprint for trapped ion quantum computing with metastable states.” *Applied*

Physics Letters, **119**(21):214002, November 2021.

- [AM13] I. Angeli and K. P. Marinova. “Table of experimental nuclear ground state charge radii: An update.” *Atomic Data and Nuclear Data Tables*, **99**(1):69–95, January 2013.
- [Arm71] Lloyd Armstrong. *Theory of the Hyperfine Structure of Free Atoms*. John Wiley & Sons, 1 edition, 1971.
- [ARS22] Fangzhao Alex An, Anthony Ransford, Andrew Schaffer, Lucas R. Sletten, John Gaebler, James Hostetter, and Grahame Vittorini. “High Fidelity State Preparation and Measurement of Ion Hyperfine Qubits with $I > 1/2$.” *Physical Review Letters*, **129**(13):130501, September 2022.
- [ASL24] Bailey Arnold, Christian Sanner, Mark Lide, and Wesley Hardin. “APS -2024 Annual Meeting of the APS Four Corners Section - Event - A novel Yb⁺ optical clock system for testing fundamental symmetries -Optimization of an ultra-stable optical cavity-.” In *Bulletin of the American Physical Society*, Flagstaff, Arizona, October 2024. American Physical Society.
- [AYJ23] A. Kramida, Yu. Ralchenko, J. Reader, and and NIST ASD Team. “The NIST Atomic Spectra Database.”, 2023. NIST Atomic Spectra Database (ver. 5.11), [Online]. Available: <https://physics.nist.gov/asd>. National Institute of Standards and Technology, Gaithersburg, MD.
- [BB02] D. J. Berkeland and M. G. Boshier. “Destabilization of dark states and optical spectroscopy in Zeeman-degenerate atomic systems.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **65**(3):13, February 2002.
- [BBF21] C. H. Baldwin, B. J. Bjork, M. Foss-Feig, J. P. Gaebler, D. Hayes, M. G. Kokish, C. Langer, J. A. Sedlacek, D. Stack, and G. Vittorini. “High-fidelity light-shift gate for clock-state qubits.” *Physical Review A*, **103**(1):012603, January 2021. Publisher: American Physical Society.
- [BBF23] Abhishek Banerjee, Dmitry Budker, Melina Filzinger, Nils Huntemann, Gil Paz, Gilad Perez, Sergey Porsev, and Marianna Safronova. “Oscillating nuclear charge radii as sensors for ultralight dark matter.”, January 2023. arXiv:2301.10784 [hep-ph].
- [BCH19] S. M. Brewer, J.-S. Chen, A. M. Hankin, E. R. Clements, C. W. Chou, D. J. Wineland, D. B. Hume, and D. R. Leibrandt. “²⁷Al⁺ Quantum-Logic Clock with a Systematic Uncertainty below 10^{−18}.” *Physical Review Letters*, **123**(3):033201, July 2019. Publisher: American Physical Society.

- [BCM19] Colin D. Bruzewicz, John Chiaverini, Robert McConnell, and Jeremy M. Sage. “Trapped-ion quantum computing: Progress and challenges.” *Applied Physics Reviews*, **6**(2):021314, May 2019.
- [BDJ08] K. Beloy, A. Derevianko, and W. R. Johnson. “Hyperfine structure of the metastable $P_{3/2}$ state of alkaline-earth-metal atoms as an accurate probe of nuclear magnetic octupole moments.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **77**(1), January 2008.
- [BHL16] C. J. Ballance, T. P. Harty, N. M. Linke, M. A. Sepiol, and D. M. Lucas. “High-Fidelity Quantum Logic Gates Using Trapped-Ion Hyperfine Qubits.” *Physical Review Letters*, **117**(6):1–6, August 2016.
- [BM92] R. W. Berends and L. Maleki. “Hyperfine structure and isotope shifts of transitions in neutral and singly ionized ytterbium.” *Journal of the Optical Society of America B*, **9**(3):332, March 1992.
- [BMB98] D. J. Berkeland, J. D. Miller, J. C. Bergquist, W. M. Itano, and D. J. Wineland. “Minimization of ion micromotion in a Paul trap.” *Journal of Applied Physics*, **83**(10):5025–5033, May 1998.
- [BS70] B. Budick and J. Snir. “Hyperfine-Structure Anomalies of Stable Ytterbium Isotopes.” *Physical Review A*, **1**(3):545–551, March 1970.
- [BSW83] Reiner Blatt, H. Schnatz, and G. Werth. “Precise Determination of the $^{171}\text{Yb}^+$ Ground State Hyperfine Separation.” *Zeitschrift für Physik A*, **312**:143–147, April 1983.
- [But20] P. A. Butler. “Pear-shaped atomic nuclei.” *Proceedings. Mathematical, Physical, and Engineering Sciences*, **476**(2239):20200202, July 2020.
- [BWV23] Matthew J. Boguslawski, Zachary J. Wall, Samuel R. Vizvary, Isam Daniel Moore, Michael Bareian, David T. C. Allcock, David J. Wineland, Eric R. Hudson, and Wesley C. Campbell. “Raman Scattering Errors in Stimulated-Raman-Induced Logic Gates in $^{133}\text{Ba}^+$.” *Physical Review Letters*, **131**(6):063001, August 2023.
- [CA65] Robert D. Cowan and Kenneth L. Andrew. “Coupling Considerations in Two-Electron Spectra.” *Journal of the Optical Society of America*, **55**(5):502, May 1965.
- [Cam20] Wesley C. Campbell. “Certified quantum gates.” *Physical Review A*, **102**(2):022426, August 2020. Publisher: American Physical Society.
- [CCV19] Stephen Crain, Clinton Cahall, Geert Vrijsen, Emma E. Wollman, Matthew D. Shaw, Varun B. Verma, Sae Woo Nam, and Jungsang Kim. “High-speed low-crosstalk detection of a $^{171}\text{Yb}^+$ qubit using superconducting nanowire single photon detectors.” *Communications Physics*, **2**(1):1–6, August 2019.

- [CGG17] William B. Cairncross, Daniel N. Gresh, Matt Grau, Kevin C. Cossel, Tanya S. Roussy, Yiqi Ni, Yan Zhou, Jun Ye, and Eric A. Cornell. “Precision Measurement of the Electron’s Electric Dipole Moment Using Trapped Molecular Ions.” *Physical Review Letters*, **119**(15):153001, October 2017. Publisher: American Physical Society.
- [CHA20] Ian Counts, Joonseok Hur, Diana P.L. Aude Craik, Honggi Jeon, Calvin Leung, Julian C. Berengut, Amy Geddes, Akio Kawasaki, Wonho Jhe, and Vladan Vuletić. “Evidence for nonlinear isotope shift in Yb^+ search for new boson.” *Physical Review Letters*, **125**(12), September 2020.
- [Chr20] Justin E Christensen. “*High-fidelity operation of a radioactive trapped-ion qubit, $^{133}\text{Ba}^+$* .”. PhD thesis, University of California, Los Angeles, 2020.
- [CNE24] Jwo-Sy Chen, Erik Nielsen, Matthew Ebert, Volkan Inlek, Kenneth Wright, Vandiver Chaplin, Andrii Maksymov, Eduardo Páez, Amrit Poudel, Peter Maunz, and John Gamble. “Benchmarking a trapped-ion quantum computer with 30 qubits.” *Quantum*, **8**:1516, November 2024. Publisher: Verein zur Förderung des Open Access Publizierens in den Quantenwissenschaften.
- [Col05] E Collett. “The Polarization Ellipse.”, 2005.
- [Cow70] Robert D. Cowan. “Cancellation Effects in Computed Atomic Transition Probabilities.” *Le Journal de Physique Colloques*, **31**(C4):C4–191–C4–201, January 1970.
- [CPZ11] Ye Cao, Shi-Guo Peng, Chao Zheng, and Gui-Lu Long. “Quantum Fourier Transform and Phase Estimation in Qudit System.” *Communications in Theoretical Physics*, **55**:790–794, May 2011. Publisher: IOP ADS Bibcode: 2011CoTPh..55..790C.
- [CTV17] Earl T. Campbell, Barbara M. Terhal, and Christophe Vuillot. “Roads towards fault-tolerant universal quantum computation.” *Nature*, **549**(7671):172–179, September 2017. arXiv:1612.07330 [quant-ph].
- [Der10] Andrei Derevianko. “Theory of magic optical traps for Zeeman-insensitive clock transitions in alkali-metal atoms.” *Physical Review A*, **81**(5):051606, May 2010.
- [DF16] V. A. Dzuba and V. V. Flambaum. “Hyperfine-induced electric dipole contributions to the electric octupole and magnetic quadrupole atomic clock transitions.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **93**(5):1–6, May 2016.
- [DFB10] V. A. Dzuba, V. V. Flambaum, K. Beloy, and A. Derevianko. “Hyperfine-mediated static polarizabilities of monovalent atoms and ions.” *Physical Review A*, **82**(6):062513, December 2010. Publisher: American Physical Society.

- [DFS16] V. A. Dzuba, V. V. Flambaum, M. S. Safronova, S. G. Porsev, T. Pruttivarasin, M. A. Hohensee, and H. Häffner. “Strongly enhanced effects of Lorentz symmetry violation in entangled Yb+ ions.” *Nature Physics*, **12**(5):465–468, May 2016.
- [DiV00] David P. DiVincenzo. “The Physical Implementation of Quantum Computation.” *Fortschritte der Physik*, **48**(9-11):771–783, 2000.
- [DKK21] R. P. De Groote, S. Kujanpaa, A. Koszorus, J. G. Li, and I. D. Moore. “Magnetic octupole moment of Yb 173 using collinear laser spectroscopy.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **103**(3):1–9, March 2021.
- [ET96] D. Engelke and Chr. Tamm. “Dark times in the resonance fluorescence of trapped ^{171}Yb ions caused by spontaneous quantum jumps to the $2D_{3/2}$ ($F=2$) state.” *Europhysics Letters*, **33**(5):347–352, February 1996.
- [ETM21] C. L. Edmunds, T. R. Tan, A. R. Milne, A. Singh, M. J. Biercuk, and C. Hempel. “Scalable hyperfine qubit state detection via electron shelving in the $^2D_{5/2}$ and $^2F_{7/2}$ manifolds in $^{171}\text{Yb}^+$.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **104**(1):012606, July 2021.
- [FFE17] T. Feldker, H. Füst, N. V. Ewald, J. Joger, and R. Gerritsma. “Spectroscopy of the $^2S_{1/2} \rightarrow ^2P_{3/2}$ transition in Yb II: Isotope shifts, hyperfine splitting and branching ratios.” *The Astrophysical Journal*, **630**(2):1213–1216, November 2017.
- [FM23] V. V. Flambaum and A. J. Mansour. “Variation of the Quadrupole Hyperfine Structure and Nuclear Radius due to an Interaction with Scalar and Axion Dark Matter.” *Physical Review Letters*, **131**(11):113004, September 2023.
- [Foo05] C. J. Foot. *Atomic Physics*. Oxford Master Series in Atomic, Optical and Laser Physics. Oxford University Press, 2005.
- [FSL97] Peter T. H. Fisk, Matthew J. Sellars, Malcolm A. Lawn, and Colin Coles. “Accurate Measurement of the 12.6 GHz ”Clock” Transition in Trapped $^{171}\text{Yb}^+$ Ions.” *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, **44**(2), March 1997.
- [GAL21] Kevin A. Gilmore, Matthew Affolter, Robert J. Lewis-Swan, Diego Barberena, Elena Jordan, Ana Maria Rey, and John J. Bollinger. “Quantum-enhanced sensing of displacements and electric fields with two-dimensional trapped-ion crystals.” *Science*, **373**(6555):673–678, August 2021. Publisher: American Association for the Advancement of Science.
- [GC97] Neil A. Gershenfeld and Isaac L. Chuang. “Bulk Spin-Resonance Quantum Computation.” *Science*, **275**(5298):350–356, January 1997. Publisher: American Association for the Advancement of Science.

- [Gho95] Pradip Ghosh. *Ion Traps*. The International Series of Monographs on Physics. Oxford University Press, 1995.
- [GWI21] Michal Gulka, Daniel Wirtitsch, Viktor Ivády, Jelle Vodnik, Jaroslav Hruby, Goele Magchiels, Emilie Bourgeois, Adam Gali, Michael Trupke, and Milos Nesladek. “Room-temperature control and electrical readout of individual nitrogen-vacancy nuclear spins.” *Nature Communications*, **12**(1):4421, July 2021. Publisher: Nature Publishing Group.
- [HAB14] T. P. Harty, D. T.C. Allcock, C. J. Ballance, L. Guidoni, H. A. Janacek, N. M. Linke, D. N. Stacey, and D. M. Lucas. “High-fidelity preparation, gates, memory, and readout of a trapped-ion quantum bit.” *Physical Review Letters*, **113**(22):2–6, November 2014.
- [HAC22] Joonseok Hur, Diana P. L. Aude Craik, Ian Counts, Eugene Knyazev, Luke Caldwell, Calvin Leung, Swadha Pandey, Julian C. Berengut, Amy Geddes, Witold Nazarewicz, Paul-Gerhard Reinhard, Akio Kawasaki, Honggi Jeon, Wonho Jhe, and Vladan Vuletić. “Evidence of Two-Source King Plot Nonlinearity in Spectroscopic Search for New Boson.” *Physical Review Letters*, **128**(16):163201, April 2022.
- [Har22] Shannon Harvey. “Quantum Dots / Spin Qubits.”, February 2022. arXiv:2204.04261 [cond-mat].
- [Hof14] Matthew Hoffman. “*Observation of the Nuclear Magnetic Octupole Moment of $^{137}\text{Ba}^+$* .”. PhD thesis, University of Washington, 2014.
- [HOL12] N. Huntemann, M. Okhapkin, B. Lipphardt, S. Weyers, Chr. Tamm, and E. Peik. “High-Accuracy Optical Clock Based on the Octupole Transition in $^{171}\text{Yb}^+$.” *Physical Review Letters*, **108**(9):090801, February 2012. Publisher: American Physical Society.
- [HRW07] D. B. Hume, T. Rosenband, and D. J. Wineland. “High-Fidelity Adaptive Qubit Detection through Repetitive Quantum Nondemolition Measurements.” *Physical Review Letters*, **99**(12):120502, September 2007. Publisher: American Physical Society.
- [HSL16] N. Huntemann, C. Sanner, B. Lipphardt, Chr. Tamm, and E. Peik. “Single-Ion Atomic Clock with 3×10^{-18} Systematic Uncertainty.” *Physical Review Letters*, **116**(6):063001, February 2016.
- [HXZ24] J. Z. Han, N. C. Xin, J. W. Zhang, Y. M. Yu, J. G. Li, L. Qian, and L. J. Wang. “Determination of the ground-state hyperfine splitting of trapped $^{171}\text{Yb}^+$ ions.” *Applied Physics Letters*, **125**(8):084002, August 2024.

- [Ip18] Michael Ip. “*Laser cooling Yb+ ions with optical frequency comb.*”. PhD thesis, UC Los Angeles, June 2018.
- [IRJ18] Michael Ip, Anthony Ransford, Andrew M. Jayich, Xueping Long, Conrad Roman, and Wesley C. Campbell. “Phonon Lasing from Optical Frequency Comb Illumination of Trapped Ions.” *Physical Review Letters*, **121**(4):043201, July 2018.
- [Ita01] W. M. Itano. “External-field shifts of the $^{199}\text{Hg}^+$ optical frequency standard.”, March 2001. arXiv:physics/0010056.
- [Ita06] Wayne M. Itano. “Quadrupole moments and hyperfine constants of metastable states of Ca^+ , Sr^+ , Ba^+ , Yb^+ , Hg^+ , and Au .” *Physical Review A - Atomic, Molecular, and Optical Physics*, **73**:1–11, February 2006.
- [ITV12] S. S. Ivanov, H. S. Tonchev, and N. V. Vitanov. “Time-efficient implementation of quantum search with qudits.” *Physical Review A*, **85**(6):062321, June 2012. Publisher: American Physical Society.
- [Jam98] Daniel F. V. James. “Quantum dynamics of cold trapped ions, with application to quantum computation.” *Applied Physics B: Lasers and Optics*, **66**(2):181–190, February 1998. arXiv:quant-ph/9702053.
- [JHC23] Shubham P. Jain, Eric R. Hudson, Wesley C. Campbell, and Victor V. Albert. “ $\mathbb{A}$ codes.”, November 2023.
- [Joh06] Walter R. Johnson. “Atomic structure theory: Lectures on atomic physics.” Lecture Notes, January 2006.
- [KGB16] Michaela Kleinert, M. E. Gold Dahl, and Scott Bergeson. “Measurement of the $\text{Yb } ^1S_0 - ^1P_1$ transition frequency at 399 nm using an optical frequency comb.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **94**(5):1–9, November 2016.
- [KH87] Kenneth S. Krane and David Halliday. *Introductory nuclear physics*. Wiley, New York, 1987.
- [KL97] Emanuel Knill and Raymond Laflamme. “Theory of quantum error-correcting codes.” *Physical Review A*, **55**(2):900–911, February 1997. Publisher: American Physical Society.
- [KL99] V. Alan Kostelecký and Charles D. Lane. “Constraints on Lorentz violation from clock-comparison experiments.” *Physical Review D*, **60**(11):116010, November 1999. Publisher: American Physical Society.

- [KLR08] E. Knill, D. Leibfried, R. Reichle, J. Britton, R. B. Blakestad, J. D. Jost, C. Langer, R. Ozeri, S. Seidelin, and D. J. Wineland. “Randomized benchmarking of quantum gates.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **77**(1):012307, January 2008.
- [KMV88] V. K. Khersonskii, A. N. Moskalev, and D. A. Varshalovich. *Quantum Theory Of Angular Momentum*. World Scientific Publishing Company, 1988. Accepted: 2021-08-20T11:39:57Z.
- [KN56] K. Krebs and H. Nelkowski. “Die Isotopieverschiebung in den Resonanzlinien des Yb II und im Singulettssystem des Yb I.” *Zeitschrift für Physik*, **145**(5):543–559, March 1956.
- [Kot19] Nikhil Kotibhaskar. “*Design and construction of an ion trapping apparatus for quantum simulation experiments.*”. PhD thesis, University of Waterloo, September 2019.
- [LPH21] R. Lange, A. A. Peshkov, N. Huntemann, Chr Tamm, A. Surzhykov, and E. Peik. “Lifetime of the $^2F_{7/2}$ Level in Yb^+ for Spontaneous Emission of Electric Octupole Radiation.” *Physical Review Letters*, **127**(21):213001, November 2021.
- [LSP10] N. Lundblad, M. Schlosser, and J. V. Porto. “Experimental observation of magic-wavelength behavior of ^{87}Rb atoms in an optical lattice.” *Physical Review A*, **81**(3):031611, March 2010.
- [LSR13] Fam Le Kien, Philipp Schneeweiss, and Arno Rauschenbeutel. “Dynamical polarizability of atoms in arbitrary light fields: general theory and application to cesium.” *The European Physical Journal D*, **67**(5):92, May 2013.
- [LWL19] K. A. Landsman, Y. Wu, P. H. Leung, D. Zhu, N. M. Linke, K. R. Brown, L. Duan, and C. Monroe. “Two-qubit entangling gates within arbitrarily long chains of trapped ions.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **100**(2):022332, August 2019.
- [LWM23] B.-W. Li, Y.-K. Wu, Q.-X. Mei, R. Yao, W.-Q. Lian, M.-L. Cai, Y. Wang, B.-X. Qi, L. Yao, L. He, Z.-C. Zhou, and L.-M. Duan. “Probing Critical Behavior of Long-Range Transverse-Field Ising Model through Quantum Kibble-Zurek Mechanism.” *PRX Quantum*, **4**(1):010302, January 2023. Publisher: American Physical Society.
- [LWS23] Pei Jiang Low, Brendan White, and Crystal Senko. “Control and Readout of a 13-level Trapped Ion Qudit.”, June 2023. arXiv:2306.03340 [quant-ph].
- [LZZ19] Yao Lu, Shuaining Zhang, Kuan Zhang, Wentao Chen, Yangchao Shen, Jialiang Zhang, Jing-Ning Zhang, and Kihwan Kim. “Global entangling gates on arbitrary ion qubits.” *Nature*, **572**(7769):363–367, August 2019.

- [MA10] B. K. Mani and D. Angom. “Atomic properties calculated by relativistic coupled-cluster theory without truncation: Hyperfine constants of Mg+, Ca+, Sr +, and Ba+.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **81**(4), April 2010.
- [MA11] B. K. Mani and D. Angom. “Fock-space relativistic coupled-cluster calculations of two-valence atoms.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **83**(1), January 2011.
- [MBA23] S. A. Moses, C. H. Baldwin, M. S. Allman, R. Ancona, L. Ascarrunz, C. Barnes, J. Bartolotta, B. Bjork, P. Blanchard, M. Bohn, J. G. Bohnet, N. C. Brown, N. Q. Burdick, W. C. Burton, S. L. Campbell, J. P. Campora, C. Carron, J. Chambers, J. W. Chan, Y. H. Chen, A. Chernoguzov, E. Chertkov, J. Colina, J. P. Curtis, R. Daniel, M. DeCross, D. Deen, C. Delaney, J. M. Dreiling, C. T. Ertsgaard, J. Esposito, B. Estey, M. Fabrikant, C. Figgatt, C. Foltz, M. Foss-Feig, D. Francois, J. P. Gaebler, T. M. Gatterman, C. N. Gilbreth, J. Giles, E. Glynn, A. Hall, A. M. Hankin, A. Hansen, D. Hayes, B. Higashi, I. M. Hoffman, B. Horning, J. J. Hout, R. Jacobs, J. Johansen, L. Jones, J. Karcz, T. Klein, P. Lauria, P. Lee, D. Liefer, S. T. Lu, D. Lucchetti, C. Lytle, A. Malm, M. Matheny, B. Mathewson, K. Mayer, D. B. Miller, M. Mills, B. Neyenhuis, L. Nugent, S. Olson, J. Parks, G. N. Price, Z. Price, M. Pugh, A. Ransford, A. P. Reed, C. Roman, M. Rowe, C. Ryan-Anderson, S. Sanders, J. Sedlacek, P. Shevchuk, P. Siegfried, T. Skripka, B. Spaun, R. T. Sprenkle, R. P. Stutz, M. Swallows, R. I. Tobey, A. Tran, T. Tran, E. Vogt, C. Volin, J. Walker, A. M. Zolot, and J. M. Pino. “A Race-Track Trapped-Ion Quantum Processor.” *Physical Review X*, **13**(4):041052, December 2023.
- [MBG87] A. Munch, M. Berkler, Ch. Gerz, D. Wilsdorf, and G. Werth. “Precise ground-state hyperfine splitting in $^{173}\text{Yb II}$.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **35**(10):4147–4150, May 1987.
- [MGH94] Ann-Marie Martensson-Pendrill, David S Gough, and Peter Hannaford. “Isotope shifts and hyperfine structure in the 369.4-nm 6s-6p $_{1/2}$ resonance line of single ionized ytterbium.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **49**(5), May 1994.
- [MS59] W. W. Macalpine and R. O. Schildknecht. “Coaxial Resonators with Helical Inner Conductor.” *Proceedings of the IRE*, **47**(12):2099–2105, December 1959. Conference Name: Proceedings of the IRE.
- [MWK24] Niklas Mueller, Tianyi Wang, Or Katz, Zohreh Davoudi, and Marko Cetina. “Quantum Computing Universal Thermalization Dynamics in a (2+1)D Lattice Gauge Theory.”, July 2024. arXiv:2408.00069.

- [NMM14] Daniel Nigg, Markus Mueller, Esteban A. Martinez, Philipp Schindler, Markus Hennrich, Thomas Monz, Miguel A. Martin-Delgado, and Rainer Blatt. “Experimental Quantum Computations on a Topologically Encoded Qubit.” *Science*, **345**(6194):302–305, July 2014. arXiv:1403.5426 [quant-ph].
- [NVG13] Rachel Noek, Geert Vrijsen, Daniel Gaultney, Emily Mount, Taehyun Kim, Peter Maunz, and Jungsang Kim. “High speed, high fidelity detection of an atomic hyperfine qubit.” *Optics Letters*, **38**(22):4735–4738, November 2013.
- [OIB07] R. Ozeri, W. M. Itano, R. B. Blakestad, J. Britton, J. Chiaverini, J. D. Jost, C. Langer, D. Leibfried, R. Reichle, S. Seidelin, J. H. Wesenberg, and D. J. Wineland. “Errors in trapped-ion quantum gates due to spontaneous photon scattering.” *Physical Review A*, **75**(4):042329, April 2007.
- [Olm09] S. Olmschenk. “*Quantum teleportation between distant matter qubits.*”. PhD thesis, University of Michigan, 2009.
- [Per13] Jonas R. Persson. “Table of hyperfine anomaly in atomic systems.” *Atomic Data and Nuclear Data Tables*, **99**(1):62–68, January 2013. arXiv:1110.5991 [nucl-ex, physics:physics].
- [PK15] Thaned Pruttivarasin and Hidetoshi Katori. “Compact field programmable gate array-based pulse-sequencer and radio-frequency generator for experiments with trapped atoms.” *Review of Scientific Instruments*, **86**(11), July 2015.
- [PS24] Octavian Postavaru and Mariana M. Stanescu. “Chaotic Zeeman effect: a fractional diffusion-like approach.” *Scientific Reports*, **14**(1):6356, March 2024.
- [PSW11] M. J. Petراسiunas, E. W. Streed, T. J. Weinhold, B. G. Norton, and D. Kielpinski. “Optogalvanic spectroscopy of metastable states in Yb+.” *Applied Physics B: Lasers and Optics*, **107**(4):1053–1059, November 2011.
- [QP20] Pascal Quinet and Patrick Palmeri. “Current status and developments of the atomic database on rare-earths at mons university (DREAM).” *Atoms*, **8**(2), February 2020.
- [RG21] B. M. Roberts and J. S. M. Ginges. “Hyperfine anomaly in heavy atoms and its role in precision atomic searches for new physics.” *Physical Review A*, **104**(2):022823, August 2021.
- [RGD09] P. Rosenbusch, S. Ghezali, V. A. Dzuba, V. V. Flambaum, K. Beloy, and A. Derevianko. “ac Stark shift of the Cs microwave atomic clock transitions.” *Physical Review A*, **79**(1):013404, January 2009.

- [RH07] Robert Raussendorf and Jim Harrington. “Fault-Tolerant Quantum Computation with High Threshold in Two Dimensions.” *Physical Review Letters*, **98**(19):190504, May 2007.
- [Rom21] Conrad Roman. “*Expanding the $^{171}\text{Yb}^+$ toolbox: the $^2F_{7/2}$ state as a resource for quantum information science.*”. PhD thesis, University of California, Los Angeles, June 2021.
- [Roo00] Roos, Christian. “*Controlling the quantum state of trapped ions.*”. PhD thesis, Leopold-Franzens-Universität Innsbruck, 2000.
- [RRD21] Anthony Ransford, Conrad Roman, Thomas Dellaert, Patrick McMillin, and Wesley C. Campbell. “Weak dissipation for high-fidelity qubit-state preparation and measurement.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **104**(6):3–7, 2021.
- [RRI20] Conrad Roman, Anthony Ransford, Michael Ip, and Wesley C. Campbell. “Coherent control for qubit state readout.” *New Journal of Physics*, **22**(7):073038, July 2020.
- [RTG99] M. Roberts, P. Taylor, S. V. Gateva-Kostova, R. B. M. Clarke, W. R. C. Rowley, and P. Gill. “Measurement of the $^2S_{1/2} - ^2D_{5/2}$ clock transition in a single $^{171}\text{Yb}^+$ ion.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **60**(4):2867–2872, October 1999.
- [SAN13] Alok K. Singh, D. Angom, and Vasant Natarajan. “Observation of the nuclear magnetic octupole moment of ^{173}Yb from precise measurements of the hyperfine structure in the 3P_2 state.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **87**(1), January 2013.
- [SAS11] Alexandre M. Souza, Gonzalo A. Álvarez, and Dieter Suter. “Robust Dynamical Decoupling for Quantum Computing and Quantum Memory.” *Physical Review Letters*, **106**(24):240501, June 2011.
- [SBS19] R. Srinivas, Burd, S. C., Sutherland, R. T., Wildon, A. C., Wineland, J. D., Leibfried, D., Allcock, D. T. C., and Slichter, D. H. “Trapped-Ion Spin-Motion Coupling with Microwaves and a Near-Motional Oscillating Magnetic Field Gradient.” *Physical Review Letters*, **122**(16), 2019.
- [Sch55] Charles Schwartz. “Theory of Hyperfine Structure.” *Physical Review*, **97**(2):380–395, January 1955.
- [SDR15] M. Schacht, J. R. Danielson, S. Rahaman, J. R. Torgerson, J. Zhang, and M. M. Schauer. “ $^{171}\text{Yb}^+ 5D_{3/2}$ Hyperfine State Detection and $F = 2$ Lifetime.” *Journal of Physics B: Atomic, Molecular and Optical Physics*, **48**(6), March 2015.

- [SHL19] Christian Sanner, Nils Huntemann, Richard Lange, Christian Tamm, Ekkehard Peik, Marianna S. Safronova, and Sergey G. Porsev. “Optical clock comparison for Lorentz symmetry testing.” *Nature*, **567**(7747):204–208, March 2019.
- [Sho97] Peter W. Shor. “Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer.” *SIAM Journal on Computing*, **26**(5):1484–1509, October 1997. arXiv:quant-ph/9508027.
- [SLM24] M. C. Smith, A. D. Leu, K. Miyanishi, M. F. Gely, and D. M. Lucas. “Single-qubit gates with errors at the 10^{-7} level.”, December 2024. arXiv:2412.04421 [quant-ph].
- [SLV24] A. S. Sotirova, J. D. Leppard, A. Vazquez-Brennan, S. M. Decoppet, F. Pokorny, M. Malinowski, and C. J. Ballance. “High-fidelity heralded quantum state preparation and measurement.”, September 2024. arXiv:2409.05805 [physics, physics:quant-ph].
- [Sob96] Igor Sobelman. *Atomic Spectra and Radiative Transitions*. Springer US, 1996.
- [SSL24] Ana S. Sotirova, Bangshan Sun, Jamie D. Leppard, Andong Wang, Mohan Wang, Andres Vazquez-Brennan, David P. Nadlinger, Simon Moser, Alexander Jesacher, Chao He, Fabian Pokorny, Martin J. Booth, and Christopher J. Ballance. “Low cross-talk optical addressing of trapped-ion qubits using a novel integrated photonic chip.” *Light: Science & Applications*, **13**(1):199, August 2024.
- [SSW12] J. D. Siverns, L. R. Simkins, S. Weidt, and W. K. Hensinger. “On the application of radio frequency voltages to ion traps via helical resonators.” *Applied Physics B*, **107**(4):921–934, June 2012.
- [Sto05] N.J. Stone. “Table of nuclear magnetic dipole and electric quadrupole moments.” *Atomic Data and Nuclear Data Tables*, **90**(1):75–176, May 2005.
- [TEM21] T. R. Tan, C. L. Edmunds, A. R. Milne, M. J. Biercuk, and C. Hempel. “Precision characterization of the $^2D_{5/2}$ state and the quadratic Zeeman coefficient in $^{171}\text{Yb}^+$.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **104**(1):L010802, July 2021.
- [Tho13] Thomas P. Harty. “*High-Fidelity Microwave-Driven Quantum Logic in Intermediate-Field 43Ca^+* .”. PhD thesis, Oxford University, 2013.
- [TPL21] A. Tretiakov, C. A. Potts, Y. Y. Lu, J. P. Davis, and L. J. LeBlanc. “Engineering atomic polarization with microwave-assisted optical pumping.”, October 2021. arXiv:2110.10673 [physics, physics:quant-ph].
- [TRB98] P. Taylor, M. Roberts, G. P. Barwood, and P. Gill. “Combined optical–infrared single-ion frequency standard.” *Optics Letters*, **23**(4):298–300, February 1998.

- [TRM99] P. Taylor, M. Roberts, G. M. Macfarlane, G. P. Barwood, W. R. C. Rowley, and P. Gill. “Measurement of the infrared $^2F_{7/2} - ^2D_{5/2}$ transition in a single $^{171}\text{Yb}^+$ ion.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **60**(4):2829–2833, October 1999.
- [TSB95] C. Tamm, D. Schnier, and A. Bauch. “Radio-frequency laser double-resonance spectroscopy of trapped ^{171}Yb ions and determination of line shifts of the ground-state hyperfine resonance.” *Applied Physics B*, **60**(1):19–29, January 1995.
- [VWB23] Samuel R. Vizvary, Zachary J. Wall, Matthew J. Boguslawski, Michael Bareian, Andrei Derevianko, Wesley C. Campbell, and Eric R. Hudson. “Eliminating qubit type cross-talk in the *omg* protocol.”, October 2023. arXiv:2310.10905 [physics, physics:quant-ph].
- [WBB23] Jonathan Wurtz, Alexei Bylinskii, Boris Braverman, Jesse Amato-Grill, Sergio H. Cantu, Florian Huber, Alexander Lukin, Fangli Liu, Phillip Weinberg, John Long, Sheng-Tao Wang, Nathan Gemelke, and Alexander Keesling. “Aquila: QuEra’s 256-Qubit Neutral-Atom Quantum Computer.”, June 2023.
- [WBD19] K. Wright, K. M. Beck, S. Debnath, J. M. Amini, Y. Nam, N. Grzesiak, J.-S. Chen, N. C. Pienti, M. Chmielewski, C. Collins, K. M. Hudek, J. Mizrahi, J. D. Wong-Campos, S. Allen, J. Apisdorf, P. Solomon, M. Williams, A. M. Ducore, A. Blinov, S. M. Kreikemeier, V. Chaplin, M. Keesan, C. Monroe, and J. Kim. “Benchmarking an 11-qubit quantum computer.” *Nature Communications*, **10**(1):5464, November 2019.
- [WHS20] Yuchen Wang, Zixuan Hu, Barry C. Sanders, and Sabre Kais. “Qudits and High-Dimensional Quantum Computing.” *Frontiers in Physics*, **8**, November 2020. Publisher: Frontiers.
- [Wil27] Edwin B. Wilson. “Probable Inference, the Law of Succession, and Statistical Inference.” *Journal of the American Statistical Association*, June 1927. Publisher: Taylor & Francis Group.
- [Wil62] S. A. Williams. “Magnetic Octupole Moments of Axially Symmetric Deformed Nuclei.” *Physical Review*, **125**(1):340–346, January 1962. Publisher: American Physical Society.
- [WLQ21] Pengfei Wang, Chun-Yang Luan, Mu Qiao, Mark Um, Junhua Zhang, Ye Wang, Xiao Yuan, Mile Gu, Jingning Zhang, and Kihwan Kim. “Single ion qubit with estimated coherence time exceeding one hour.” *Nature Communications*, **12**(1):233, January 2021. Publisher: Nature Publishing Group.
- [Won22] Thomas G. Wong. “Unstructured Search by Random and Quantum Walk.” *Quantum Information and Computation*, **22**(1&2):53–85, January 2022. arXiv:2011.14533 [quant-ph].

- [XL24] Xueping Xu and Yi Liu. “The active magnetic compensation coil.” *Review of Scientific Instruments*, **95**(8):081501, August 2024.
- [XLC20] Di Xiao, Jiguang Li, Wesley C. Campbell, Thomas Dellaert, Patrick McMillin, Anthony Ransford, Conrad Roman, and Andrei Derevianko. “Hyperfine structure of $^{173}\text{Yb}^+$: Toward resolving the ^{173}Yb nuclear-octupole-moment puzzle.” *Physical Review A - Atomic, Molecular, and Optical Physics*, **102**(2):1–10, 2020.
- [YGY24] Jialiang Yu, Kai C. Grensemann, Chih-Han Yeh, Ikbāl Ahamed Biswas, Abhishashi Singh, Laura S. Dreissen, Henning A. Fürst, and Tanja E. Mehlstäubler. “Precision Spectroscopy in Yb^+ ions.” In *2024 European Frequency and Time Forum (EFTF)*, pp. 334–336. IEEE, 2024.
- [YMW22] H.-X. Yang, J.-Y. Ma, Y.-K. Wu, Y. Wang, M.-M. Cao, W.-X. Guo, Y.-Y. Huang, L. Feng, Z.-C. Zhou, and L.-M. Duan. “Realizing coherently convertible dual-type qubits with the same ion species.” *Nature Physics*, **18**(9):1058–1061, September 2022.
- [YSD15] Bryce Yoshimura, Marybeth Stork, Danilo Dadic, Wesley C. Campbell, and James K. Freericks. “Creation of two-dimensional Coulomb crystals of ions in oblate Paul traps for quantum simulations.” *EPJ Quantum Technology*, **2**(1):2, December 2015.
- [Zar91] Richard M Zare. *Angular Momentum: Understanding Spatial Aspects in Chemistry and Physics*. Wiley, 1 edition, 1991.